Wednesday Weekly Seminar

Transverse momentum dependent of charged pion, kaon, and proton/antiproton fragmentation functions from $e^+ e^-$ annihilation process

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School of Particles and Accelerators

July 22, 2020

Outline

- **Motivation**

- **Hadronization**

  - **Single inclusive electron-positron annihilation (SIA)**
  - **Semi inclusive deep inelastic scattering (SIDIS)**
  - **Single inclusive hadron collisions**

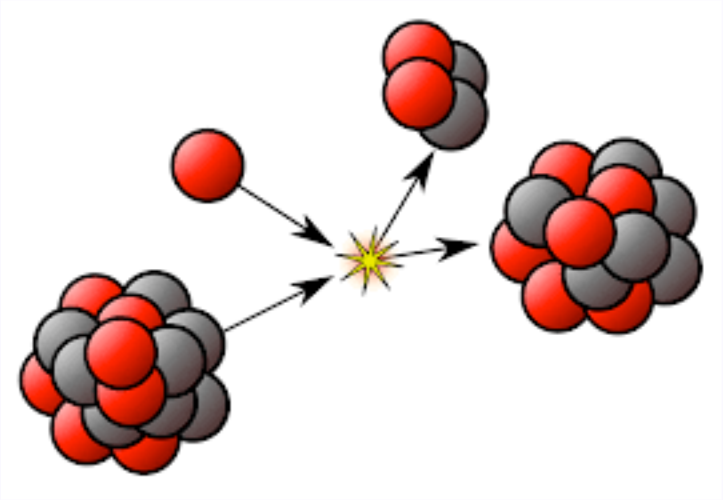
- **Different kinds of distribution functions**

  - **Integrated FFs**
  - **TMD FFs**
  - **TMD PDFs**
  - **Collins FFs**

- **Our Analysis**

  - **Belle data set**
  - **Factorization and Fit Methodology**
  - **Kinematical cuts**
  - **Results**

Hadron physics, or the quest for the nucleon structure



Hadronization process turns partons produced in hard-scattering reactions into the physical, colorless, non- perturbative hadronic bound states detected in experiments.

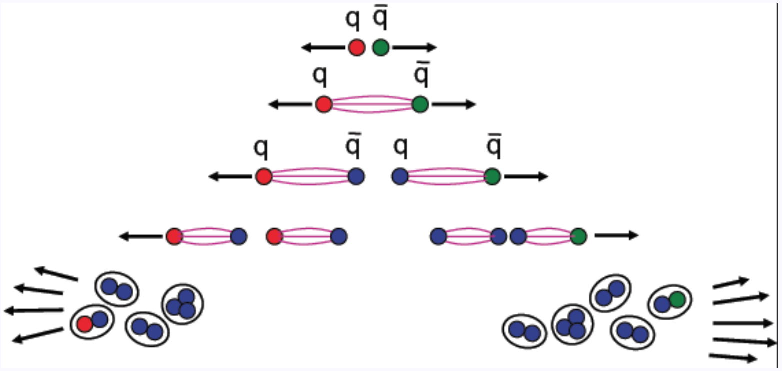
Processes with an observed hadron in the final state can be described in terms of perturbative hard-scattering cross sections and certain non- perturbative but universal functions:

1-parton distributions, accounting for the partonic structure of the hadrons in the initial state just before the interaction.

2-Fragmentation functions, encoding the details of the subsequent hadronization process .

Purposes of investigating fragmentation functions

1. Understanding the generation of hadrons from quarks and gluons (partons).

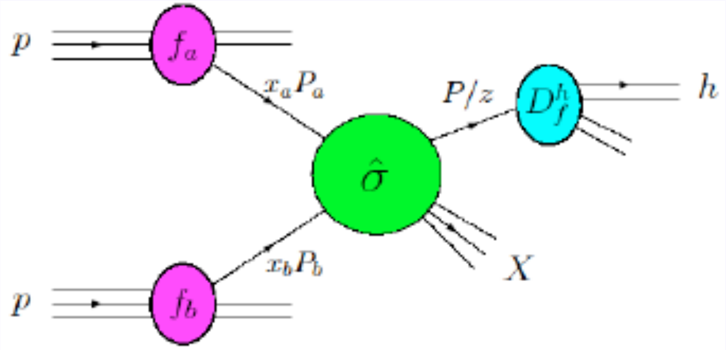


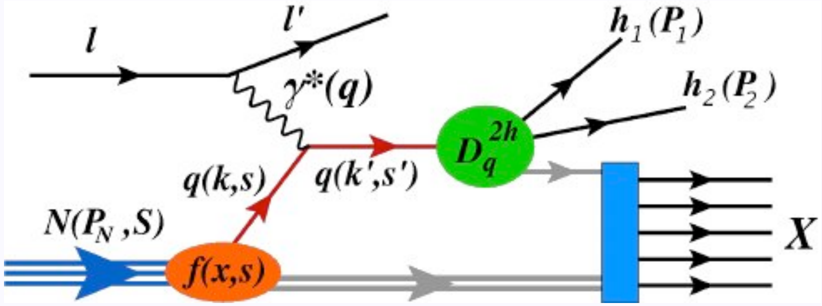
2. An essential tool in the description of a number of processes used to examine the internal structure of nucleons. For example, processes probing nucleon momentum, spin, flavor and spatial distributions.
[arXiv:0804.2021]

3. An essential tool in the description of the dynamics of cold (nuclear DIS) and hot nuclear matter (high-energy nucleus-nucleus collisions) .
[Phys. Rev. D **81**, 054001 (2010)]

Purposes of investigating fragmentation functions

4. **Essential ingredients of theoretical predictions for the present or future hadron colliders such as LHC and LHeC.** [arXiv:2004.04213]

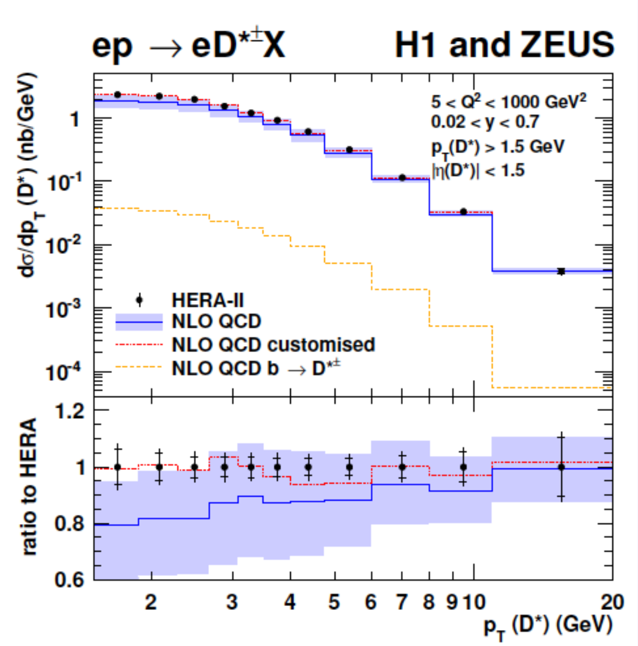




5. **The FFs are a basic ingredient for the calculation of the production of high transverse-momentum particles at collider energies within perturbative QCD.** [J. High Energy Phys.06 (2019) 51]

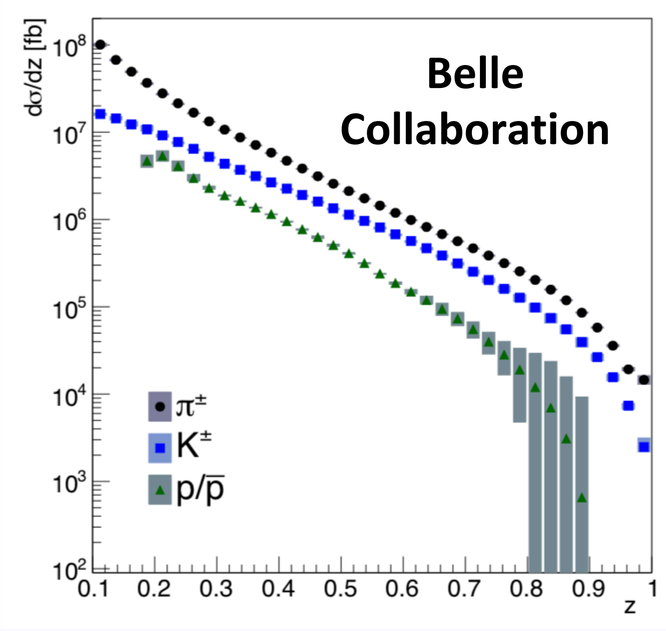
Purposes of investigating fragmentation functions

Precise determination of fragmentation functions (FFs) including their experimental uncertainties had become an active topic for many hard Processes.



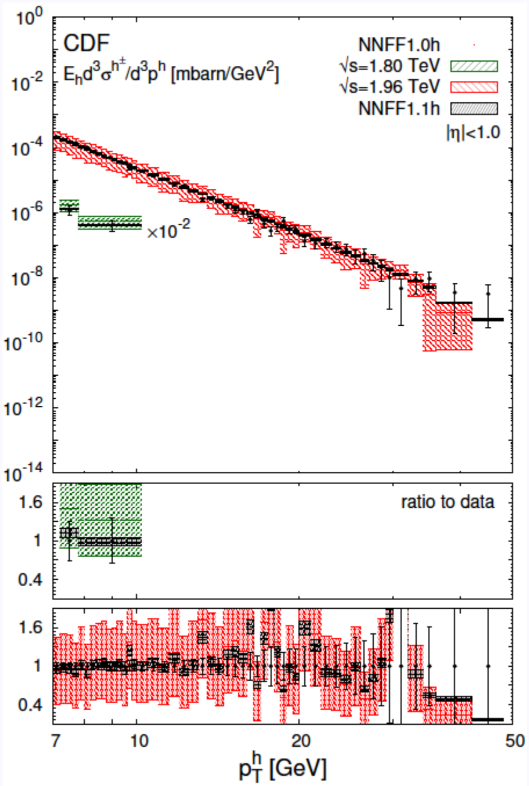
[JHEP 1509 (2015) 149]

$e^+e^- \to (\gamma, Z) \to H + X$



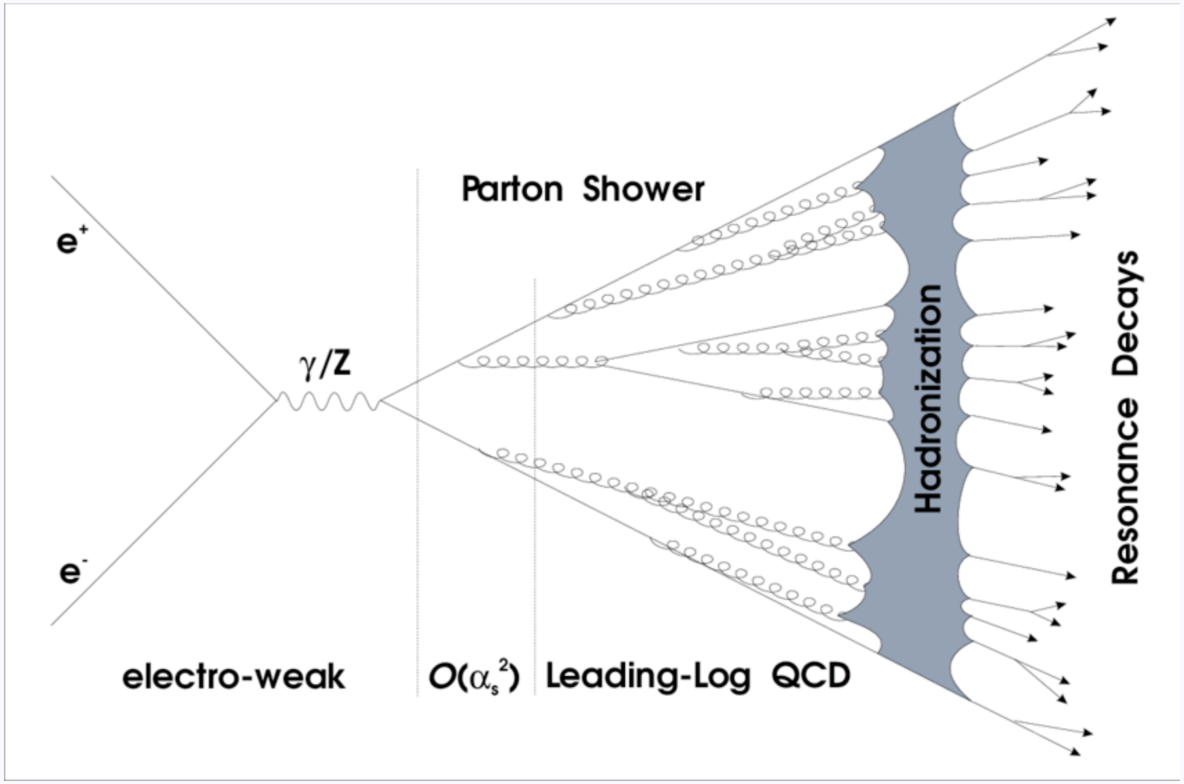
[arXiv:2001.10194v2]

