



DEPARTMENT OF
PHYSICS



MARYLAND CENTER
for Fundamental Physics

Holography of the Cosmological Phase Transition of Composite Higgs Confinement

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In collaboration with Kaustubh Agashe, Peizhi Du, Soubhik Kumar
& Raman Sundrum

based on *JHEP* 05(2020)086 [arXiv 1910.06238] and 2010.04083.

Introduction- composite Higgs

Composite Higgs models:

- Highly motivated since they can explain the large hierarchies
- Attractive targets for the LHC and future colliders
- Higgs can be a confined composite state of strong dynamics at or above the TeV scale.

Introduction- phase transitions

Early universe 1st order phase transitions (PT):

- Stochastic gravitational wave background from the PT can be observed.
- PT affects baryon and dark matter genesis:
 - Generation of baryon asymmetry during the transition (e.g. electroweak baryogenesis)
 - A supercooled PT dilutes matter abundances (similar to inflation)

Introduction

- Early universe 1st order phase transitions (PT):
 - Stochastic gravitational wave background from the PT can be observed.
 - PT affects baryon and dark matter genesis
- Composite Higgs: a confinement-deconfinement PT
- *Strongly coupled, non-perturbative!*

How to analyze the confinement PT?

Composite Higgs models: nearly conformal, large N

Holography (5D)

- 4D strong coupling \rightarrow weakly coupled 5D Gravity ($G_N^{(5D)} \sim \frac{1}{N^2}$)
- PT dynamics in 5D EFT control

Spontaneous confinement (4D)

- Deconfined theory approximately scale invariant
- Confinement breaks scale invariance, but spontaneously
- Corresponding pNGB is dilaton
- Dynamics of dilaton dominates the PT in some regime

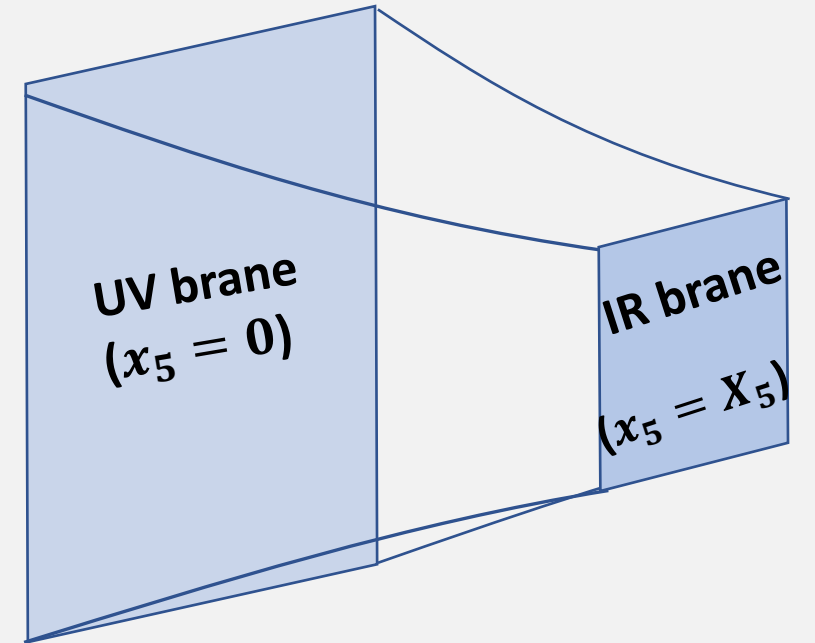
Holographic dual

Low temperature phase: RS1

Randall & Sundrum 1999
Akrani-Hamed, Porrati
& Randall 2000
Rattazzi & Zaffaroni 2000

Dual of the confined phase

- Control parameter: large N , $\frac{N^2}{16\pi^2} = \left(\frac{M_5}{k}\right)^3$
- Confinement/ compositeness scale $\sim M_{\text{Pl}} e^{-kX_5}$



$$ds^2 = e^{-2kx_5} \eta_{\mu\nu} dx^\mu dx^\nu + dx_5^2$$
$$0 < x_5 < X_5$$

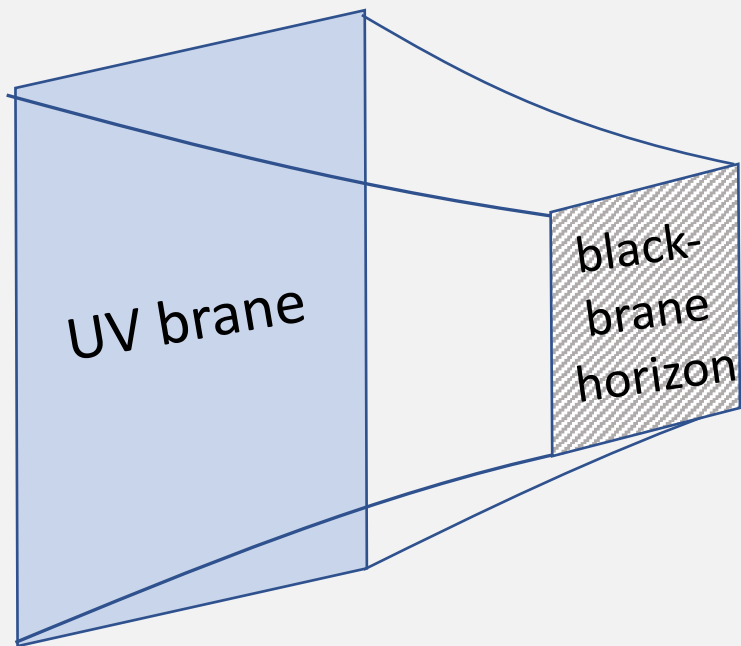
Holographic dual

Creminelli, Nicolis & Rattazzi 2002

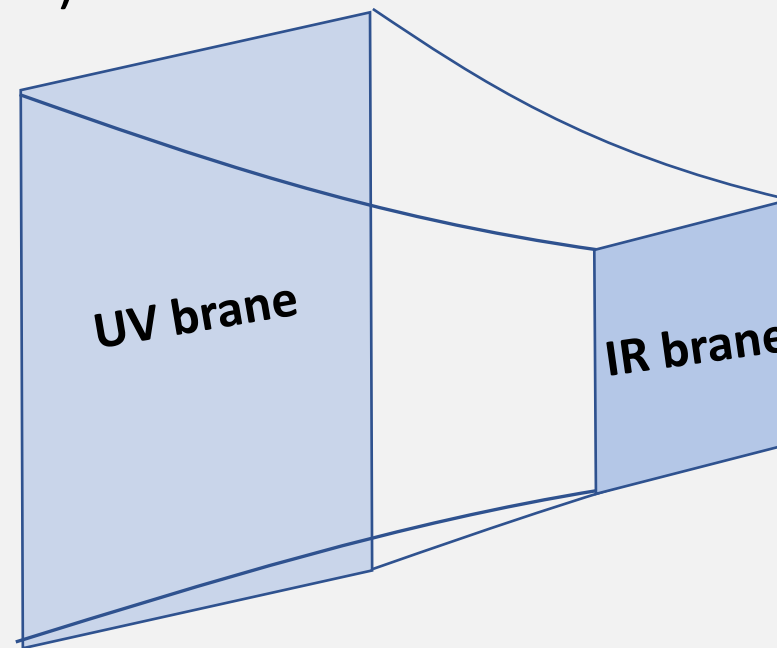
Black-brane phase \rightarrow RS1

Control parameter: large N , $\frac{N^2}{16\pi^2} = \left(\frac{M_5}{k}\right)^3$

(Poincare patch analog of Hawking-Page PT)



