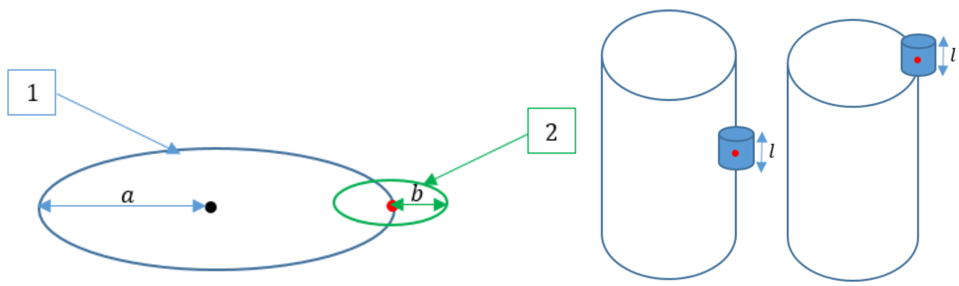
- **Gauss's Law:**

$$\oint_S \vec{E}(\vec{r}') \cdot \overrightarrow{ds}' = \frac{1}{\varepsilon_0} \int_V \rho(\vec{r}') \cdot \overrightarrow{dv}' \quad (1)$$

HW:

Show the ratio of the intersected area between the two circles over the total area of the smaller circle becomes 1/2 in the limit of $b \to 0$.



- **Vector Analysis:**

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \quad (2)$$

$$df = \vec{\nabla} f \cdot d\vec{r} \quad (3)$$

$$d\vec{r} = h_1 dq_1 \hat{q}_1 + h_2 dq_2 \hat{q}_2 + h_3 dq_3 \hat{q}_3 \quad (4)$$

$$\vec{\nabla} f = \frac{\partial f}{h_1 \partial q_1}\hat{q}_1 + \frac{\partial f}{h_2 \partial q_2}\hat{q}_2 + \frac{\partial f}{h_3 \partial q_3}\hat{q}_3 \quad (5)$$

HW:

Find an expression for the three dimensional delta function in a general orthogonal coordinate system.

- **Divergence Operator:**

$$\vec{\nabla} \cdot \vec{f} = \lim_{\Delta V, \Delta S \to 0} \frac{1}{\Delta V} \oint_{\Delta S} \vec{F} \cdot \overrightarrow{ds}' \quad (6)$$

$$\vec{\nabla} \cdot \vec{f} = \frac{1}{h_1 h_2 h_3}\left\{\frac{\partial}{\partial q_1}(F_1 h_2 h_3) + \frac{\partial}{\partial q_2}(F_2 h_1 h_3) + \frac{\partial}{\partial q_3}(F_3 h_1 h_2)\right\} \quad (7)$$

$$\nabla^2 f = \frac{1}{h_1 h_2 h_3}\left\{\frac{\partial}{\partial q_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial F_1}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_1 h_3}{h_2}\frac{\partial F_2}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial F_3}{\partial q_3}\right)\right\} \quad (8)$$

Divergence theorem:

$$\oint_S \vec{f} \cdot \overrightarrow{ds} = \int_V \vec{\nabla} \cdot \vec{f}\, dv \quad (9)$$

HW:

Using the divergence theorem prove the following identity:

$$\oint_S \vec{\mathrm{F}}\left(\vec{\mathrm{G}} \cdot \overrightarrow{ds}\right) = \int_V \left(\vec{\mathrm{F}}\left(\vec{\nabla} \cdot \vec{G}\right) + \left(\vec{G} \cdot \vec{\nabla}\right)\vec{\mathrm{F}}\right) dv \tag{10}$$