$N_1 + N_2 \to h + X$



[Phys. Rev. D 82 (2010) 119903]

Hadronization

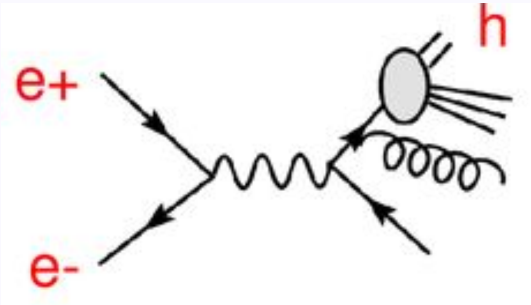


Perturbative phase
$a_s<1$ (Parton Level)

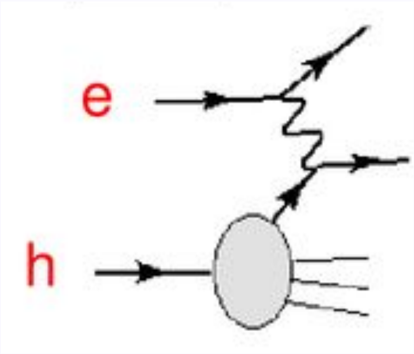
Non-perturbative phase
$a_s \geq 1$

$$\frac{d\sigma}{dz}(e^-e^+ \to hX) = \sum_q \sigma(e^-e^+ \to q\bar{q})\left[D_q^h(z) + D_{\bar{q}}^h(z)\right].$$

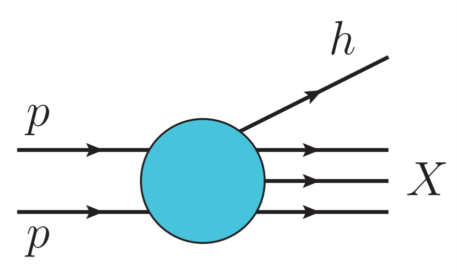
Parton-model and factorization formula



$$\sigma^{e^+e^-\to hX} = \hat{\sigma}\otimes FF\,,$$

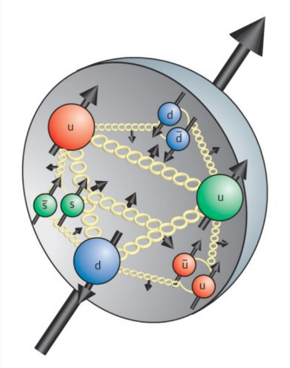


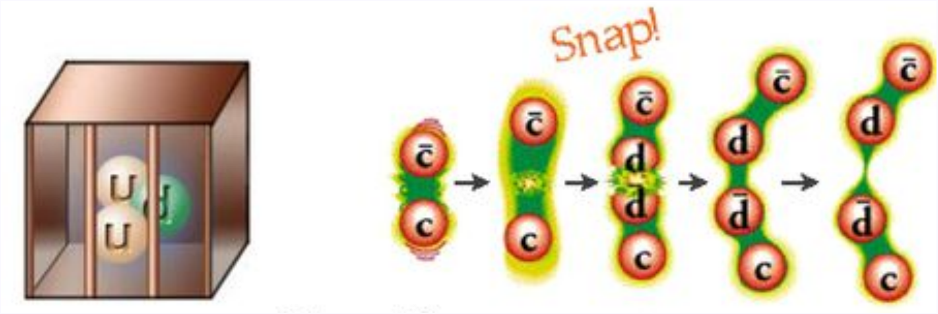
$$\sigma^{\ell N\to \ell hX} = \hat{\sigma}\otimes PDF\otimes FF\,,$$



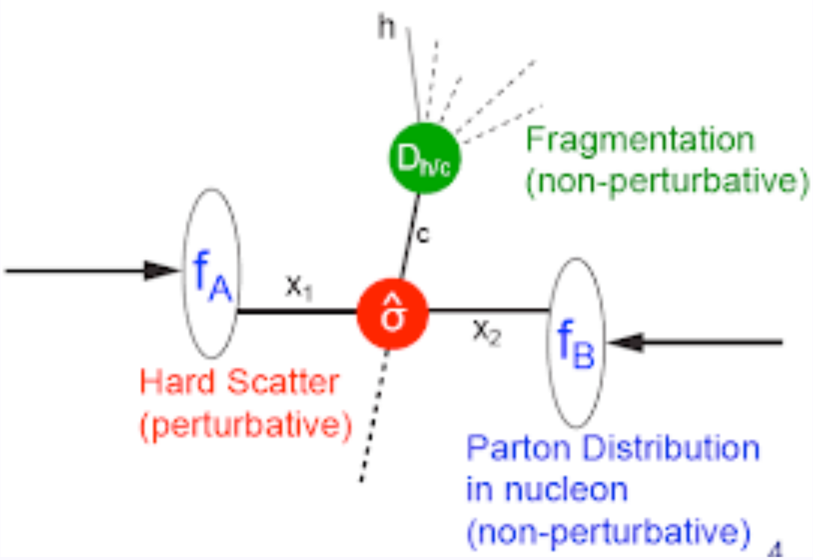
$$\sigma^{pp\to hX} = \hat{\sigma}\otimes PDF\otimes PDF\otimes FF\,,$$

PDFs and FFs

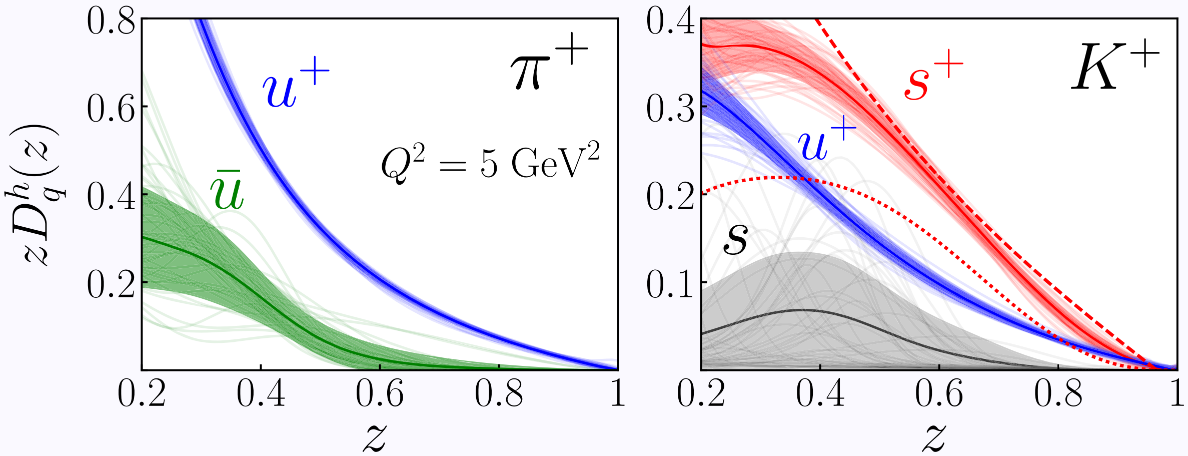




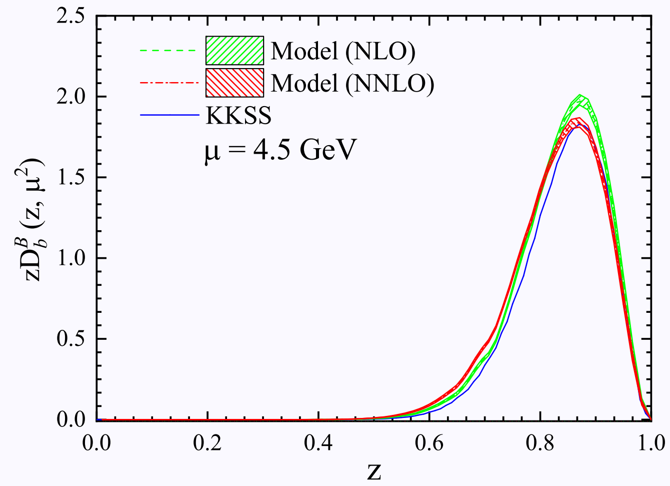
While PDFs were understood as probability densities for finding partons, with a given momentum, inside color-neutral particles, FFs were understood as probability densities for finding color-neutral particles inside partons



Heavy Flavored Hadrons



[PRL 119, 132001 (2017)]



[PRD 99, 114001 (2019)]

In the limit of a very heavy quark, one expects the fragmentation function for a heavy quark to go into any heavy hadron to be peaked near z = 1 → Different parameterization Forms

Energy sum rule →

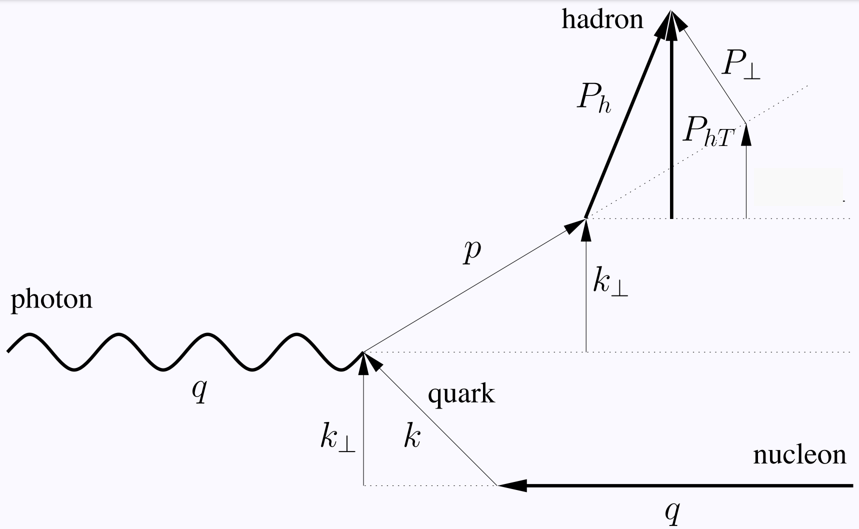
$$\sum_H \int_0^1 dz z D_i^H(z, Q^2) = 1,$$

The energy fraction for the hadron h which is created from the parton i.

Each parton will fragment with 100% probability into some hadron H.

The sum rule should be dominated by the fragmentation into the lightest hadrons such as pions and kaons.

TMD PDFs and TMD FFs

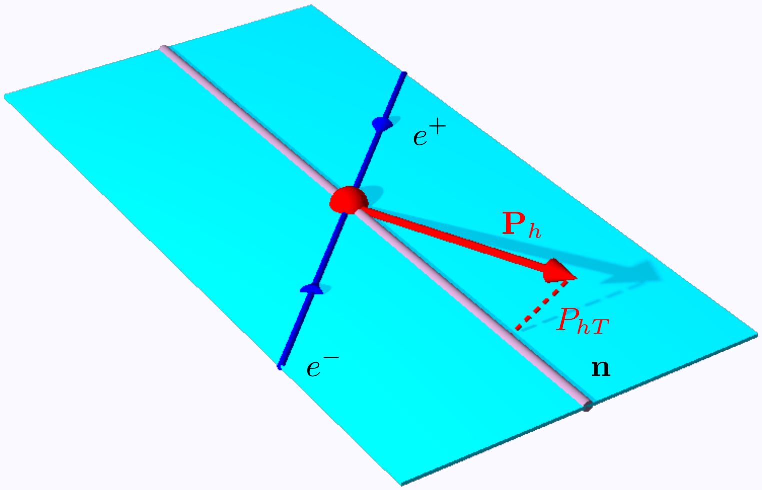


$f_1^a(x, \boldsymbol{k}_\perp^2; Q^2)$ **is the TMD PDF of unpolarized partons with flavor a in an unpolarized proton, carrying longitudinal momentum fraction x and transverse momentum $k_\perp$**

$D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; Q^2)$ **is the TMD FF describing the fragmentation of an unpolarized parton with flavor a into an unpolarized hadron h carrying longitudinal momentum fraction z and transverse momentum $P_\perp$**

Transverse-momentum dependent (TMD) FFs:

$$D_1^{h/q}(z, \vec{P}_{hT}^{\,2})$$



In addition to the z dependence, the dependence on P_T is considered, where P_T is the transverse momentum of the detected hadron.