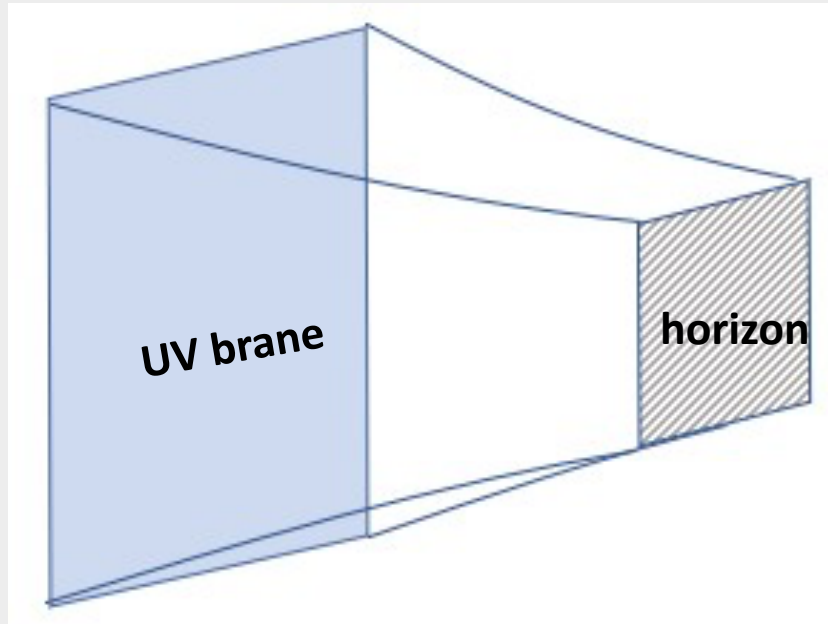
Dual of the deconfined phase



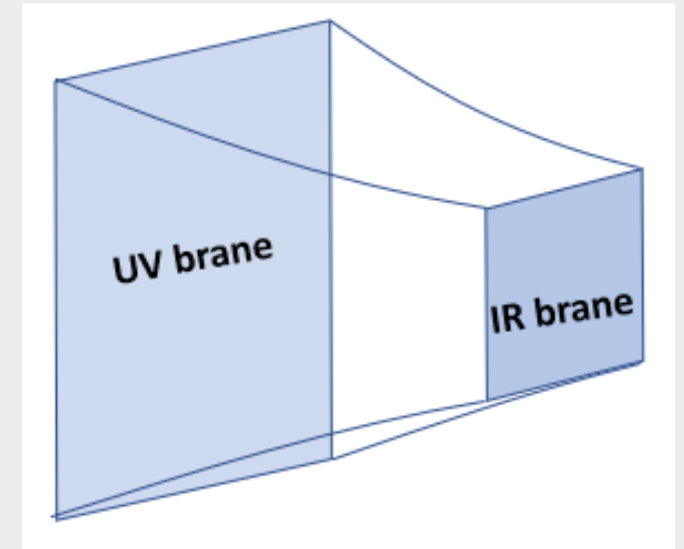
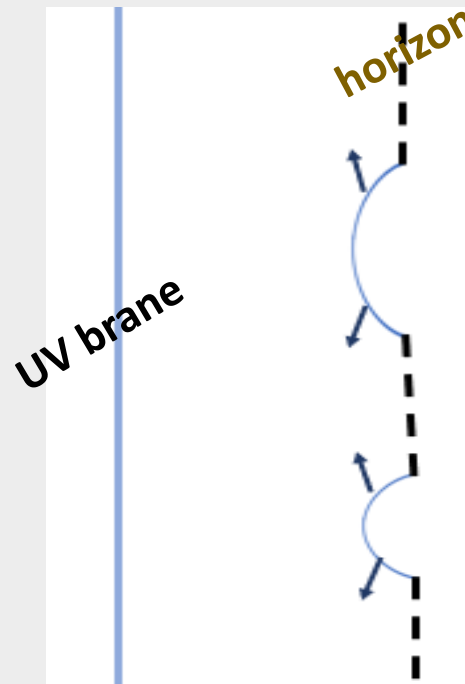
Dual of the confined phase

Holography

Black-brane phase \rightarrow RS1



IR brane emerges from the horizon



(Non-perturbative)

Tunneling rate:

$$\Gamma \sim e^{-1/G_N^{(5D)}} \sim e^{-N^2}$$

How to analyze the confinement PT?

Composite Higgs models: nearly conformal, large N

Holography (5D)

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How to analyze the confinement PT?

Composite Higgs models: nearly conformal, large N

Holography (5D)

Focus of this talk

- 4D strong coupling \rightarrow weakly coupled 5D Gravity ($G_N^{(5D)} \sim \frac{1}{N^2}$)
- PT dynamics in 5D EFT control Agashe, Du, M.E., Kumar, Sundrum 2020

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Agashe, Du, M.E., Kumar, Sundrum 2019

Outline

Theory

- Equilibrium description
- Bubble nucleation rate
 - The 5D bounce

Application to cosmology of composite Higgs

- Phase transition in the minimal model
 - (Slow, resulting in empty universe or large supercooling and dilution)
- Faster transition rate? Beyond the minimal model
- Supercooled phase transition

Phenomenology

- Gravitational waves
- Dilution of matter and baryogenesis

Questions to answer about the PT

- Is it 1st order, 2nd order, cross over?
- What is the critical/transition temperature?

PT Dynamics

- What is the rate of bubble nucleation?
- Does the PT complete? If yes, at what temperature? Is it prompt or supercooled?
- How do the bubbles/bounce solutions look like?
- What are the features of the gravitational waves generated by the PT?

The two phases

Low temperature phase: RS1

- RS1 stabilization: Goldberger-Wise mechanism

High temperature black-brane phase

Low temperature phase: RS1

- In the RS1 model, hierarchies are related to the position of the IR brane:

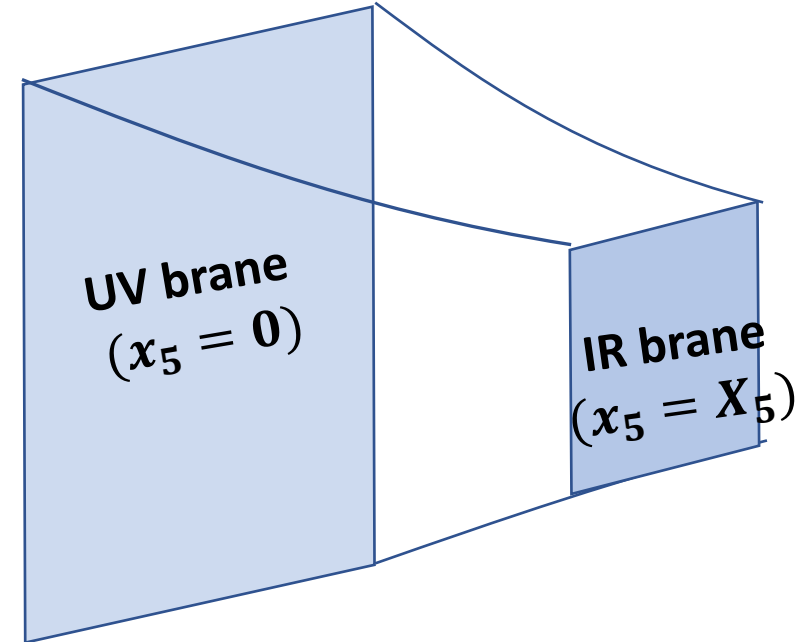
$$\frac{\text{TeV}}{M_{\text{Pl}}} \sim e^{-kX_5}$$

- An isometry of the bulk space-time: 4D scaling + x_5 translation

$$x_5 \rightarrow x_5 + \delta, \quad x^\mu \rightarrow e^{k\delta} x^\mu$$

- Spontaneously broken by the presence of the IR brane
 - Corresponding Glosdtone boson: radion $\varphi = k e^{-kX_5}$
- What sets the position of the IR-brane (X_5)?

Randall & Sundrum 1999



$$ds^2 = e^{-2kx_5} \eta_{\mu\nu} dx^\mu dx^\nu + dx_5^2$$
$$0 < x_5 < X_5$$

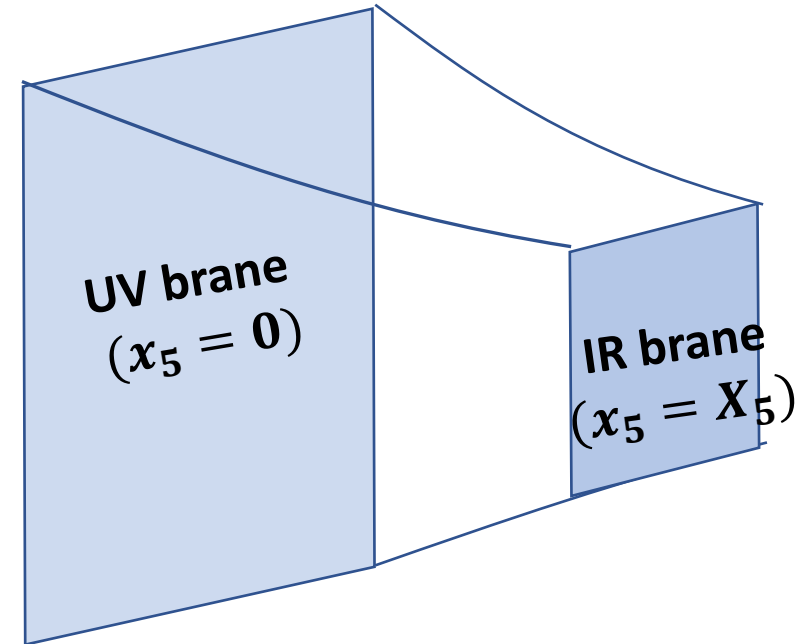
RS1 - Stabilization

- RS1: a solution to GR for specific values of bulk CC and brane tensions
- With this choice:

$$V_{\text{radion}}(\varphi) = 0$$

Position of the IR-brane (X_5) is a free parameter

Randall & Sundrum 1999



$$ds^2 = e^{-2kx_5} \eta_{\mu\nu} dx^\mu dx^\nu + dx_5^2$$
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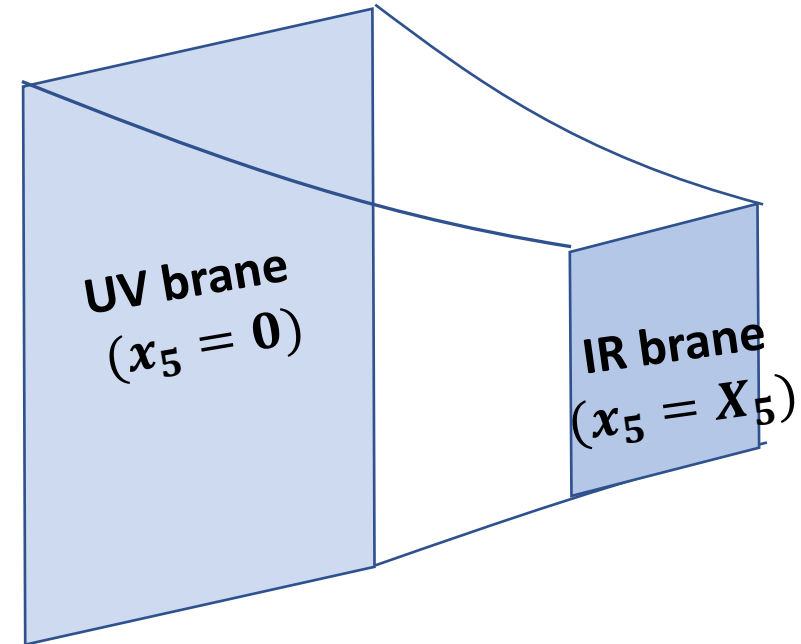
RS1- Stabilization

- RS1: a solution to GR for specific values of bulk CC and brane tensions
- With this choice: $V_{\text{radion}}(\varphi) = 0$
- If IR brane tension detuned:

$$V_{\text{radion}}(\varphi) = \delta T_{IR} \varphi^4$$

Finite, nonzero $\langle \varphi \rangle$ not stable!

Randall & Sundrum 1999



$$ds^2 = e^{-2kx_5} \eta_{\mu\nu} dx^\mu dx^\nu + dx_5^2$$
$$0 < x_5 < X_5$$

Stabilization: Goldberger-Wise mechanism

Goldberger & Wise 1999

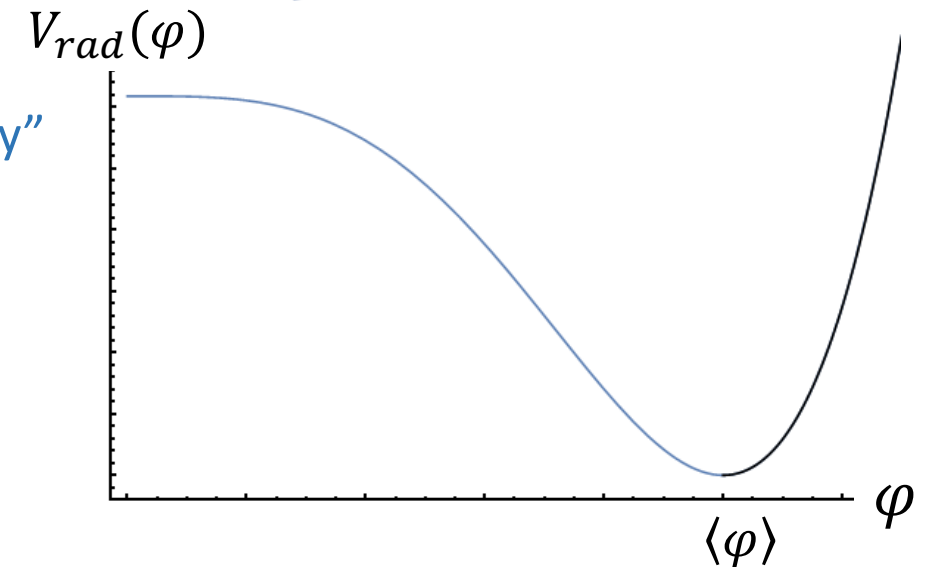
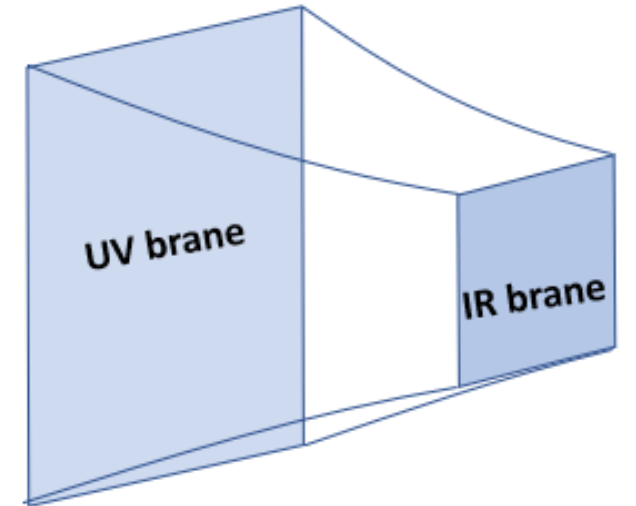
- RS1: a solution to GR for specific values of bulk CC and brane tensions
- If IR brane tension detuned: $V_{\text{radion}}(\varphi) = \delta T_{IR} \varphi^4$
- With a bulk scalar (Goldberger-Wise) field, minimally with the potential:

$$V_{\text{GW,bulk}}(\Phi) = \frac{1}{2} m^2 \Phi^2$$

explicit breaking of “4D scale invariance/ x_5 -translation symmetry”

$$V_{\text{radion}}(\varphi) = \frac{3N^2}{4\pi^2} \lambda \varphi^4 \left(1 - \omega \left(\frac{\varphi}{\Lambda_{UV}} \right)^\epsilon \right)$$

Hierarchy is set mainly by $\epsilon \approx \frac{m^2}{4k^2}$: $\ln \frac{M_{\text{Pl}}}{\text{TeV}} \sim \frac{1}{\epsilon}$

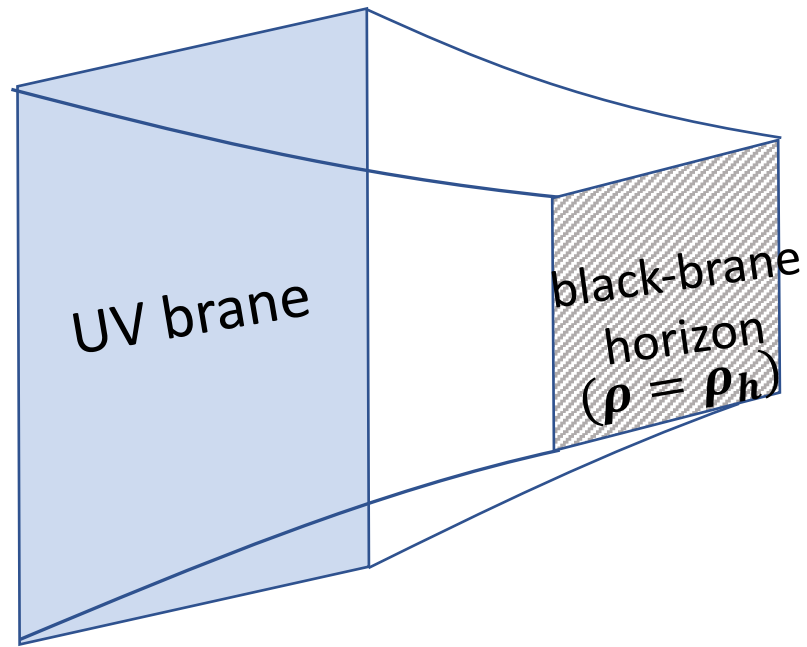


Control parameters

$$\begin{aligned} V_{radion}(\varphi) &= \frac{3N^2}{4\pi^2} \lambda \varphi^4 \left(1 - \omega \left(\frac{\varphi}{\Lambda_{UV}} \right)^\epsilon \right) \\ &= \frac{3N^2}{4\pi^2} \lambda \varphi^4 \left(1 - \frac{1}{1+\epsilon/4} \left(\frac{\varphi}{\langle \varphi \rangle} \right)^\epsilon \right) \end{aligned}$$

- Large N , $\frac{N^2}{16\pi^2} = \left(\frac{M_5}{k} \right)^3$: 5D GR perturbative
- Small λ : small backreaction to geometry
- Small ϵ : small explicit breaking of scale invariance

High temperature black-brane phase \approx AdS-Schwarzschild



Hawking temperature: $T = \frac{\hbar}{\pi l^2} \rho_h$

$$ds^2 = -\left(\frac{\rho^2}{l^2} - \frac{\rho_h^4/l^2}{\rho^2}\right) dt^2 + \frac{d\rho^2}{\left(\frac{\rho^2}{l^2} - \frac{\rho_h^4/l^2}{\rho^2}\right)} + \frac{\rho^2}{l^2} \sum_i dx_i^2$$

