Beam Cooling

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- Introduction to cooling, temperature, phase space and Liouville
- Stochastic cooling
- Electron cooling
- Laser cooling
- Radiation damping
- Ionisation and other cooling

cooling is <u>not</u>: collimation (loss of particles) adiabatic damping

Beam cooling

- Since beamcooling is "slow", it is mostly used in storage rings;
- however, ionisation and "stochastic cooling" has been or will be used for e.g. for muons

Introduction

What is cooling? What is Temperature?

$$\left(\frac{3}{2}k\right)T_{\perp//} = \frac{1}{2}m\langle \vec{v}_{\perp//}^2 \rangle$$

v is the velocity relative to the reference particle moving with the average ion velocity. (internal temperature)

Temperature is a measure of the disordered motion.

Cooling is hence a reduction of the temperature, i.e. of the disordered motion. Clearly, other parts of the total system will get warmer.



My old thermodynamics teacher

• How do you measure the temperature of an ant?

Introduction

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v is the velocity relative to the reference particle moving with the average ion velocity.Temperature is a measure of the disordered motion.

In an accelerator

$$T_{\prime\prime} = Mc^{2}\beta^{2}\langle\Delta p/p\rangle^{2}$$
$$T_{\perp} = Mc^{2}\beta^{2}\gamma^{2}\varepsilon \left(\frac{1}{\langle\beta_{H}\rangle} + \frac{1}{\langle\beta_{V}\rangle}\right)$$



Why beam cooling?

DOCTOR FUN



Deep within the atomic supercollider, the search continues for the elusive elephantino.

11 May 94

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Why beam cooling?

Improve beam quality

- beam size, emittance
- energy spread
- intensity of beam, accumulation, stacking
- lifetime of beam

Counteract degradation of beam quality

due to interaction of beam particles with

- other particles (intrabeam scattering)
- rest-gas (internal targets)
- non-ideal fields, resonances, instabilities injection errors

Stacking by cooling



Stacking by cooling 2

ASTRID SR source: ~200 mA accumulated from many injections of ~5 mA Fermilab antiproton accumulator stacking for 1 hour

 $10^{8}/2$ sec pbar at 8 GeV







Liouville formelt

Et system bestående af *N* partikler beskrives i 6*N* dimensionalt faserum. Hvis partiklerne ikke vekselvirker kan systemet beskrives som *N* partikler i 6 dimensionalt faserum med såkaldte konjugerede koordinater \vec{q} , \vec{p} , $p_i = \partial L/\partial q_i$, hvor $L(q, \dot{q}, t) \equiv T - U$ er Lagrangefunktionen F.eks. *x*, *y*, *z* og de tilhørende impulser p_x, p_y, p_z Hamiltonfunktion $H = H(q, p, t) \equiv \sum p_i \dot{q}_i - L$ Hamiltons ligninger siger $\dot{p} = -\partial H/\partial q + Q$ $\dot{q} = \partial H/\partial p$, hvor $\partial H/\partial q$ er den konservative eller Hamiltonske del, der kan udledes fra et potential, og *Q* er ikke Hamiltonske kræfter som friktion etc.

Kontinuitetetsligning $\frac{d\rho}{dt} = -\rho \sum \frac{\partial Q_i}{\partial p_i} \Rightarrow$ Liouville for konservative kræfter $d\rho/dt = 0 \Rightarrow \rho = konstant$

Phase space and Liouville

Liouville: For hamiltonian systems, the phase space density is constant (when measured along a trajectory) The phase space volume (emittance) is conserved

Often the two transverse and the longitudinal degrees of freedom are decoupled



Phase space, Liouville and cooling

Liouvilles theorem means that cooling is <u>not</u> possible for Hamiltonian systems, that is systems with forces that can be derived from potentials. In addition particles cannot be injected into already filled areas of Phase space.

All you can do is to change the form of phase space.

However, with velocity-dependent forces drag, friction (dissipative) forces

electron, radiation, Laser, ionisation cooling

cooling is indeed possible!!

Coffee, cream, Liouville and Stochastic cooling



Stochastic cooling principle





Maxwells demon

Stochastic cooling

Liouville: Cooling is not possible with electromagnetic forces deflecting the particles (continous fluid, og $N=\infty$). When single particles can be observed, and a corresponding correction applied, cooling is possible! This is the secret of stochastic cooling!



Stochastic cooling



Stochastic cooling exercise

- 1) Ask for 5 random numbers with $\langle x \rangle = 0$ and $\sigma = 1$
- 2) Find actual $\langle x \rangle$ (in general $\langle x \rangle \neq 0$)
- 3) Subtract error in mean to restore mean to zero
- 4) Calculate new σ
- 5) Goto 1)
- 6) Watch σ as function of time
- 7) What is the cooling time?
- 8) Include electronical noise!