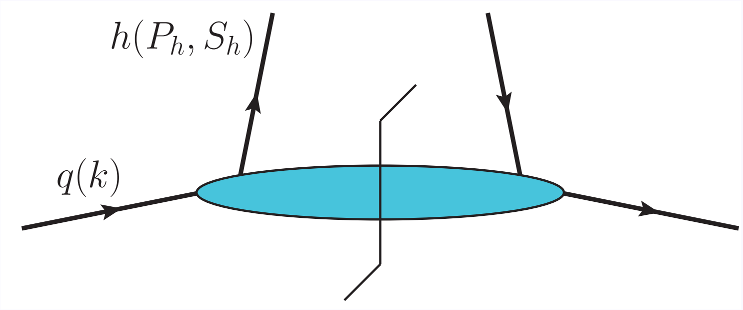
Significant experimental and theoretical efforts have been focused on the intrinsic transverse momentum dependence of the FFs.

At present people are interested in this function mainly because in SIDIS it couples to the transversity PDF.

Important role in the future electron-ion collider to pin down the transverse momentum structure of the nucleon and its transverse spin.

Integrated Fragmentation Functions

$$D_1^{h/q}(z) \;=\; z^2 \int d^2\vec{k}_T\, D_1^{h/q}(z, z^2\vec{k}_T^{\,2}) = \int d^2\vec{P}_{hT}\, D_1^{h/q}(z, \vec{P}_{hT}^{\,2})\,,$$



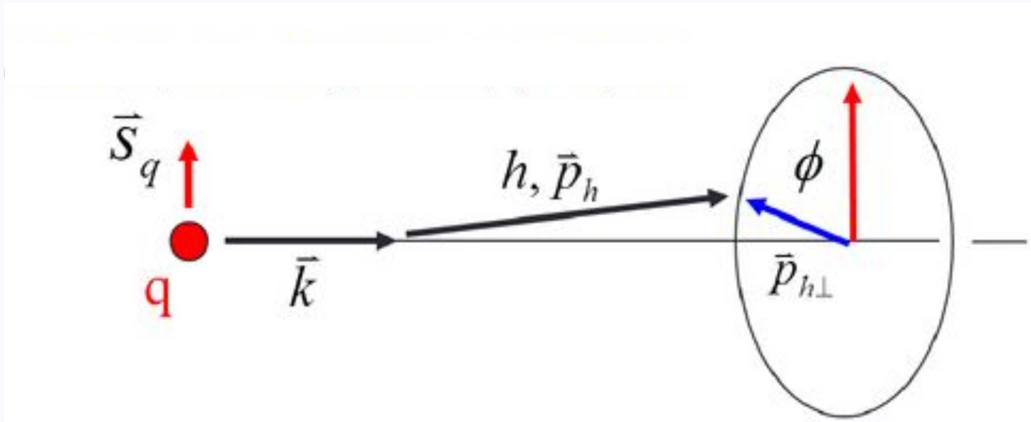
$$P_h \;=\; (P_h^+, P_h^-, \vec{0}_T) = \left(\frac{M_h^2}{2P_h^-}, P_h^-, \vec{0}_T\right),$$

$$k \;=\; (k^+, k^-, \vec{k}_T) = \left(z\,\frac{k^2 + \vec{k}_T^{\,2}}{2P_h^-}, \frac{P_h^-}{z}, \vec{k}_T\right),$$

Integrated FFs:

Integrated FF is the number density for finding an unpolarized hadron with momentum P_h = zk inside an unpolarized quark with longitudinal momentum k.

Collins Fragmentation Functions



$\vec{k}$: quark momentum

$\vec{s}_q$: quark spin

$\vec{p}_h$: hadron momentum

$\vec{p}_{h\perp}$: transverse hadron momentum

$z_h = E_h / E_q$

$= 2 E_h / \sqrt{s}$: relative hadron momentum

The Collins FF H_1^q describes a correlation between the transverse polarization of the fragmenting quark and the transverse momentum of the produced unpolarized hadron

The possibilities for finding a hadron produced from transversely polarized quark:

$$D_{hq^{\uparrow}}(z, P_{h\perp}) = D_1^q(z, P_{h\perp}^2) + H_1^{\perp q}(z, P_{h\perp}^2) \frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{z M_h},$$

- **D_1: the unpolarized Fragmention Function**
- ***H*: Collins Fragmentation Function**

Different kinds of TMD fragmentation functions

H \ q	U	L	T
U	$D_1^{h/q}$		$H_1^{\perp\, h/q}$
L		$G_1^{h/q}$	$H_{1L}^{\perp\, h/q}$
T	$D_{1T}^{\perp\, h/q}$	$G_{1T}^{h/q}$	$H_1^{h/q}$ $H_{1T}^{\perp\, h/q}$

If the transverse-momentum (k_T) dependence of fragmentation functions is considered, there are eight types of functions, defined by the correlations among the hadronic and partonic spin vectors and transverse-momentum vectors they represent.

$D^{i/h}$: unpolarized TMD fragmentation functions

Observables for TMD FFs

TMD FF $D_1(z, k_T)$		
$e^+e^- \to h_a h_b X$	$\sum_q e_q^2 D_1^{h_a/q}(z_a, k_{aT}) \otimes D_1^{h_b/\bar{q}}(z_b, k_{bT})$ $+\{q \leftrightarrow \bar{q}\}$	back-to-back production of hadron pair
$pp \to h_a h_b X$	$\sum_{i,j,k,l} f_1^{i/p_a}(x_a, p_{aT}) \otimes f_1^{j/p_b}(x_b, p_{bT})$ $\otimes D_1^{h_a/k}(z_a, k_{aT}) \otimes D_1^{h_b/l}(z_b, k_{bT})$	back-to-back production of hadron pair
$pp \to \gamma h X$	$\sum_{i,j,k} f_1^{i/p_a}(x_a, p_{aT}) \otimes f_1^{j/p_b}(x_b, p_{bT})$ $\otimes D_1^{h/k}(z, k_T)$	back-to-back production of hadron with direct γ
$e^+e^- \to$ $(h, \text{jet/thrust axis})X$	$\sum_q e_q^2 D_1^{h/q}(z, k_T)$	can access z, k_T
$\ell p \to \ell h X$	$\sum_q e_q^2 f_1^{q/p}(x, p_T) \otimes D_1^{h/q}(z, k_T)$	
$pp \to (h, \text{jet})X$	$\sum_{i,j,k} f_1^{i/p_a}(x_a)\, f_1^{j/p_b}(x_b)\, D_1^{h/k}(z, k_T)$	hadron in jet: can access z, k_T

At present just TASSO and Belle data

HERMES and COMPASS

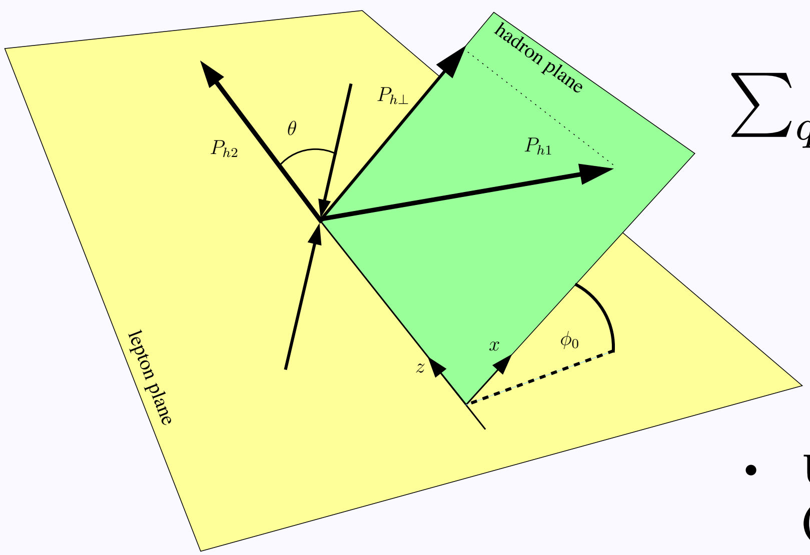
K_T Dependence of FFs in e^+e^-

The cleanest process can be achieved in e^+e^- annihilation because there is no contribution from transverse dependence of PDFs.

Two ways to obtain transverse momentum dependence:

Single or double inclusive electron positron annihilations

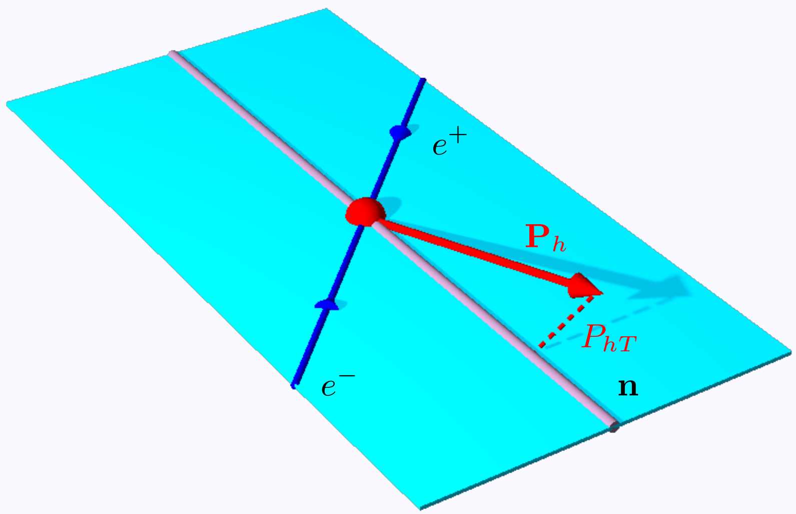
1- Back-to-back production of hadron pair $e^+e^- \to h_a h_b X$



$$\sum_q e_q^2 D_1^{h_a/q}(z_a, k_{aT}) \otimes D_1^{h_b/\bar{q}}(z_b, k_{bT}) + \{q \leftrightarrow \bar{q}\}$$

- **Useing transverse momentum between two hadrons (in opposite hemispheres)**
- **Usual convolution of two transverse momenta**

K_T Dependence of unpolarized FFs in e^+e^-



Single-hadron FF wrt to Thrust

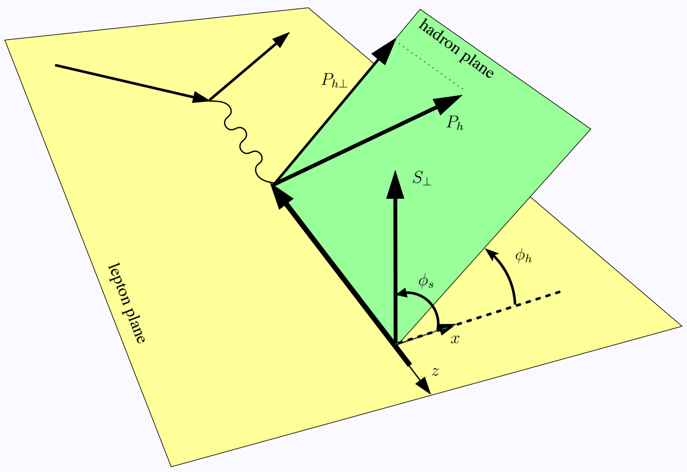
$$\sum_q e_q^2 D_1^{h/q}(z, k_T)$$

- **No convolution**
- **Poorly measured**

TASSO Collaboration [Z. Phys. C 47 (1990) 187–198]**: old datasets for single unidentified light charged hadron productions in electron-positron annihilation**

Belle Collaboration [PRD 99, 112006 (2019)]**: first measurements of the production unpolarized cross sections of pions, kaons, as well as protons in SIA process at 10.58 GeV**

K_T Dependence of SIDIS Data



$$\ell p \to \ell h X$$

$$\sum_q e_q^2 f_1^{q/p}(x, p_T) \otimes D_1^{h/q}(z, k_T)$$

Transverse momenta convoluted between FF and PDF

HERMES Collaboration at HERA [Phys. Rev. D87, 074029 (2013), 1212.5407]
COMPASS Collaboration at CERN [Eur. Phys. J.C75,no.2,94(2015)]

At present only poorly measured at best because:

- **Convolution of several transverse momenta is involved**
- **The statistical precision of several measurements is not sufficient, yet**

Workshop on Novel Probes of the Nucleon Structure in SIDIS, e+e- and pp (FF2019)

chaired by Anselm Vossen (Duke University), Harut Avagyan (Jefferson Lab)

from Thursday, 14 March 2019 at **08:00** to Saturday, 16 March 2019 at **18:00** (US/Eastern)
at **Bostock Library (127)**

Go to day

Thursday, 14 March 2019

08:30 - 09:00 Registration

09:00 - 09:10 Welcome *10'* (127 Bostock Library)
Speaker: Dr. Anselm Vossen (Duke University)
Material: **Slides**

09:10 - 09:50 Recent Belle Results *40'*
Speaker: Dr. Ralf Seidl (RIKEN)
Material: **Slides**

09:50 - 10:30 Recent SIDIS results *40'*
Speaker: Dr. Gunar Schnell (University of the Basque Country UPV/EHU)
Material: **Slides**

10:30 - 11:00 Coffee!

