- **1-Prove the following equations representing Dirac delta function:**

$$\delta(x) = \lim_{\sigma\to 0} \frac{e^{-\left(\frac{x}{\sqrt{2}\sigma}\right)^2}}{\sqrt{2\pi}\sigma}$$

$$\delta(x) = \lim_{\sigma\to 0} \frac{1}{\pi\sigma\left(1+\left(\frac{x}{\sigma}\right)^2\right)}$$

$$\delta(x) = \lim_{\sigma\to 0} \frac{Sin\left(\frac{x}{\sigma}\right)}{\pi x}$$

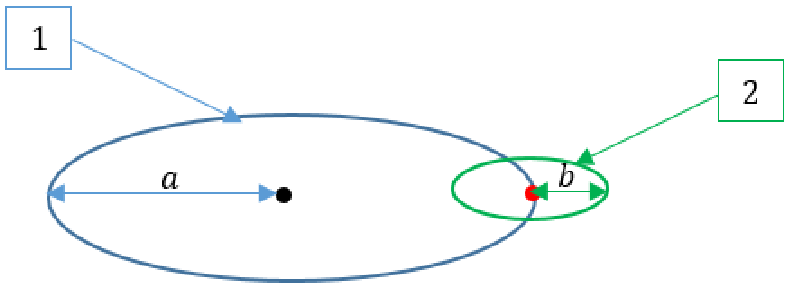
$$\delta(x) = \lim_{\sigma\to 0} \frac{e^{-\left|\frac{x}{\sigma}\right|}}{2\sigma}$$

- **2-Find an expression for the following expression:**

$$\delta\big(f(x)\big)$$

- **3-find an expression for the delta Dirac function in general orthogonal coordinate system.**

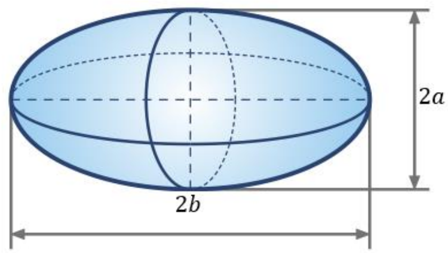
- **4-Show the ratio of the intersected area between the two circles over the total area of the smaller circle becomes 1/2 in the limit of $b \to 0$.**




- **5-Prove the following identity:**

$$\oint_S \vec{F}(\vec{G}\cdot\vec{ds}) = \int_V \left(\vec{F}(\vec{\nabla}\cdot\vec{G}) + (\vec{G}\cdot\vec{\nabla})\vec{F}\right) dv$$

- **6-For a uniform charged spheroidal bunch with total charge $q_b$ calculate the electrostatic potential function within the volume.**




➢ **7-Prove the mean value theorem, for a charge-free space, the value of the electrostatic potential at any point is equal to the average of the potential over the surface of any sphere centered on that point.**

➢ **8-With two different approaches prove the following equation:**

$$\nabla^2 \left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = -4\pi\delta(\vec{r}-\vec{r}')$$

➢ **9-The Green function is in general a symmetric function with respect to interchange between $\vec{r}$ and $\vec{r}'$. Prove this symmetry in Dirichlet's boundary condition and find an additional constraint which has to be imposed to the Neumman boundary condition for obtaining this symmetry.**

$$G(\vec{r}|\vec{r}') = G(\vec{r}'|\vec{r})$$

➢ **10-Using equation $U = \frac{\varepsilon_0}{2}\int\left|\vec{E}(\vec{r})\right|^2 d^3r$ with direct integration, calculate the potential energy of a system composed of two point charge particles $q_1$ and $q_2$ located at $\vec{r}_1$ and $\vec{r}_2$, respectively.**

➢ **11-As a simple application, consider here the two-dimensional problem of a hallow grounded circular cylinder with radius $a$ centered on the z axis with an interior source density $\rho = -\frac{5\varepsilon_0}{a^2}\left(1-\frac{r}{a}\right) + \frac{10^4\varepsilon_0}{a^2}\left(\frac{r}{a}\right)^5\left(1-\left(\frac{r}{a}\right)^5\right)$. Then, First: find the exact potential function $\varphi$ within the volume using numerical calculations with Mathematica, Matlab or etc. Second: find the variational coefficients for a test function $\psi = \alpha\left(\frac{r}{a}\right)^2 + \beta\left(\frac{r}{a}\right)^3 + \gamma\left(\frac{r}{a}\right)^4 - (\alpha+\beta+\gamma)$ which can provide the best fit to the real potential function $\varphi(\vec{r})$ within the volume.**

➢