Cooling time 2

$$\frac{1}{\tau} = \frac{gW}{N} \left[1 - \frac{g}{2} (\Gamma + \upsilon) \right]$$

optimum gain $g = 1/(\Gamma + \upsilon) < 1$

optimum cooling time

$$\frac{1}{\tau} = \frac{W}{N(\Gamma + \upsilon)}$$

 $au \propto N$

Decrease gain as cooling proceeds

Good mixing, $\Gamma = 1$, by designing storage ring so $\eta = \partial (\Delta T/T) / \partial (\Delta p/p)$ is large. However small mixing PU \rightarrow K Large bandwidth (W> GHz, $N_s \sim 10^{-3}N$)

Weak dependence on energy

Z dependence in v

Stochastic cooling

Betatron cooling: 2 systems (hor. and vert.) dist. PU \rightarrow kicker = odd number of $\lambda/4$

Momentum cooling:

acc. gap instead of transverse kicker

(*i*) PU in high-dispersion region $\Delta x/x=D \Delta p/p$ (*ii*) detect $\Delta f/f=\eta \Delta p/p$ and correct $\Delta p/p$



Stochastic Cooling





Stochastic Cooling

AD at CERN



Fermilab antiproton accumulator stacking for 1 hour

 $10^{8}/2$ sec pbar at 8 GeV





Electron Cooling



Invented by Budker in Novosibirsk

NAP-M ring at INP, 1974, 68 MeV p

> magnets: solenoids toroids

MSL in Stockholm



MSL electron cooler



ASTRID electron cooler



Electron cooling 2

Initially
$$\overline{v_I^2} \ge \overline{v_e^2}$$

 $T_I^i \equiv \frac{1}{2} M \overline{v_I^2} >> \frac{1}{2} m \overline{v_e^2} \equiv T_e^i$

Finally
$$T_I^f = T_e^f$$
 no heating
 $v_I^{rms} \equiv \sqrt{\overline{v_I^2}} = \sqrt{\frac{m}{M}} v_e^{rms} \approx \frac{1}{43} \sqrt{\frac{m}{M}} v_e^{rms}$



Electron cooling time

$$\tau \equiv \left| \frac{1}{v_I} \frac{dv_I}{dt} \right|^{-1} = \left| \frac{Mv_I}{F} \right|$$

$$\tau = \frac{\gamma^{2}}{\eta} \frac{Mm}{Z^{2}e^{4}} \frac{1}{nL} \begin{cases} \frac{1}{4\pi} (v_{I}^{PF})^{3} & (v_{I} > v_{e}^{rms})^{PF} \\ \frac{3}{2\sqrt{2\pi}} \left(\frac{T_{e}}{m}\right)^{3/2} & (v_{I} < v_{e}^{rms})^{PF} \end{cases}$$

Typically τ ~tens of seconds (*Z*=1)

Electron cooling





Electron cooling at AD

Mod. to simple description

1) Flattened distribution due to acceleration

$$T_{\prime\prime\prime}\approx 0 \Longrightarrow \tau_{\prime\prime\prime} \downarrow$$

2)

$$B \neq 0 (B = \infty) \rightarrow T_{\perp} \approx 0 \rightarrow \tau_{\perp} \downarrow (\tau_{\perp} \propto v_{I}^{-3})$$

In practice $B=\infty$ only for distant collisions

Virtues of electron cooling

Versatile cooling technique Longitudinal and transverse cooling Cooling times $\tau \approx 0.1$ -1sec A/Q^2 $T_{//} << 0.1 \text{ eV}$ $T_{\perp} \approx 0.1 \text{ eV}$ in addition: adiabatic expansion $T_{//} \propto B$





Kick from one photon absorption-emission

100 keV Li⁺



Change in momentum : $\Delta p = h / \lambda$

Change in energy : $\Delta E = p\Delta v = 12 \text{meV}$

At saturation (stimulated = spontaneous)(1mW in Ø3mm)

 $r = \frac{1}{2}\Gamma = 10^7 \,\mathrm{s}^{-1}$

in 2 m : $r \cdot 2m/v = 15$

change in 2 m : 0.2 eV

Ultimate limit : single recoil = 12meV

Laser cooling





Laser cooling

Virtues of laser cooling:

Laser cooling is fast

However:

Only effective for longitudinal cooling

Not versatile: Li⁺, Be⁺, Mg⁺, ...

Radiation damping

(for details, see lectures by L. Rivkin)

In principle: any charged particle in practise: only electrons/positrons since $\tau \approx E/(U_0/T_0)$



Ionisation cooling

Slowing down in matter Friction force $\vec{F} \propto -\vec{v}$ Not hadrons due to large inelastic cross section Not electrons due to short radiation length

Can only be used for in μ in μ -collider/ ν -factory

v's produced by decaying μ 's μ 's produced from decaying π 's π 's produced by p's on target

Since μ 's do not live forever ($\tau_0=2.2 \ \mu$ s) cooling has to be fast (even when γ is large: $\tau = \gamma \tau_0$.

Also emittances are very large!

Ionisation cooling principle

Transverse cooling: muons lose energy by dE/dx and longitudinal momentum is restored by RF



To minimize heating from Coulomb scattering: • Small β_{\perp} (high-field solenoids) • Large L_R (low-Z absorber): Liquid H_2

Ionisation energy cooling

Ionisation energy cooling using a wedge and dispersion



possible v-factory at CERN



MICE at RAL



Other cooling methods

Stimulated radiation cooling

Radiative cooling



Conclusions

	Stochastic	Electron	Radiation	Laser	Ionisation
Species	all	ions	e ⁻ /e ⁺	some ions	muons
Favoured beam velocity	high	medium 0.01<β β <0.1	very high γ>100	any (but Doppler)	any
Favoured beam intensity	low	any	any	any	any
Cooling time	<i>N</i> ·10 ⁻⁸ s	10-10 ⁻² s	>10 ⁻³ s	10 ⁻⁴ -10 ⁻⁵ s	10 ⁻⁶ s
Favoured beam temperature	high	low	any	low	any