11:00 - 11:40 News from NNPDF *40'*
Speaker: Dr. Emanuele Nocera (NIKHEF)
Material: **Slides**

11:40 - 12:20 Challenges and recent progress in SIDIS *40'*
Speaker: Nobuo Sato (ODU)
Material: **Slides**

12:20 - 12:50 Simultaneous analysis of unpolarized PDFs and fragmentation functions from JAM *30'*

RIKEN (Japan) in 2012,
Indiana University (USA) in 2013
Stresa (Italy) in 2018

PHYSICAL REVIEW D **99**, 112006 (2019)

Transverse momentum dependent production cross sections of charged pions, kaons and protons produced in inclusive e^+e^- annihilation at $\sqrt{s} = 10.58$ GeV

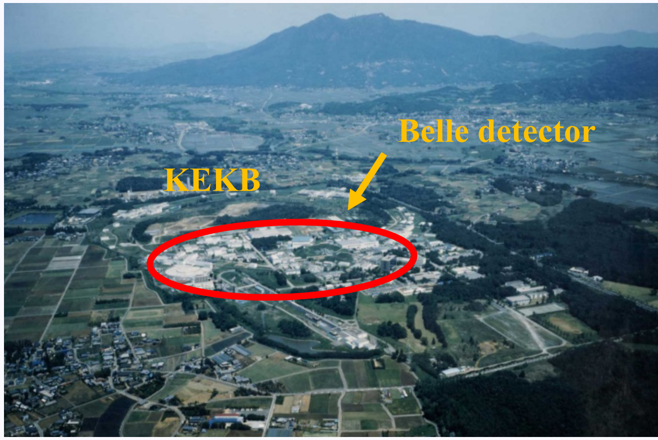
(Belle Collaboration)

(Received 5 February 2019; published 14 June 2019)

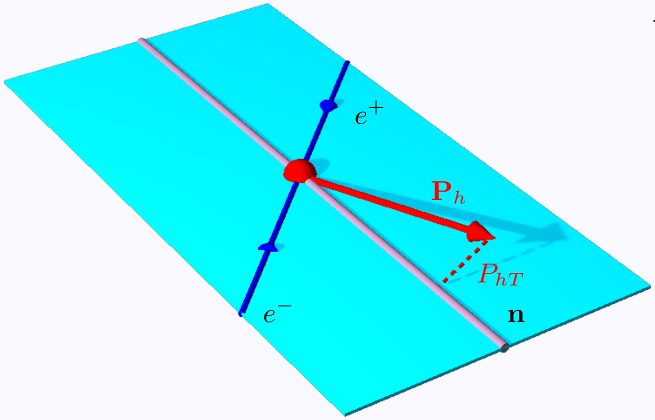
We report measurements of the production cross sections of charged pions, kaons, and protons as a function of fractional energy, the event-shape variable called thrust, and the transverse momentum with respect to the thrust axis. These measurements access the transverse momenta created in the fragmentation process, which are of critical importance to the understanding of any transverse-momentum-dependent distribution and fragmentation functions. The low transverse-momentum part of the cross sections can be well described by Gaussians in transverse momentum as is generally assumed but the fractional-energy dependence is nontrivial and different hadron types have varying Gaussian widths. The width of these Gaussians decreases with thrust and shows an initially rising, then decreasing fractional-energy dependence. The widths for pions and kaons are comparable within uncertainties, while those for protons are significantly narrower. These single-hadron cross sections and Gaussian widths are obtained from a 558 fb^{-1} data sample collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider.

TMD Production Cross section by Belle Collaboration

This single-hadron cross-section measurement is based on a data sample of 558 fb^{-1} collected with the Belle detector at the KEKB asymmetric-energy e^+e^- (3.5 GeV on 8 GeV) collider.



$$d^3\sigma(e^+e^- \to hX)/dz dP_{hT} dT$$



Experimentally, the transverse momentum of the hadron is calculated relative to the thrust axis nˆ which maximizes the event-shape variable thrust T

$$T \stackrel{\max}{=} \frac{\sum_h |\mathbf{P}_h^{\mathrm{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\mathrm{CMS}}|}$$

$$\sqrt{s} = 10.58 \text{ GeV}$$

As the thrust variable describes how collimated all particles in an event are, the results are presented in bins of this value.

Binning

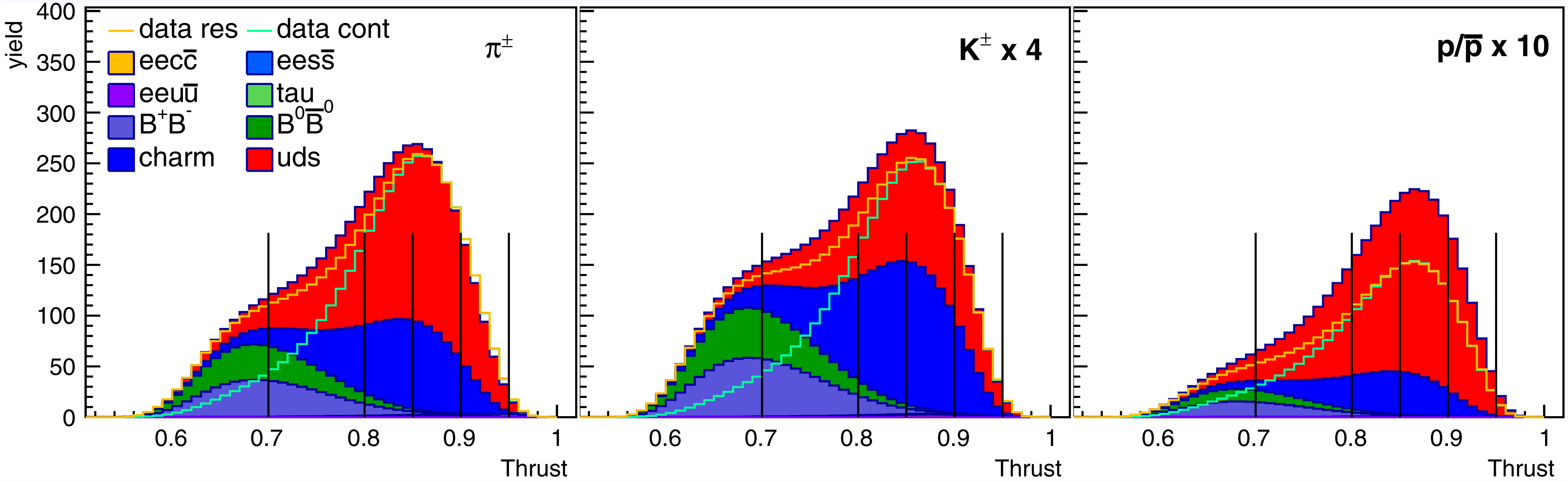
$$d^3\sigma(e^+e^- \to hX)/dzdP_{hT}dT$$

For the hadron cross section, a (z, P_{hT}) binning of 18 equidistant z bins from 0.1 to 1.0 and 20 equidistant P_{hT} bins from 0 to 2.5 GeV/c is chosen. The thrust values are separated into six bins with boundaries at 0.5, 0.7, 0.8, 0.85, 0.9, 0.95, and 1.0

#Final	weak de	cays	subtract	ed cross	secti	ons
d2sigma	/ dz dkt	in	microb/GeV	for the		pi+- data.
#thrus	t range	#zbin	range	#kbin	range	xsec[fb/G
0.5	0.7	0.1	0.15	0	0.125	1.46E+06
0.5	0.7	0.1	0.15	0.125	0.25	4.50E+06
0.5	0.7	0.1	0.15	0.25	0.375	7.43E+06
0.5	0.7	0.1	0.15	0.375	0.5	1.26E+07
0.5	0.7	0.1	0.15	0.5	0.625	1.17E+07
0.5	0.7	0.1	0.15	0.625	0.75	3.09E+06
0.5	0.7	0.1	0.15	0.75	0.875	6.92E+04
0.5	0.7	0.15	0.2	0	0.125	3.40E+05
0.5	0.7	0.15	0.2	0.125	0.25	9.76E+05
0.5	0.7	0.15	0.2	0.25	0.375	1.53E+06
0.5	0.7	0.15	0.2	0.375	0.5	1.94E+06
0.5	0.7	0.15	0.2	0.5	0.625	2.22E+06
0.5	0.7	0.15	0.2	0.625	0.75	2.54E+06
0.5	0.7	0.15	0.2	0.75	0.875	2.09E+06
0.5	0.7	0.15	0.2	0.875	1	5.22E+05
0.5	0.7	0.15	0.2	1	1.125	2.21E+04
0.5	0.7	0.2	0.25	0	0.125	1.08E+05
0.5	0.7	0.2	0.25	0.125	0.25	3.16E+05
0.5	0.7	0.2	0.25	0.25	0.375	4.63E+05
0.5	0.7	0.2	0.25	0.375	0.5	5.47E+05
0.5	0.7	0.2	0.25	0.5	0.625	5.72E+05
0.5	0.7	0.2	0.25	0.625	0.75	5.27E+05
0.5	0.7	0.2	0.25	0.75	0.875	4.20E+05
0.5	0.7	0.2	0.25	0.875	1	3.78E+05
0.5	0.7	0.2	0.25	1	1.125	2.93E+05

Binning and cross-section extraction

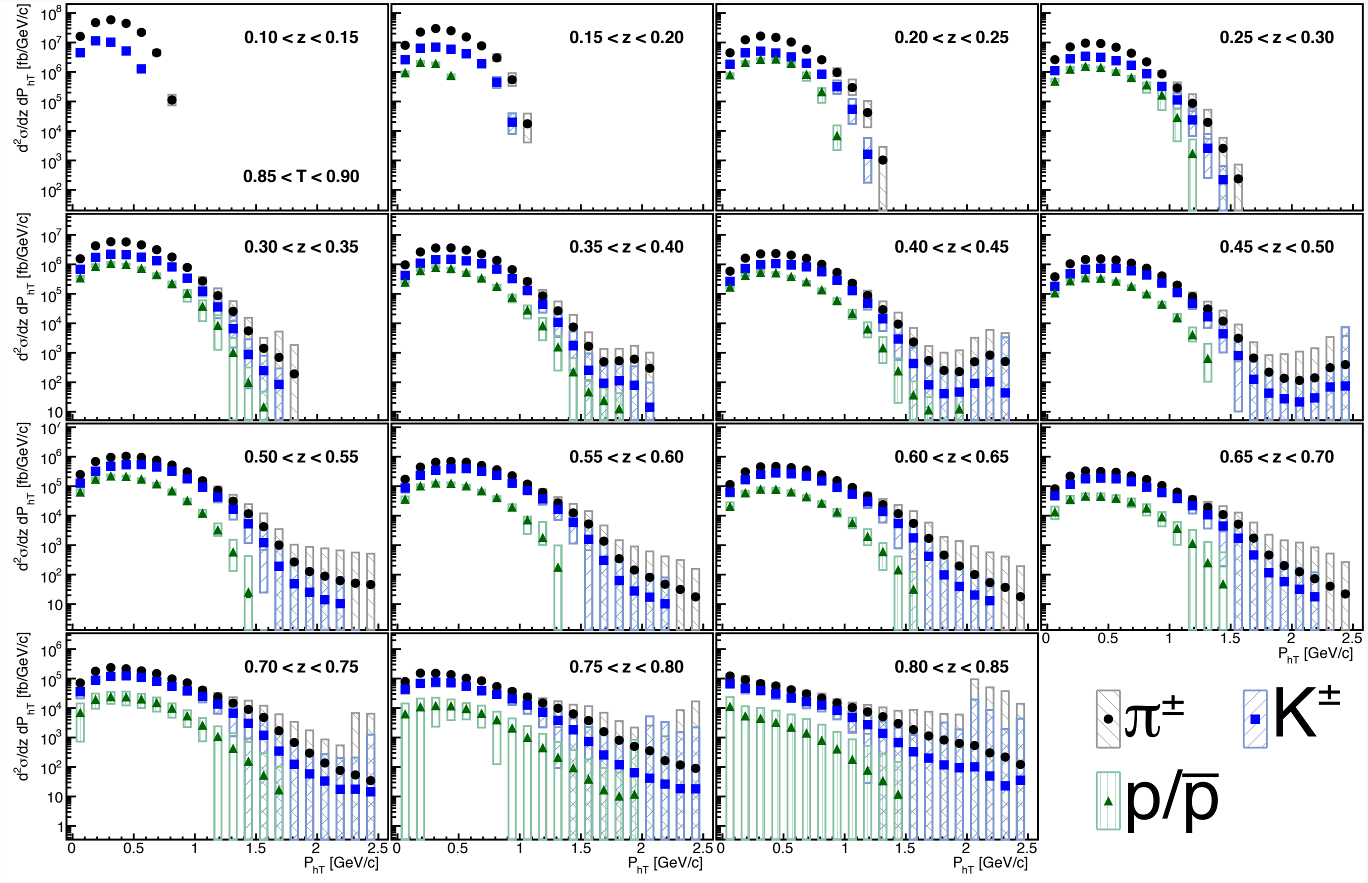
The goal of this analysis is to extract hadron cross sections from uds and charm pair events.