Equilibrium description of the phase transition

- Free energies
- Critical temperature
- Order of the phase transition

Free energy

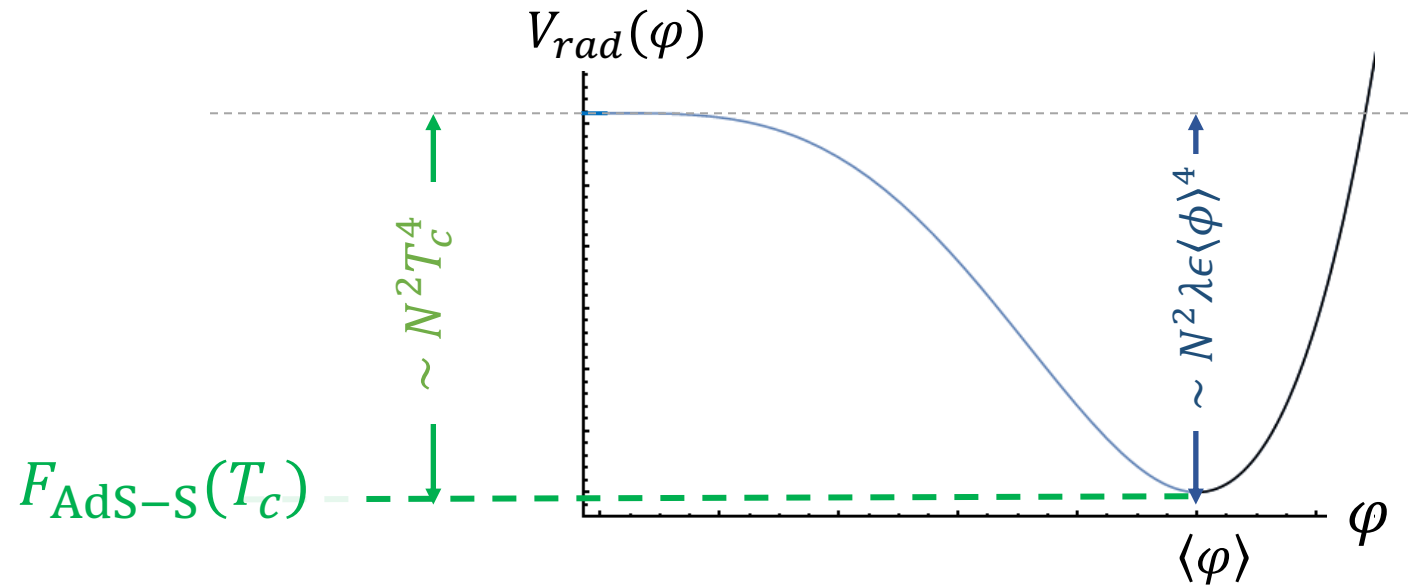
$$F_{\text{RS}} \approx V_{\text{rad}}(\langle\varphi\rangle)$$

$(T \ll \langle\varphi\rangle)$

- Critical Temperature:

$$F_{\text{RS}} = F_{\text{AdS-S}}$$

$$F_{\text{AdS-S}} - V_{\text{rad}}(0) = -\frac{\pi^2}{8} N^2 T^4$$



Creminelli, Nicolis & Rattazzi 2002

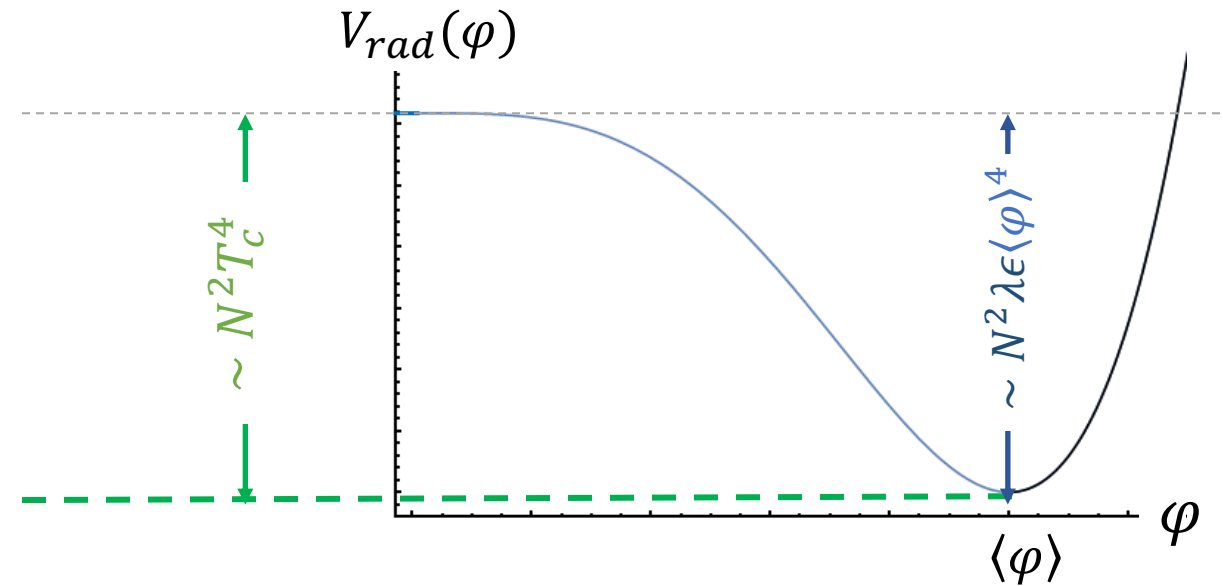
Transition/critical temperature

- Critical Temperature: $\frac{T_C}{\langle\varphi\rangle} \sim (\epsilon\lambda)^{\frac{1}{4}} \ll 1$

- PT is first order, for small ϵ or λ .

$$\left. \frac{\partial F_{RS}}{\partial T} \right|_{T_C} \neq \left. \frac{\partial F_{AdS-S}}{\partial T} \right|_{T_C}$$

1st order \Rightarrow bubble nucleation



Creminelli, Nicolis & Rattazzi 2002

Questions to answer about the PT

For small ϵ or λ :

✓ PT is 1st order.

✓ What is the critical temperature? $\frac{T_c}{\langle\phi\rangle} \sim (\epsilon\lambda)^{\frac{1}{4}} \ll 1$

PT Dynamics

- What is the rate of bubble nucleation?
- Does the PT complete? If yes, at what temperature? Is it prompt or supercooled?
- How do the bubbles/bounce solutions look like?
- What are the features of the gravitational waves generated by the PT?

Bubble nucleation: review of general formalism

Coleman 1977
Colman & De Luccia 1980
Linde 1981

Bubble Nucleation

- Probability of bubble nucleation per unit 4-volume:

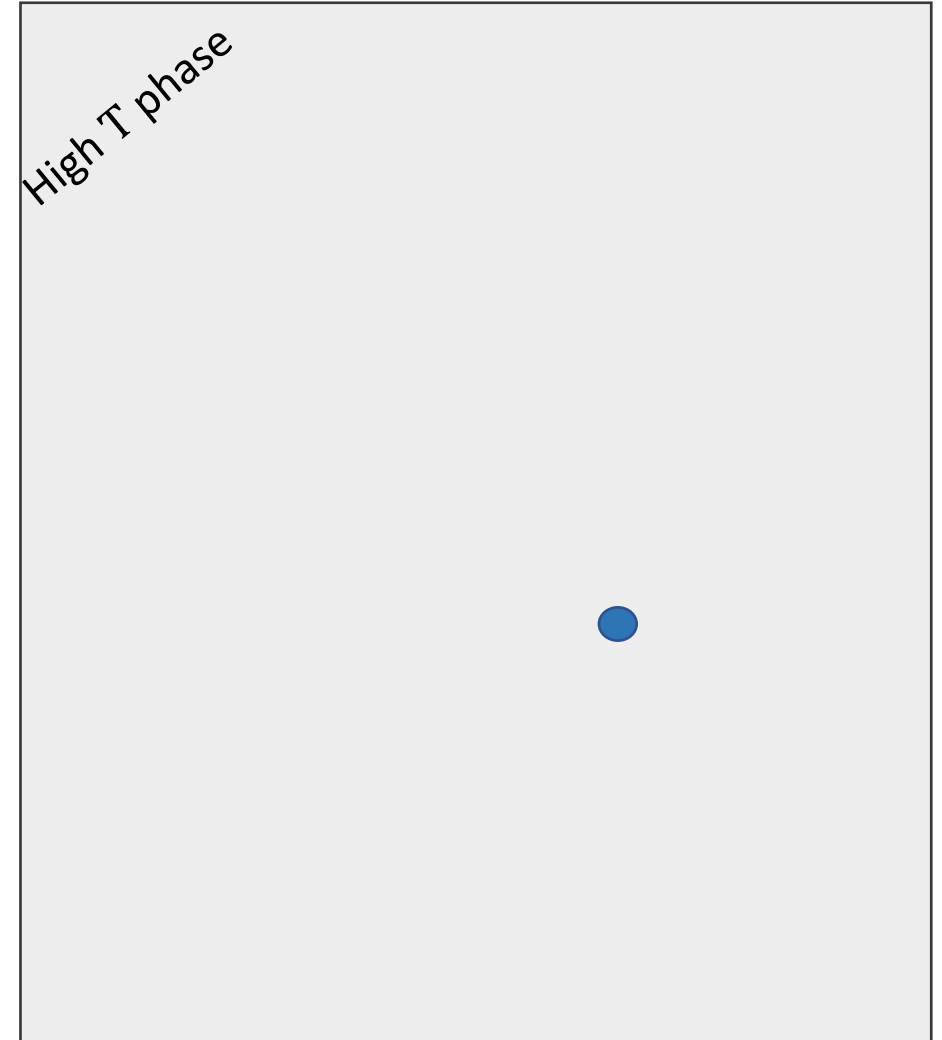
$$\Gamma \sim T^4 e^{-S_b}$$



Bubble Nucleation

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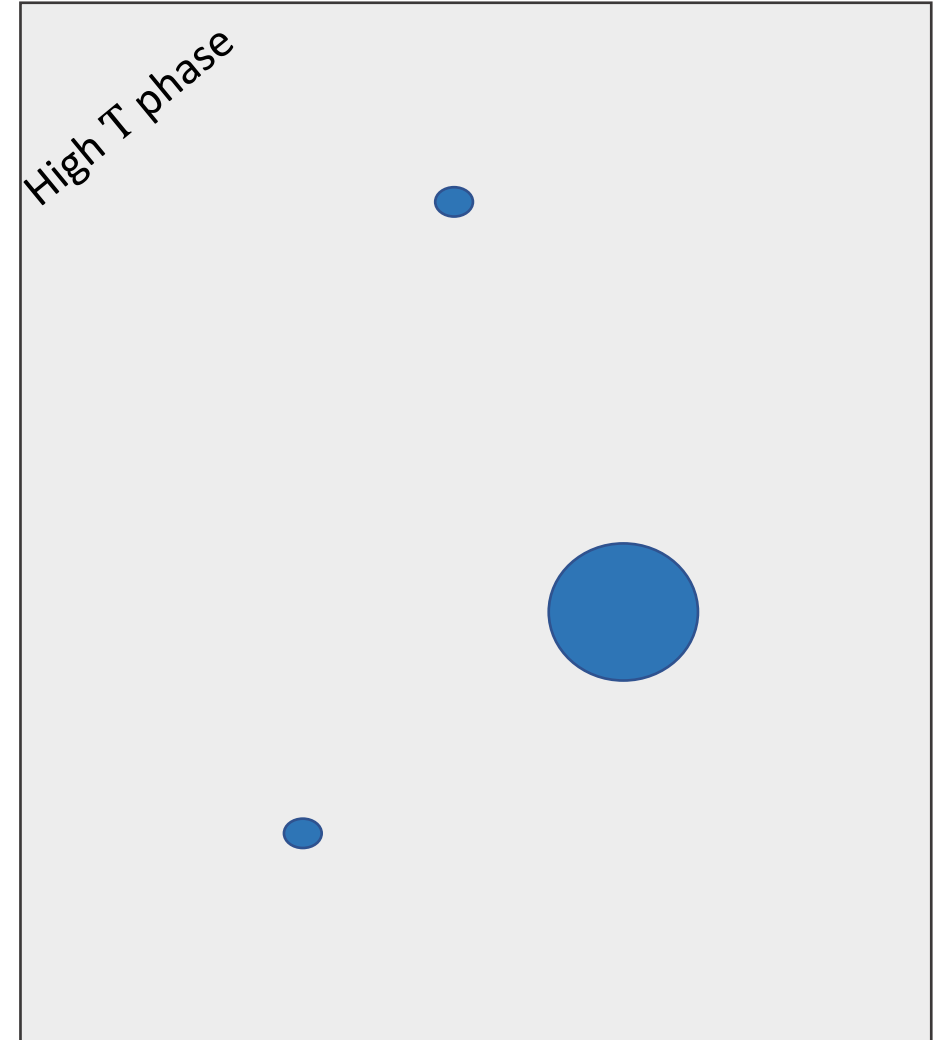
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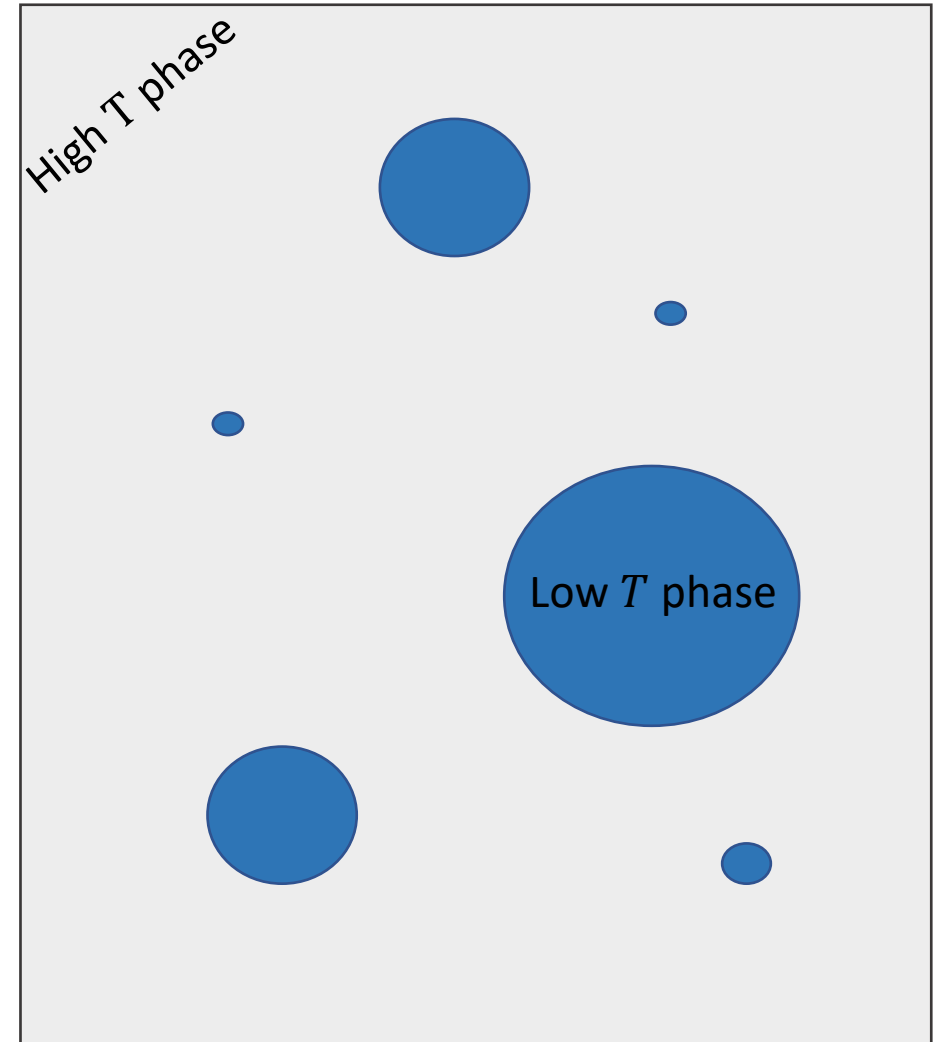
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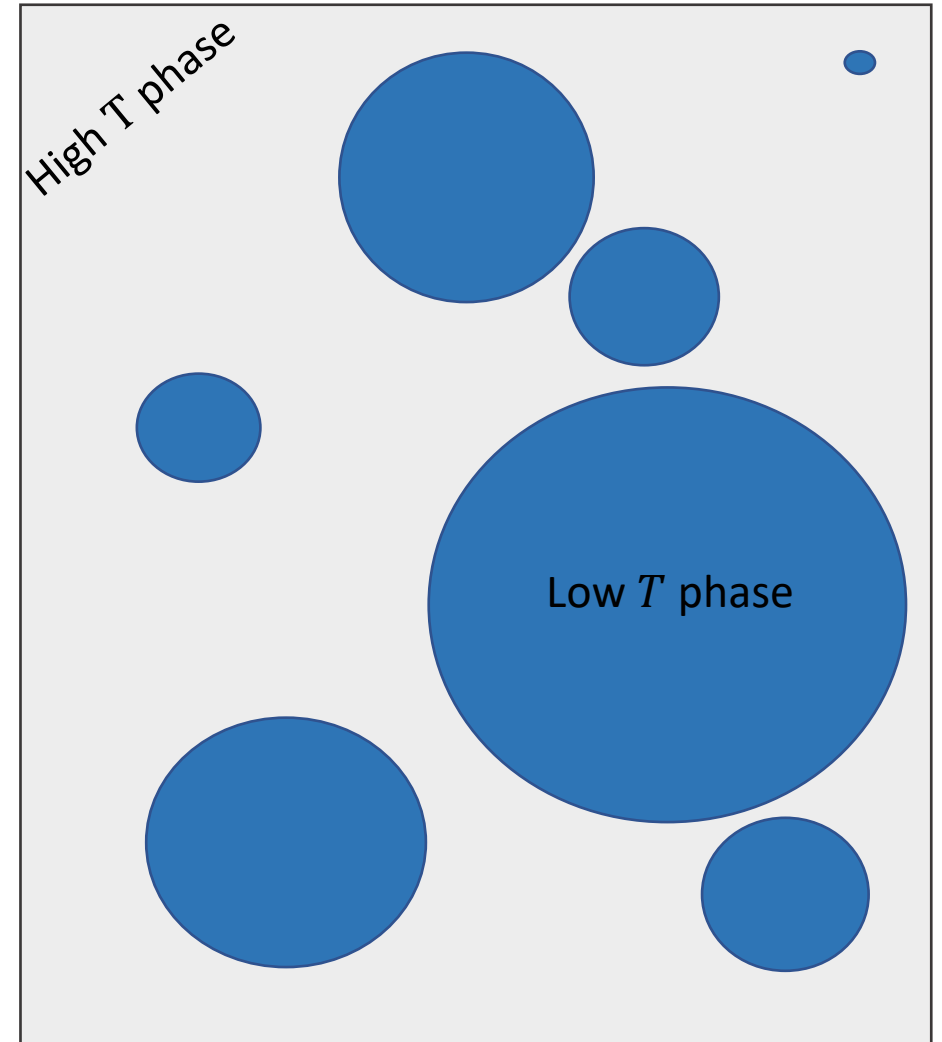