The distributions of thrust for the selected hadron samples are displayed in Fig, where the different processes are depicted. It can be seen that uds and charm events peak at high thrust values, which is why in the following most corrections and results are displayed in the 0.85 < T < 0.9 thrust bin.

TMD Production Cross section by Belle Collaboration

First direct transverse-momentum-dependent single-hadron production cross sections $e^+ e^-$ collisions at Q =10.58 GeV for pions, kaons, and protons.



TMD FFs

PHYSICAL REVIEW D **100,** 094033 (2019)

Transverse momentum dependent of charged pion, kaon, and proton/antiproton fragmentation functions from e^+e^- annihilation process

Maryam Soleymaninia[1,*] and Hamzeh Khanpour[2,1,†]

[1]*School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM), P.O.Box 19395-5531 Tehran, Iran*

[2]*Department of Physics, University of Science and Technology of Mazandaran, P.O.Box 48518-78195 Behshahr, Iran*

(Received 29 October 2019; published 26 November 2019)

Factorization of TMD FFs

Factorization of TMD for e+ e- → hX process, one observed hadron at the final state

$$\frac{d\sigma^h}{dzdP_{hT}} = 2\pi P_{hT}\frac{4\pi\alpha^2}{3s}\sum_q {e_q}^2 \mathcal{N} D_{h/f}(z, P_{hT}, Q^2) = 2\pi P_{hT}\sigma_{\text{tot}}\sum_q \mathcal{N} D_{h/f}(z, Q^2) h^h(P_{hT}),$$

$$\sigma_{\text{tot}} = \sum_q {e_q}^2 \frac{4\pi\alpha^2}{3s}.$$

TMD fragmentation function two terms:
first term is the unpolarized collinear FF $D_{h/f}(z,Q^2)$
second term corresponds to the TMD dependent $h^h(P_{hT})$

$$D_{h/f}(z, P_{hT}, Q^2) = D_{h/f}(z, Q^2)h^h(P_{hT}).$$

not dependent on the scale of energy and also the flavor

FFs by NNFF1.0 Collaboration [Eur. Phys. J. C 77, 516 (2017)]

NNFF1.0

$D_i^H(z, Q_0)$ $i = u^+, d^+, s^+, c^+, b^+, g$

$Q_0 = 5$ GeV $q^+ = q + \bar{q}$

		DHESS	HKNS	JAM	NNFF
DATA	SIA	☑	☑	☑	☑
	SIDIS	☑	☒	☒	☒
	PP	☑	☒	☒	☒
METH.	statistical treatment	Iterative Hessian 68% - 90%	Hessian $\Delta\chi^2 = 15.94$	Monte Carlo	Monte Carlo
	parametrisation	standard	standard	standard	neural network
THEORY	pert. order	(N)NLO	NLO	NLO	LO, NLO, NNLO
	HF scheme	ZM(GM)-VFN	ZM-VFN	ZM-VFN	ZM-VFN
	hadron species	$\pi^\pm$, $K^\pm$, $p/\bar{p}$, $h^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$	$\pi^\pm$, $K^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$
	latest update	PRD 91 (2015) 014035 PRD 95 (2017) 094019	PTEP 2016 (2016) 113B04	PRD 94 (2016) 114004	EPJ C77 (2017) 516

Fitting Methodology

Commonly parametrization for TMD FFs in SIDIS, Drell-Yan and SIA processes, Gaussian form at low $p_\perp$

$$D_{h/f}(z, P_{hT}, Q^2) = D_{h/f}(z, Q^2) \frac{e^{-P_{hT}{}^2/\langle P_{hT}{}^2\rangle}}{\pi\langle P_{hT}{}^2\rangle}$$

[Physics Letters B 772 (2017) 78–86]

[JHEP04(2014)005]

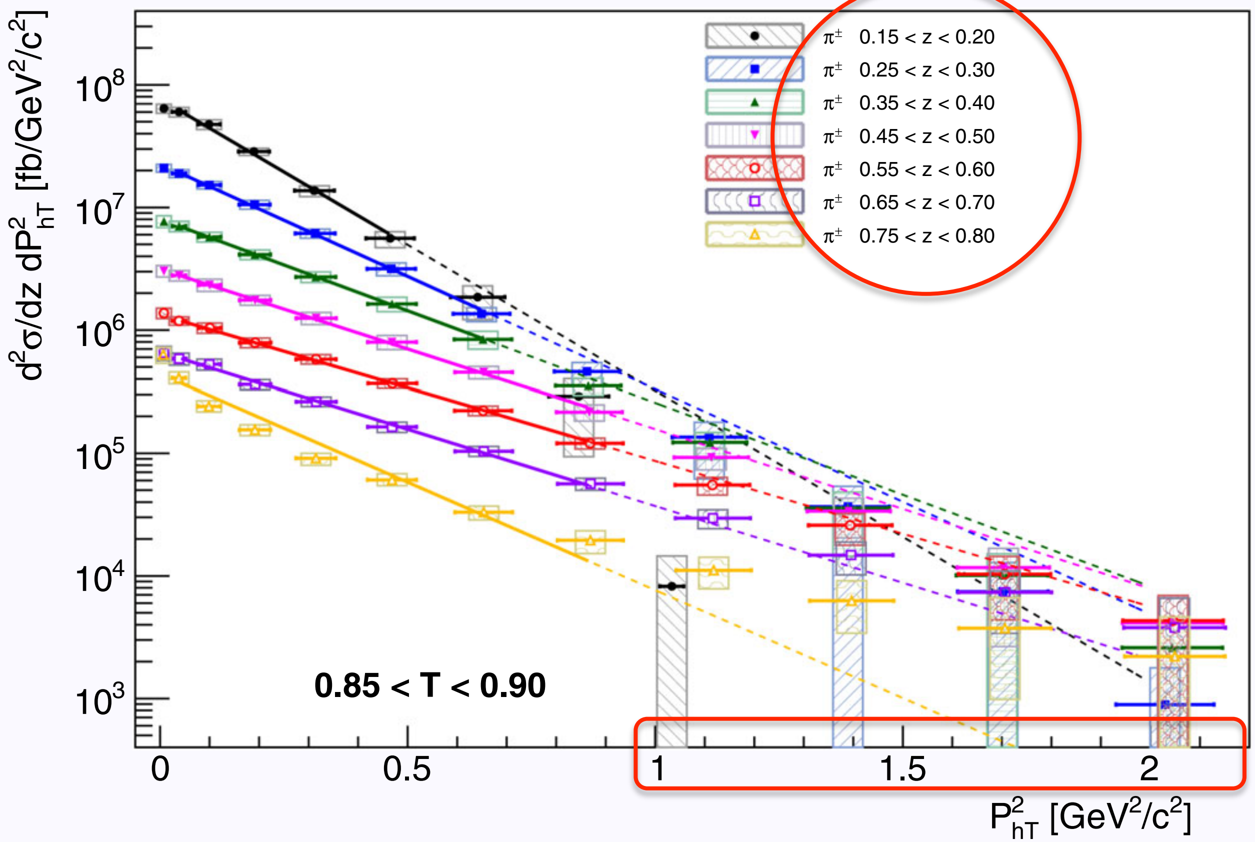
$$\langle P_{hT}^2\rangle = \alpha + z^\beta (1-z)^\gamma \qquad Q_0 = 5 \text{ GeV.}$$

$$\chi_n^2(p) = \sum_{i=1}^{N_n^{\text{data}}} \left(\frac{\mathcal{E}_i - \mathcal{T}_i(p)}{\Delta(\mathcal{E}_i)} \right)^2$$

Minimization → **CERN program library MINUIT**

[Comput. Phys. Commun. 10, 343 (1975)]

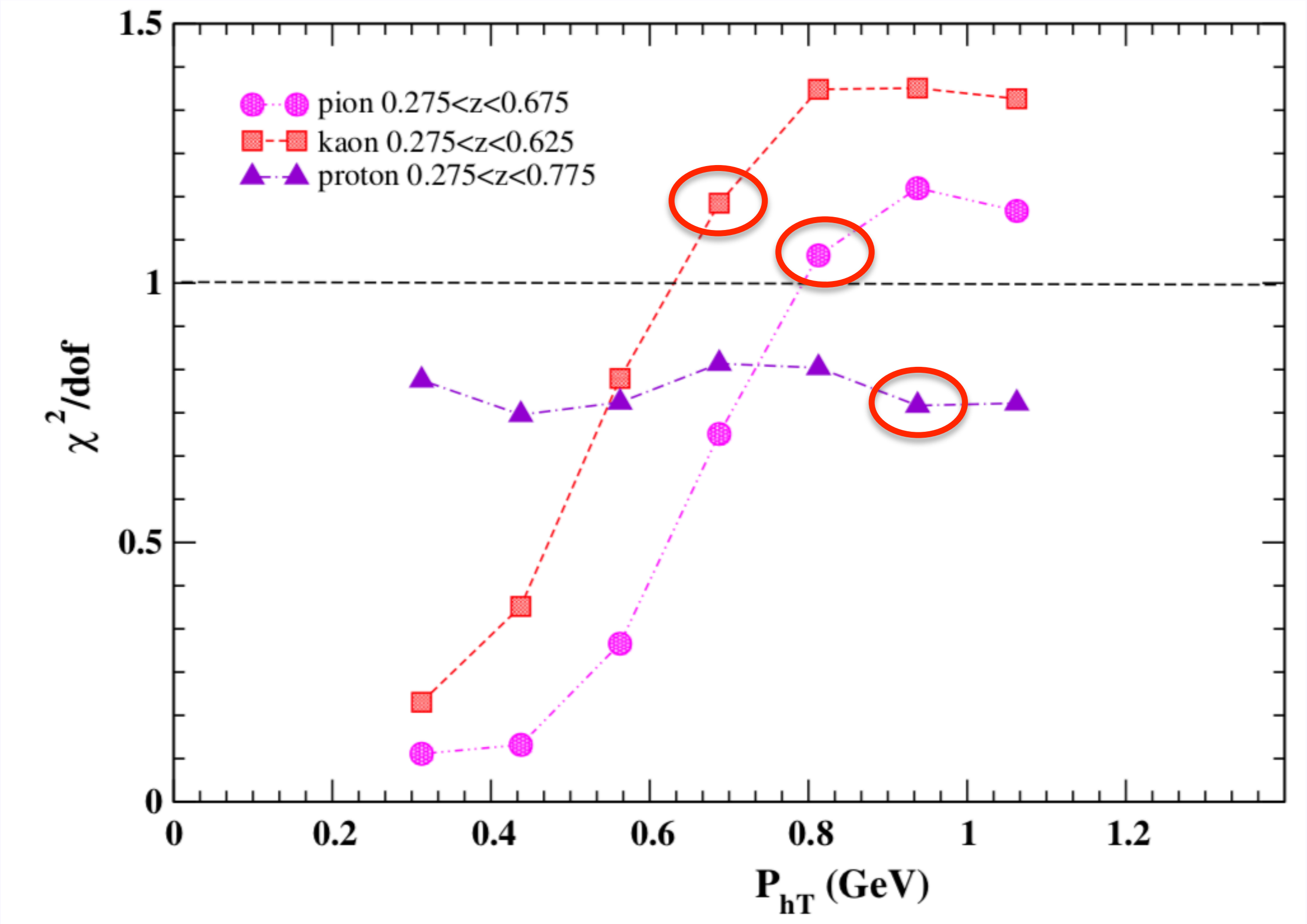
Kinematical cuts



χ^2 scans,

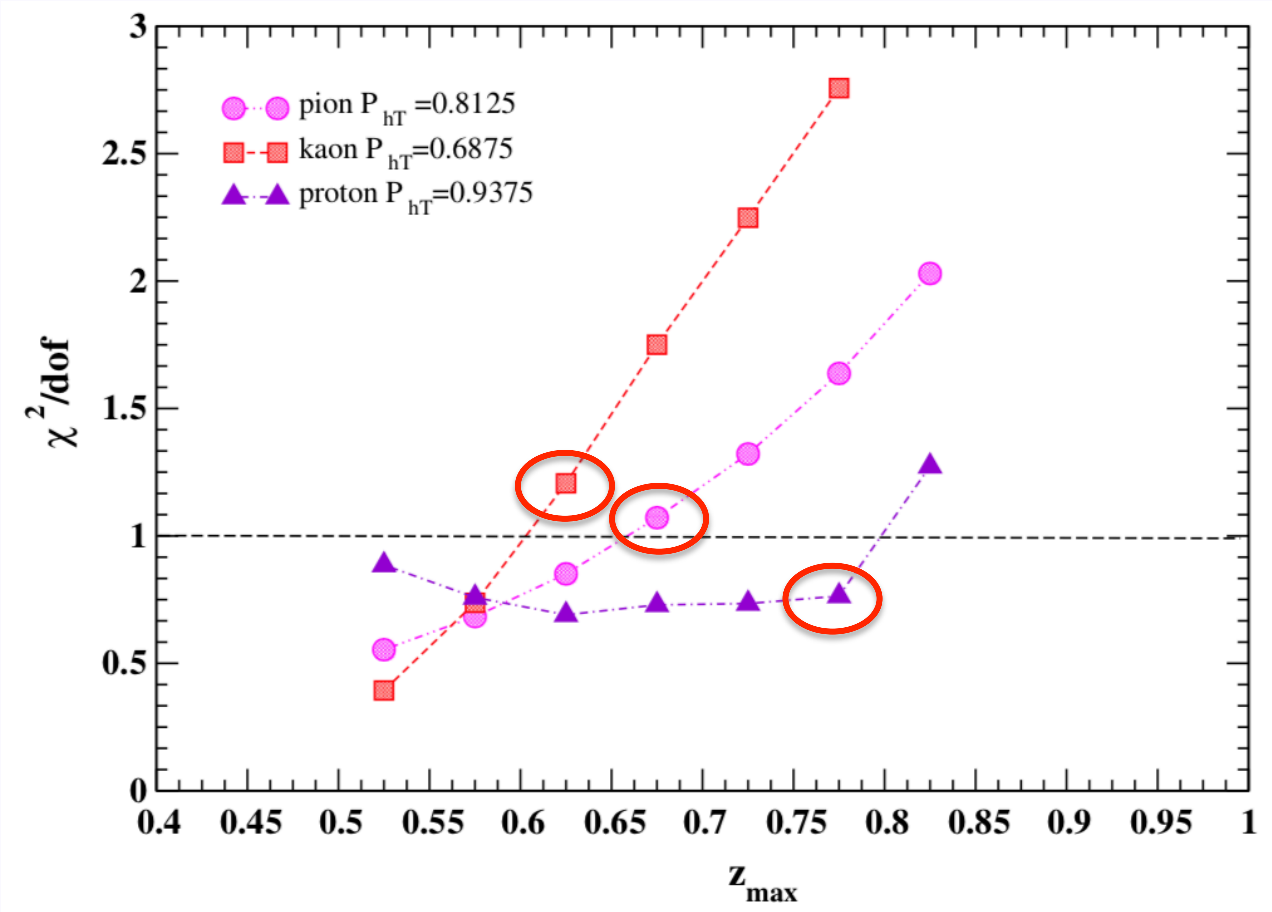
χ^2 Scanning

TMD FFs fits for each different $P_{hT} < P_{hT}^{max}$ cut



X^2 Scanning

Dependence on z parameter :
Sensitivity of χ^2 to the particular value of z_{max}
Exclude the datasets with $z > z_{max}$ from the analysis.



Results

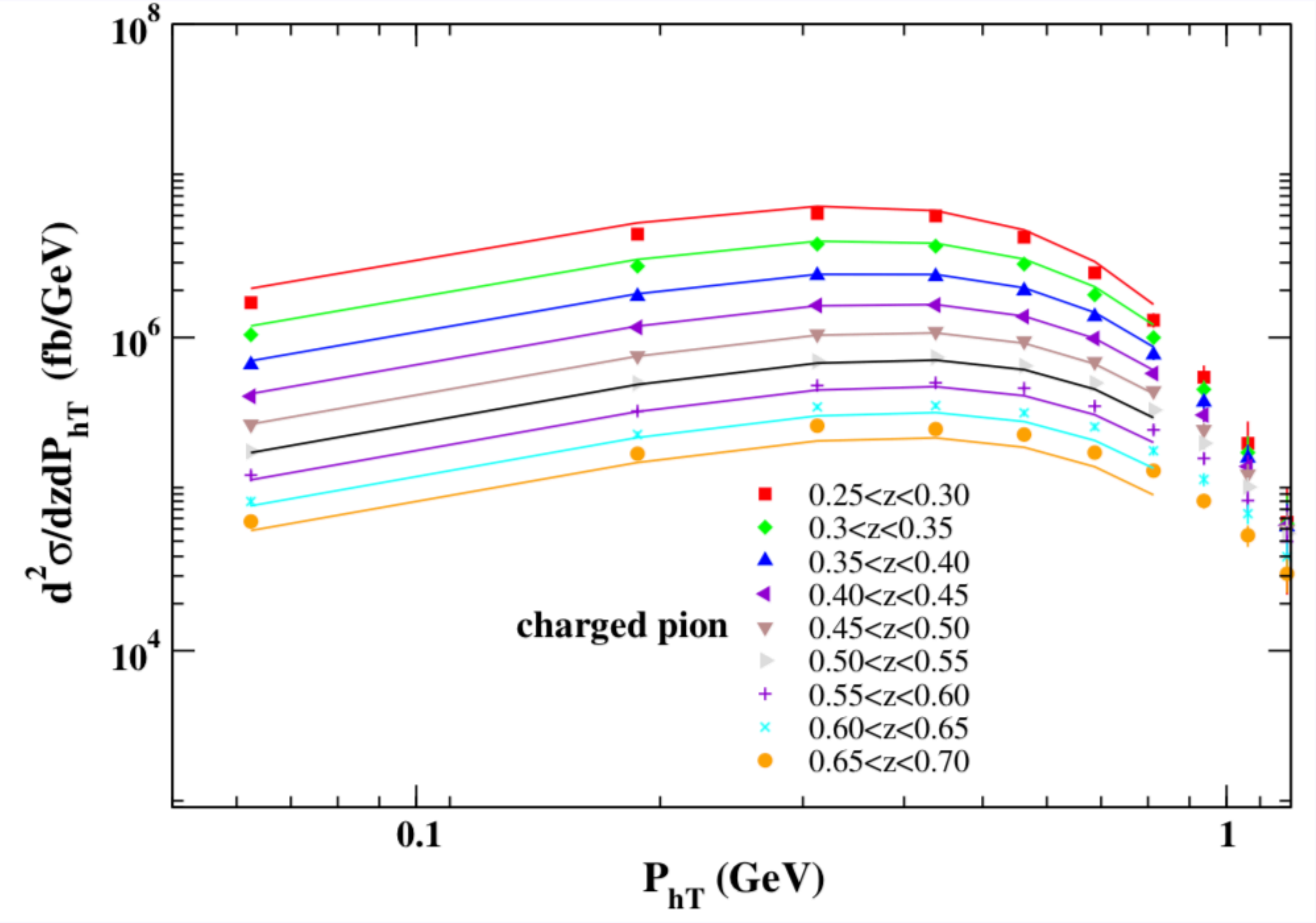
TABLE I. The input datasets included in the three individual analyses for $\pi^{\pm}$, $K^{\pm}$, and $p/\bar{p}$. For each hadron, we indicate the kinematical cuts of z and P_{hT}, number of data points in the fits, and the χ^2/d.o.f. values for each dataset.

Hadron	z cut	P_{hT} cut	Data points	χ^2/d.o.f.
$\pi^{\pm}$	[0.275–0.675]	[0–0.9]	63	1.053
$K^{\pm}$	[0.275–0.625]	[0–0.8]	48	1.154
$p/\bar{p}$	[0.275–0.775]	[0–1]	88	0.755

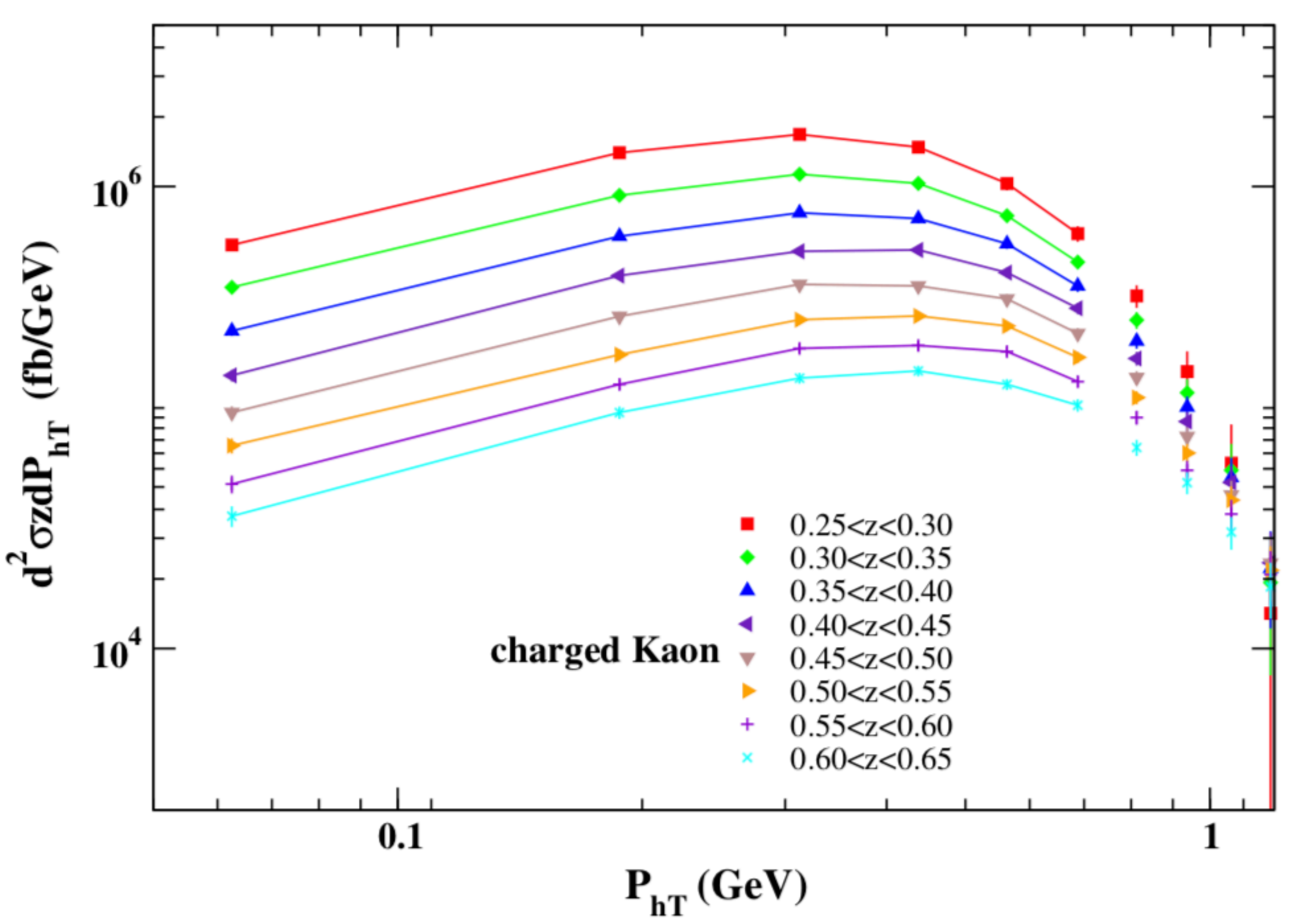
TABLE II. The best-fit parameters for the `SK19 TMD FFs` into $\pi^{\pm}$, $k^{\pm}$, and $p/\bar{p}$. The values labeled by (*) have been fixed. The details of the determination of best fit values are described in the text.