Bubble Nucleation

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S_b : action of “bounce” solutions to Euclidean equations of motion that interpolate between the two phases



Bubble Nucleation

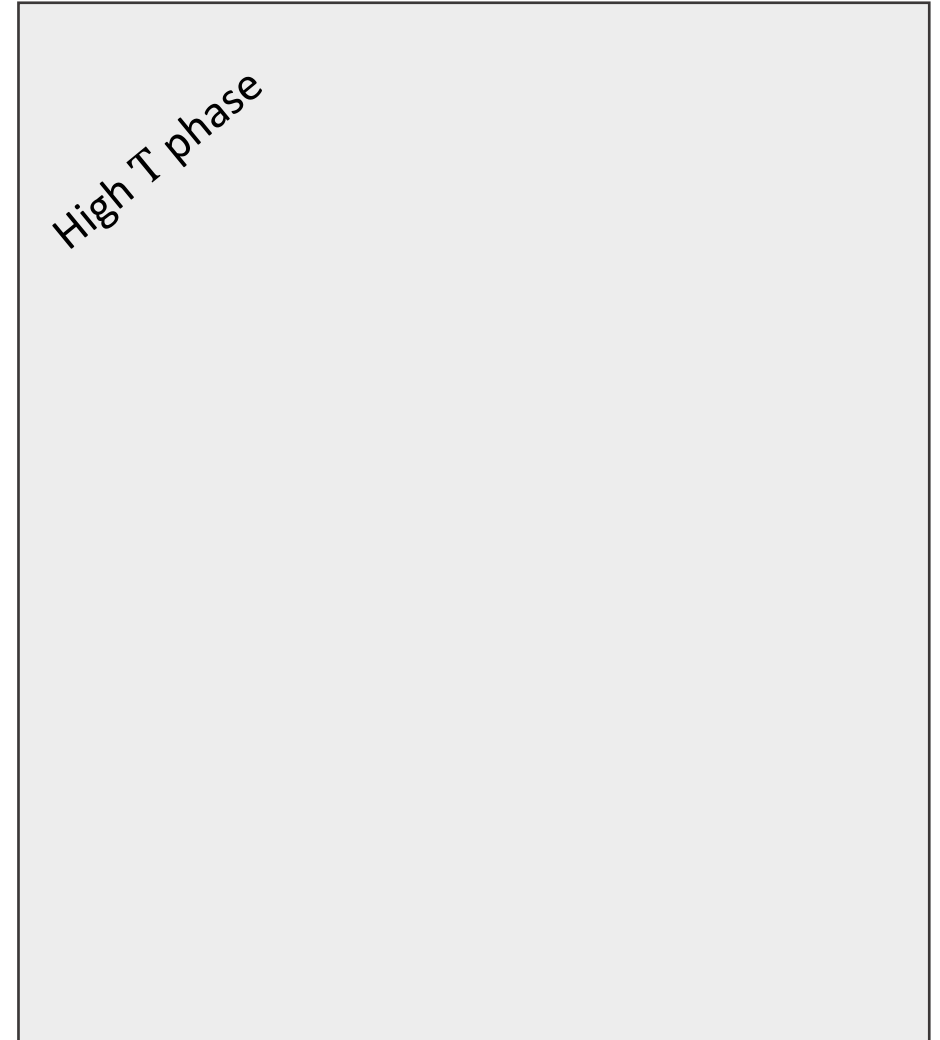
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- In an expanding universe, PT completes if