Parameters	$\pi^{\pm}$	$K^{\pm}$	$p/\bar{p}$
α	0.105	0.002	0.240
β	1.413	1.077	4.648
γ	0.854	0.739	1.153
$\mathcal{N}$	0.290*	0.166*	0.335*

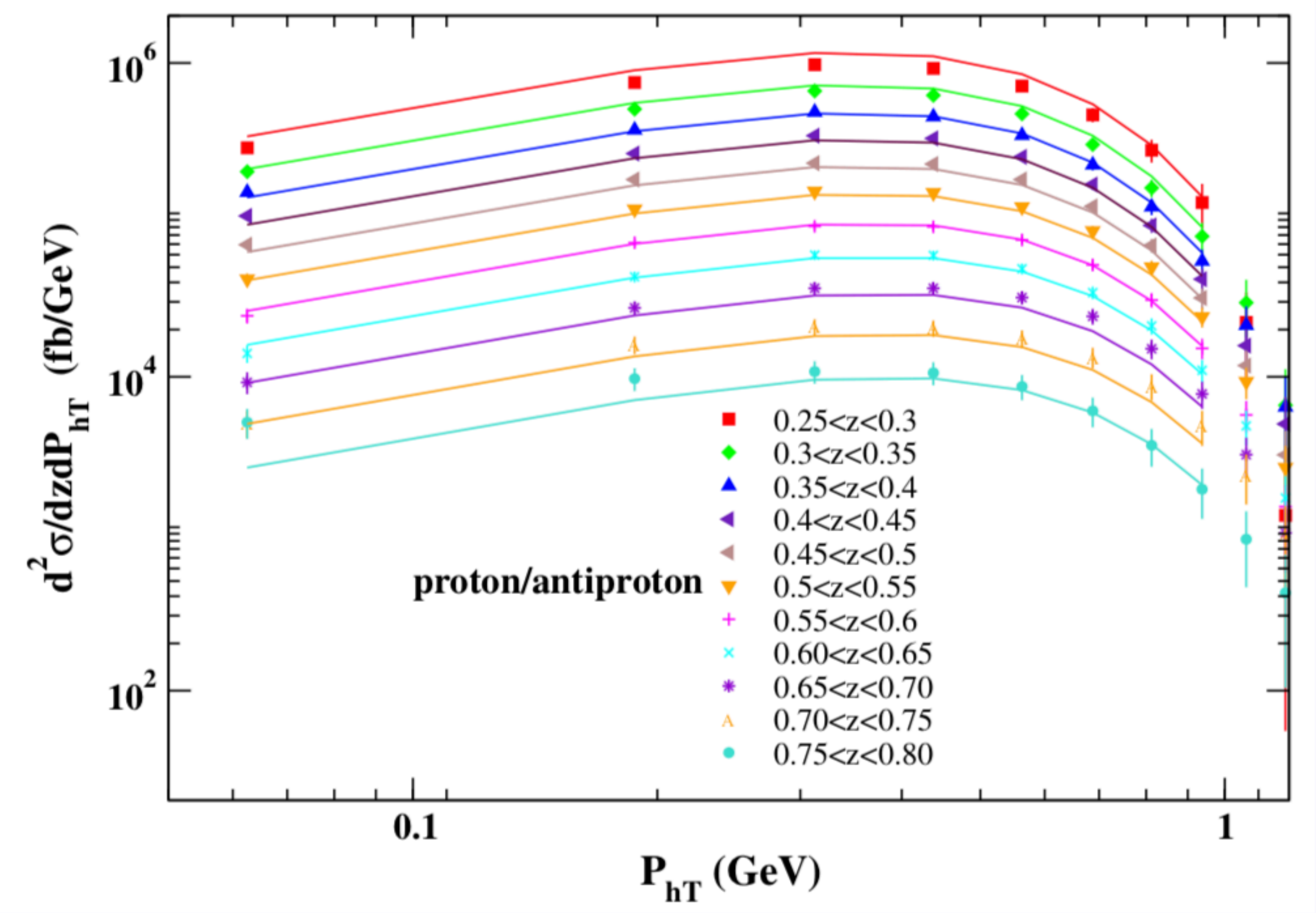
Pion Results



Kaon Results



Proton Results



Conclusion

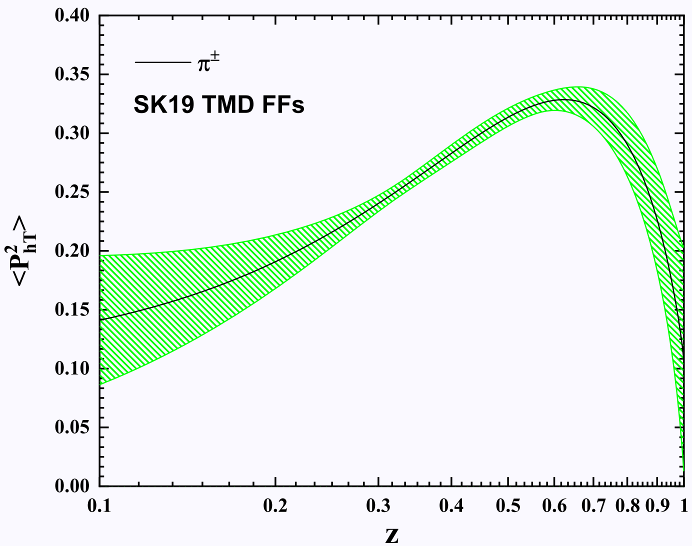
- ✓ Recently, Belle Collaboration at KEK has published the first measurements on $e^+ e^- \rightarrow hX$ differential cross sections in both z and P_{hT} space for charged pion, kaon, and proton/antiproton. Previously, there was no dataset on the transverse momentum dependence of the cross sections or multiplicities for extraction of the unpolarized TMD FFs for identified light hadrons.

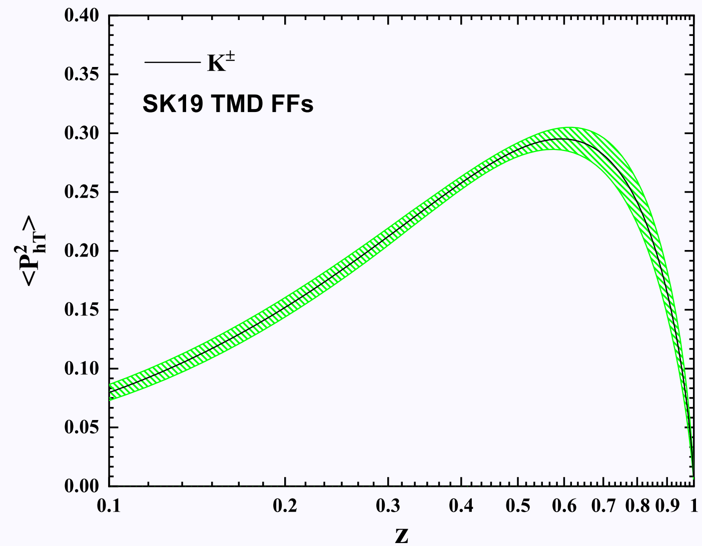
- ✓ These datasets are the only available observables in SIA process which can be used, for the first time, to determine the unpolarized TMD FFs for pin, kaon, and proton from QCD fits. These new measurements could provide enough constrains on the energy fraction z of the fragmentation process.

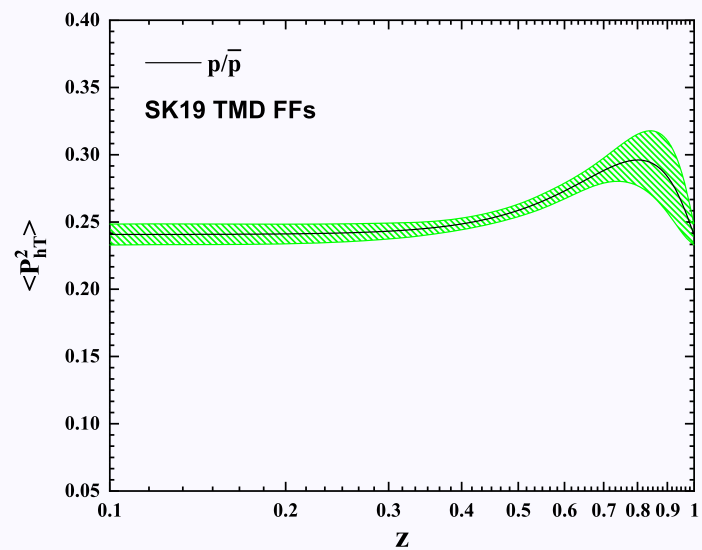
- ✓ Provide the unpolarized baseline for any polarized, transverse-momentum-dependent fragmentation functions such as the Collins FF.

- ✓ This analysis is restricted to the electron-positron annihilation processes, and hence, another possible area of future research would be to investigate the effect of another source of information on the TMD FFs which mainly come from the SIDIS processes. In terms of future work, it would be interesting to repeat the analysis described here considering the mentioned improvements.

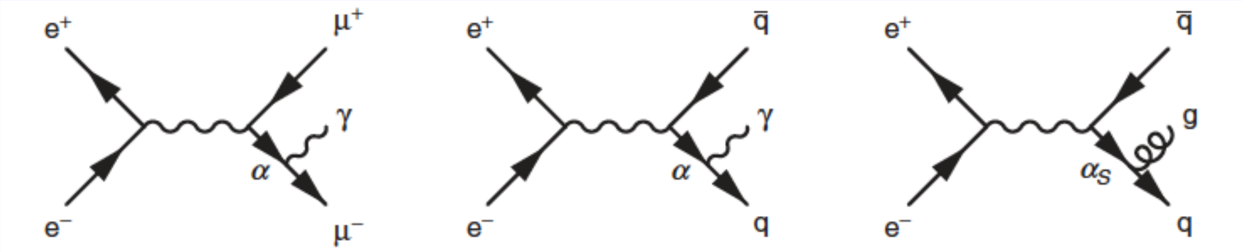
Back up


π±
SK19 TMD FFs
$<P_{hT}^2>$
z
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


K±
SK19 TMD FFs
$<P_{hT}^2>$
z
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

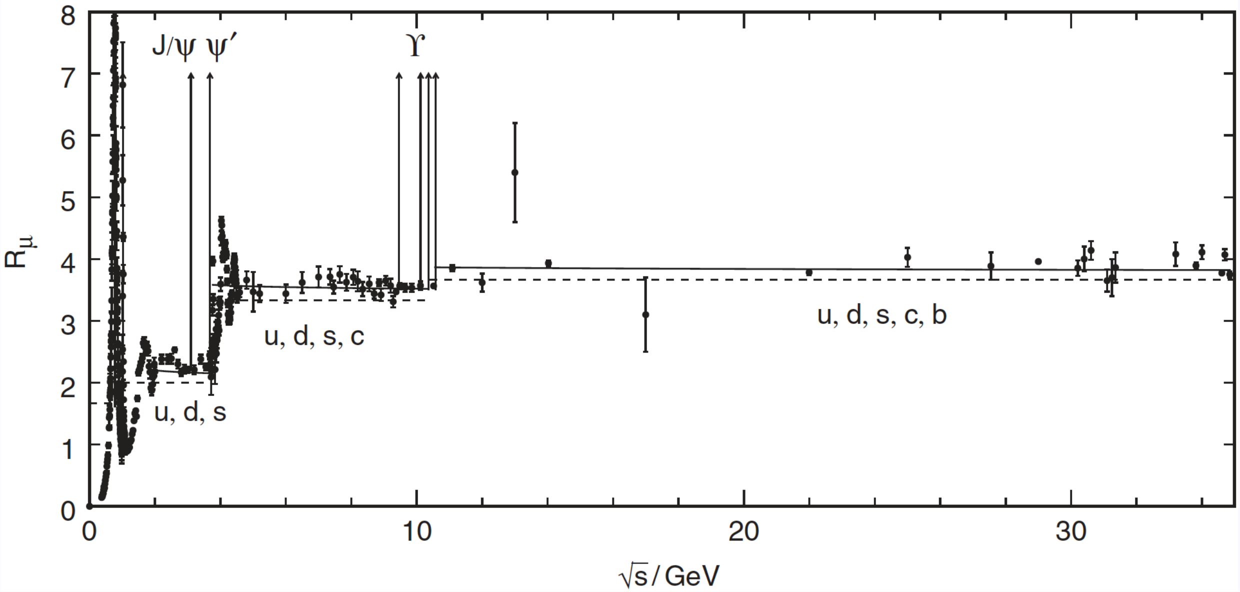

p/p̄
SK19 TMD FFs
$<P_{hT}^2>$
z
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