$$\Gamma \gtrsim H^4 \quad \left(H \sim \frac{T_C^2}{M_{Pl}} \right)$$

$$S_b \lesssim 4 \ln \frac{M_{Pl}}{T_C}$$



Bubble Nucleation

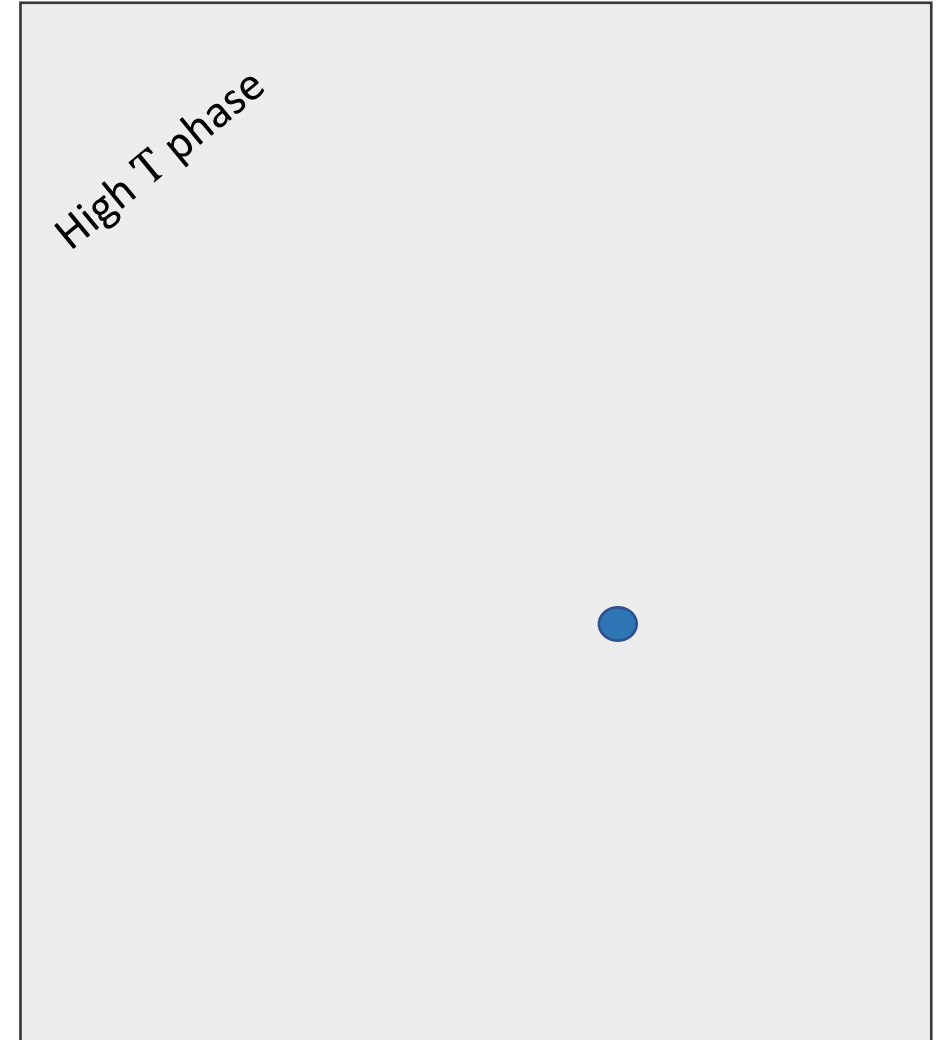
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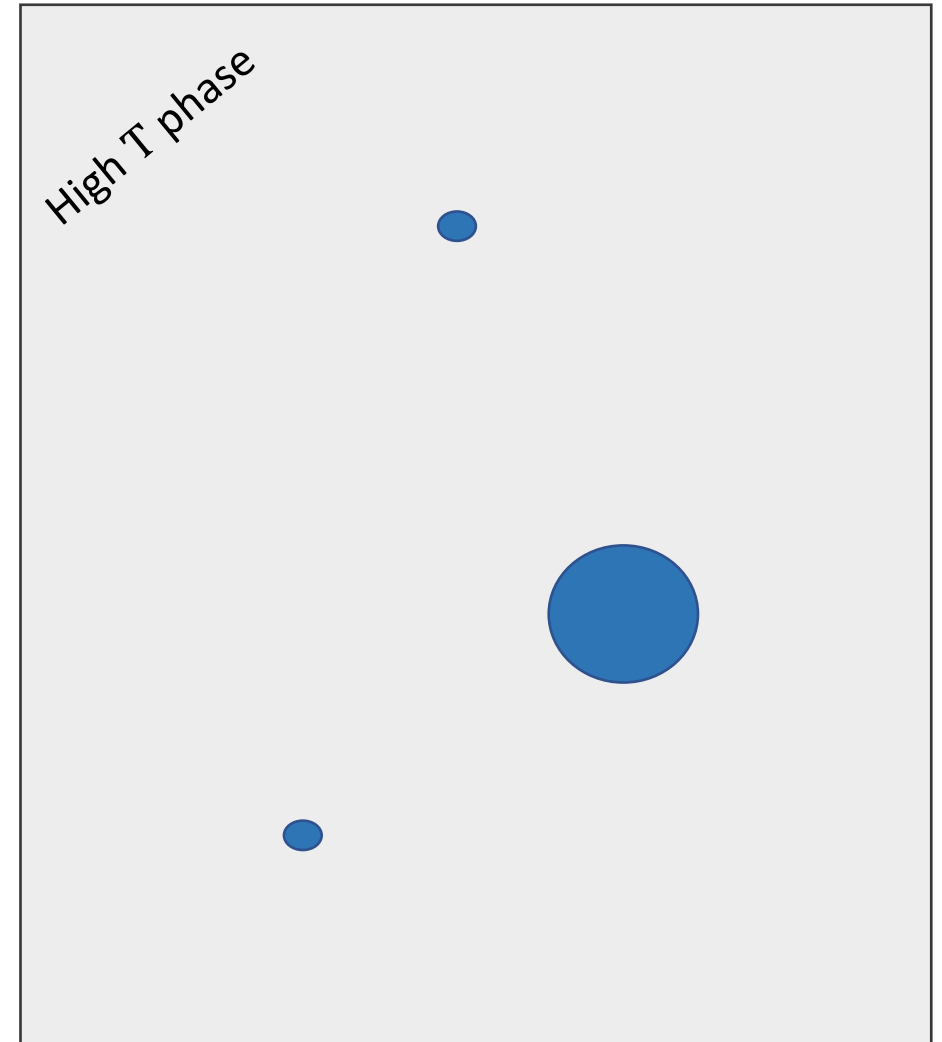
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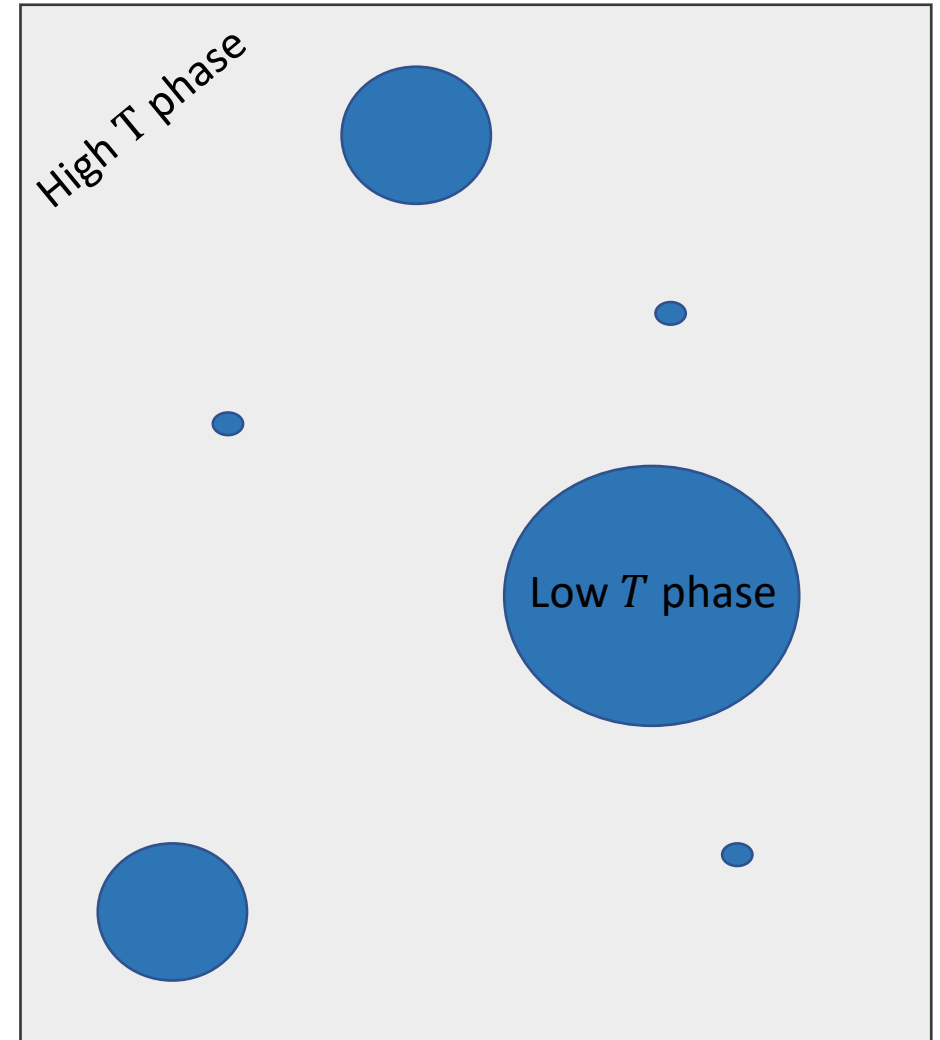
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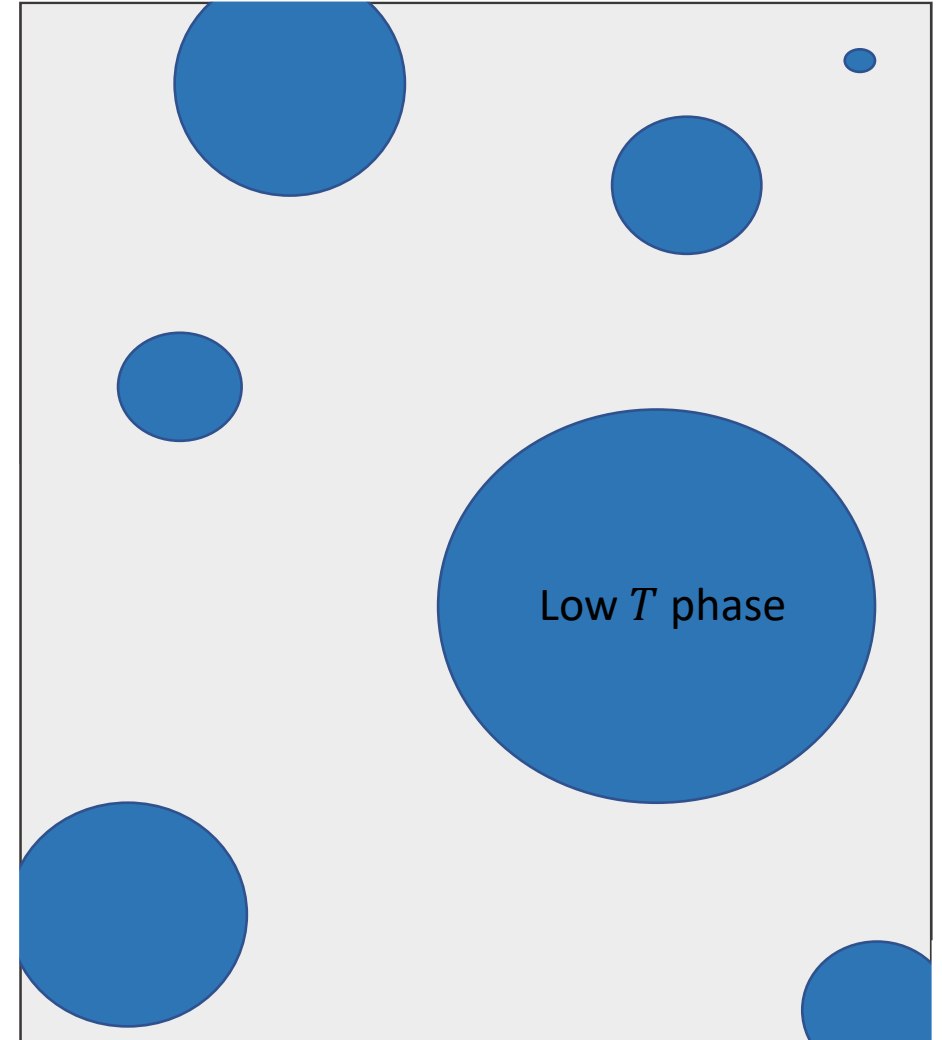
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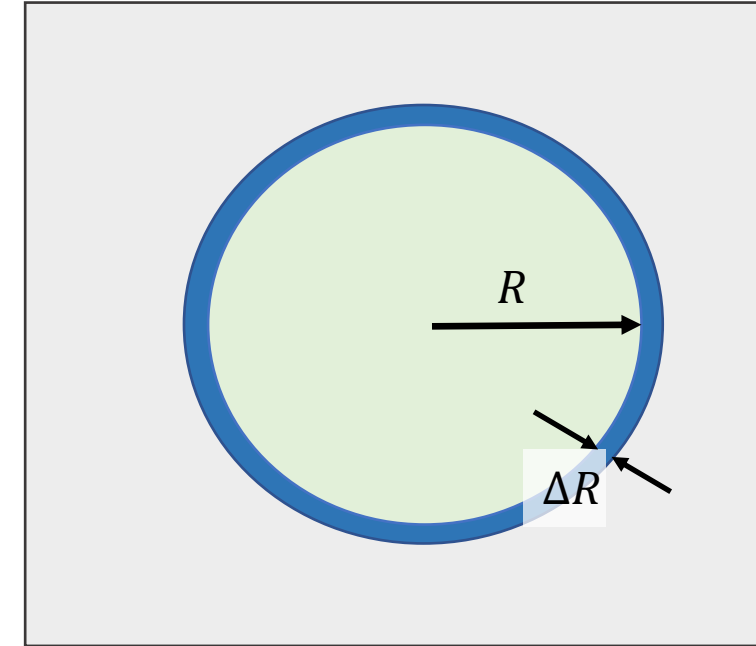
Nucleation of thin-wall bubbles