QCD Corrections

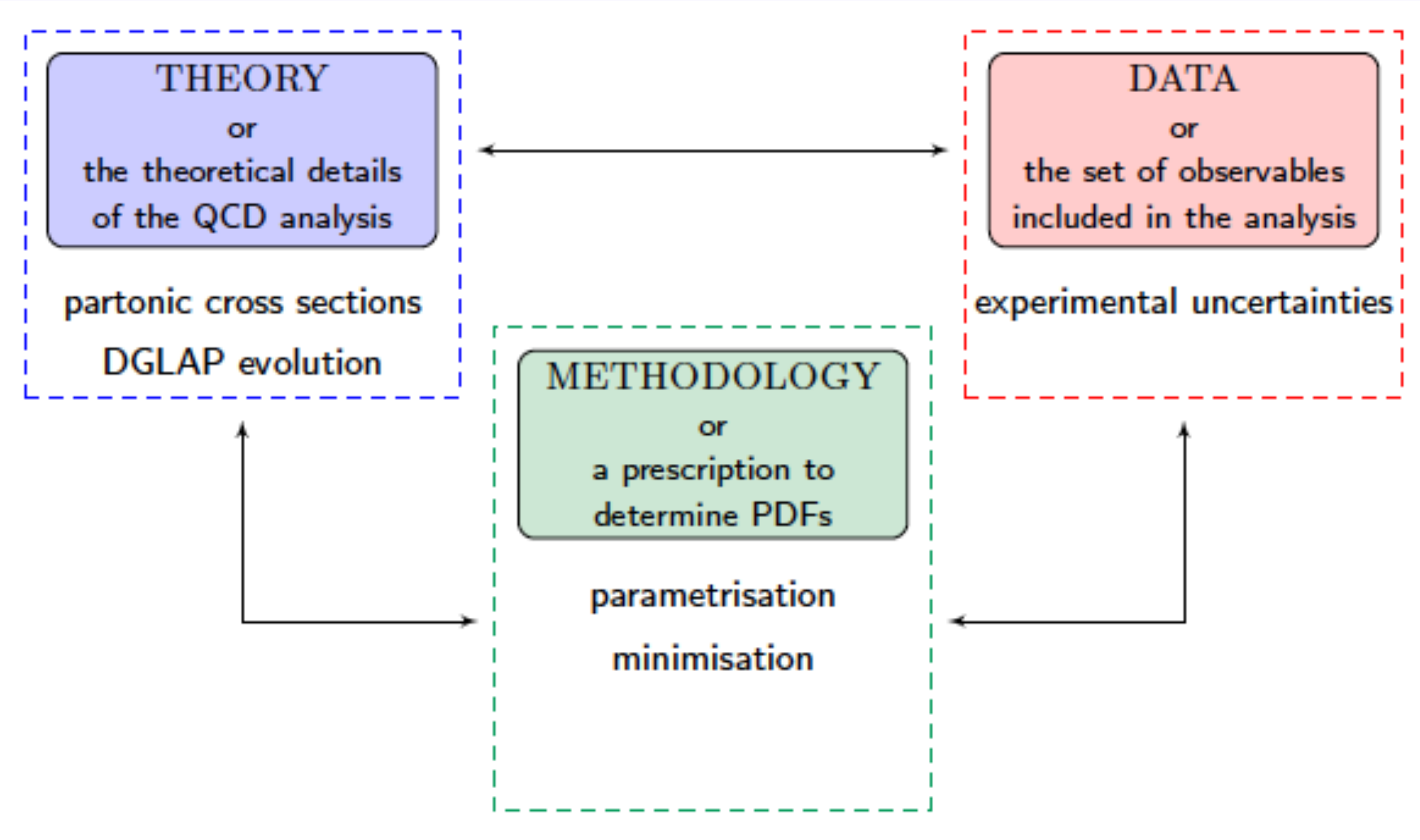


The QED corrections are relatively small, but the O(α_s) correction cannot be neglected

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\left(1 + \frac{\alpha_S(q^2)}{\pi}\right) \sum_{\text{flavours}} Q_q^2.$$



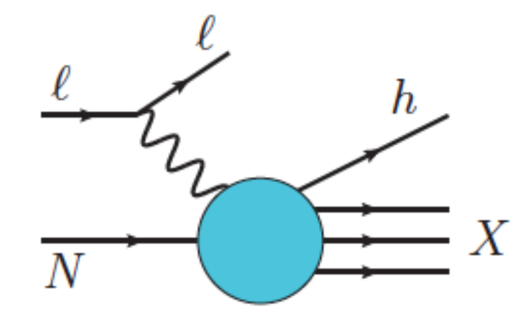
QCD analysis



Each of these ingredients is a source of uncertainty in the FF determination

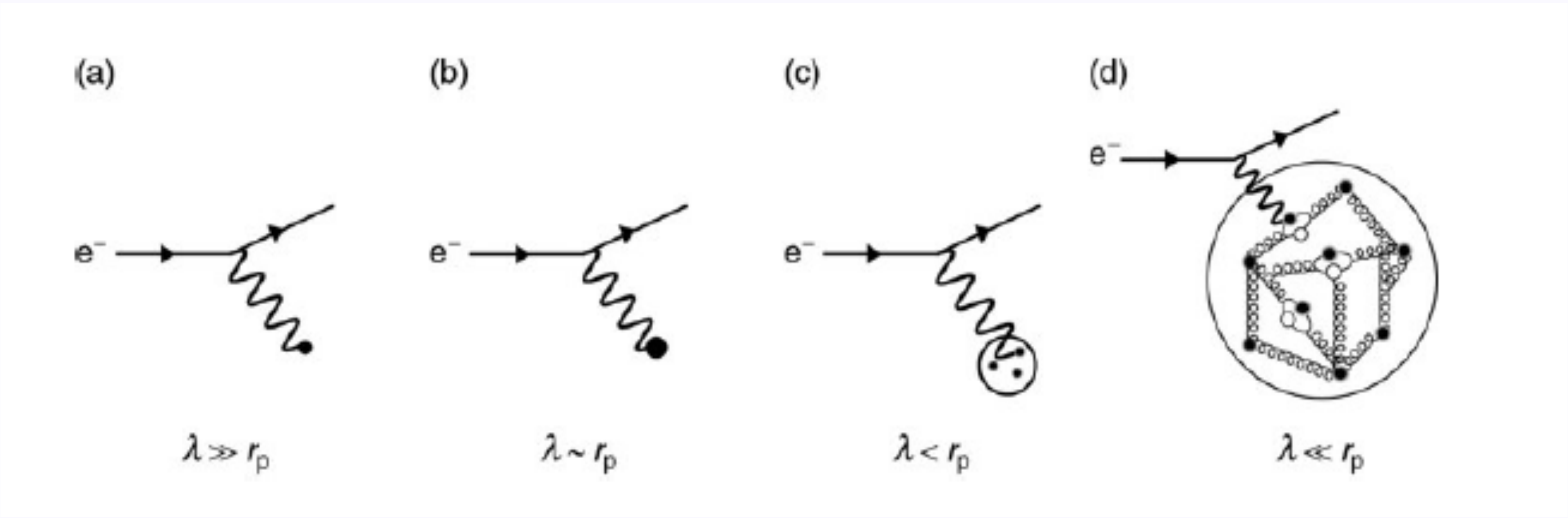
$k = (E, \vec{k}),\ k' = (E', \vec{k}')$	4-momenta of incident and scattered lepton l'
$P \overset{\text{lab}}{=} (M, \vec{0})$	4-momentum of the target nucleon
$q = k - k'$	4-momentum of the virtual photon γ^*
$\nu = \frac{P\cdot q}{M} \overset{\text{lab}}{=} E - E'$	Energy transfer to the target
$Q^2 = -q^2 \overset{\text{lab}}{\approx} 4EE' \sin^2(\frac{\theta}{2})$	Negative squared four-momentum transfer
$W^2 = (P + q)^2$	Squared invariant mass of the photon-nucleon system
$x_{\mathrm{B}} = \frac{Q^2}{2P\cdot q} \overset{\text{lab}}{=} \frac{Q^2}{2M\cdot\nu}$	Bjorken scaling variable
$y = \frac{P\cdot q}{P\cdot k} \overset{\text{lab}}{=} \frac{\nu}{E}$	Fractional energy of the virtual photon
ϕ_h	Azimuthal angle between the lepton scattering plane and the hadron production plane
$z = \frac{P\cdot P_h}{P\cdot q} \overset{\text{lab}}{=} \frac{E_h}{\nu}$	Fractional energy of hadron h
$P_{h\perp} \overset{\text{lab}}{=} \frac{\lvert\vec{q}\times\vec{P}_h\rvert}{\lvert\vec{q}\rvert}$	Component of the hadron momentum, P_h, transverse to q

TABLE II. Kinematic variables in semi-inclusive deep-inelastic scattering



$$\frac{d\sigma^h}{dz dP_{hT}} = L_{\mu\nu}(W^{\mu\nu}_{\text{TMD}} + W^{\mu\nu}_{\text{coll}}),$$

$L_{\mu\nu}$ is the leptonic tensor and W_{TMD} and W_{coll} are the hadronic tensors. The first hadronic tensor W_{TMD} has contribution in the region of small transverse momenta, while the second one contains collinear factorization.



(a)
e⁻
λ ≫ r_p
(b)
e⁻
λ ~ r_p
(c)
e⁻
λ < r_p
(d)
e⁻
λ ≪ r_p

As was the case for elastic scattering, Q^2 is defined as the negative four-momentum squared of the virtual photon,

$$Q^2 = -q^2.$$

When written in terms of the four-momenta of the initial- and final-state electrons,

$$Q^2 = -(p_1 - p_3)^2 = -2m_e^2 + 2p_1 \cdot p_3 = -2m_e^2 + 2E_1 E_3 - 2\mathrm{p}_1\mathrm{p}_3 \cos\theta.$$

In inelastic scattering, the energies are sufficiently high that the electron mass can be neglected and therefore, to a very good approximation

$$Q^2 \approx 2E_1 E_3(1 - \cos\theta) = 4E_1 E_3 \sin^2\frac{\theta}{2},$$

implying that Q^2 is always positive.

The Lorentz-invariant dimensionless quantity

$$x \equiv \frac{Q^2}{2p_2 \cdot q}, \tag{8.1}$$

will turn out to be an important kinematic variable in the discussion of the quark model of deep inelastic scattering. The range of possible values of x can be found by writing the four-momentum of the hadronic system in terms of that of the virtual photon

$$W^2 \equiv p_4^2 = (q + p_2)^2 = q^2 + 2p_2 \cdot q + p_2^2$$

$$\Rightarrow \quad W^2 + Q^2 - m_{\mathrm{p}}^2 = 2p_2 \cdot q,$$

and therefore, from the definition of (8.1),

$$x = \frac{Q^2}{Q^2 + W^2 - m_p^2}. \tag{8.2}$$

Because there are three valence quarks in the proton, and quarks and antiquarks can be produced together only in pairs, the hadronic final state in an $e^- p$ inelastic scattering process must include at least one baryon (qqq). Consequently, the invariant mass of the final-state hadronic system is always greater than the mass of the proton (which is the lightest baryon), thus

$$W^2 \equiv p_4^2 \geq m_p^2.$$

Because $Q^2 \geq 0$ and $W^2 \geq m_p$, the relation of (8.2) implies that x is always in the range

$$0 \leq x \leq 1.$$

The value of x expresses the "elasticity" of the scattering process. The extreme case of $x = 1$ is equivalent to $W^2 = m_p^2$, and therefore corresponds to elastic scattering.

$$\chi^2_{global}(\{\eta_i\}) = \sum_{n=1}^{n^{exp}} \left(\frac{1-\mathcal{N}_n}{\Delta\mathcal{N}_n} \right)^2 + \sum_{j=1}^{N_n^{data}} \left(\frac{(\mathcal{N}_n\, \mathcal{E}_j^{data} - \mathcal{T}_j^{theory}(\{\eta_i\})}{\mathcal{N}_n\, \delta\mathcal{E}_j^{data}} \right)^2$$

- ✓ E_i: The measured value of a given observable
- ✓ T_i: The corresponding theoretical estimate
- ✓ The experimental errors associated with this measurements are calculated from systematic and statistical errors added in quadrature.
- ✓ $\mathbf{n^{exp}}$: individual experimental data sets for the $\mathbf{n^{th}}$ experiment.
- ✓ $\mathbf{N_n^{data}}$ **:** refers to the number of data points in each data set.
- ✓ The optimization is done by the CERN program MINUIT.

For the nth experiment, $\mathcal{E}^{\rm data}$, $\delta\mathcal{E}^{\rm data}$, and $\mathcal{T}^{\rm theory}$ denote the data value, measurement uncertainty and theoretical value for the ith data point. Here, $\Delta\mathcal{N}$ is the experimental normalization uncertainty quoted by the experiments. The relative normalization shift $\mathcal{N}_n$ in above equation can be fitted along with the fitted parameters $\{\eta_i\}$ of Eqs. (15) and (17) and then keep fixed. In order to illustrate the effects