$$\Gamma \sim T^4 e^{-S_b}$$

- For T close to T_c
 - Transition dominated by nucleation of critical thermal bubbles
 - Critical bubbles have a thin wall ($\Delta R \ll R$)
 - In the thin-wall regime:

$$S_b = \frac{S_3}{T} = \frac{16\pi}{3} \frac{S_1^3}{(\Delta F)^2 T}$$

S_1 : tension of the wall



$$S_3 \approx -\frac{4\pi}{3} R^3 \Delta F + 4\pi R^2 S_1$$

extremize $S_3 \Rightarrow R \sim \frac{S_1}{\Delta F}$

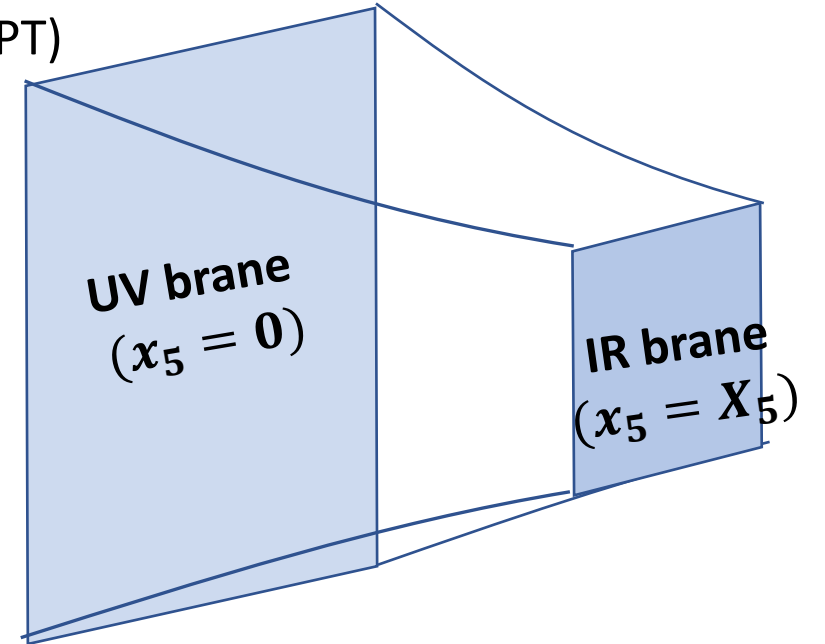
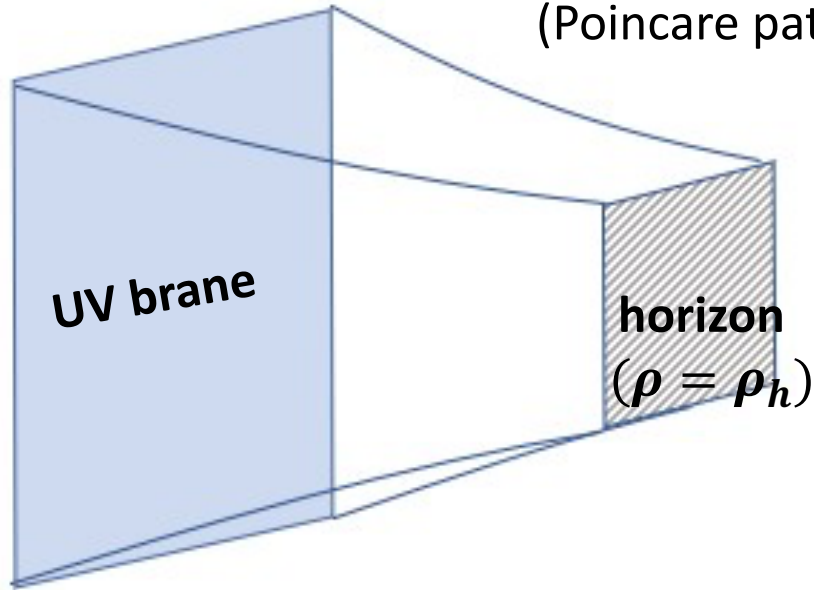
Coleman 1977
Linde 1981

The 5D bounce

- Topology of the phases
- The bounce configuration

AdS-Schwarzschild \rightarrow RS-I

(Poincare patch analog of Hawking-Page PT)



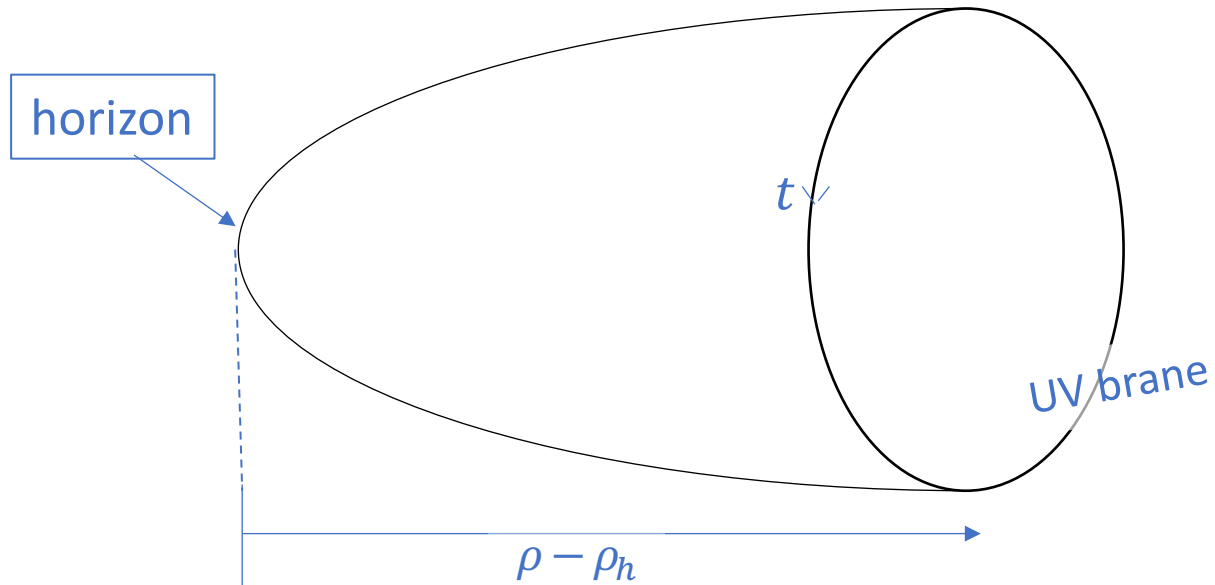
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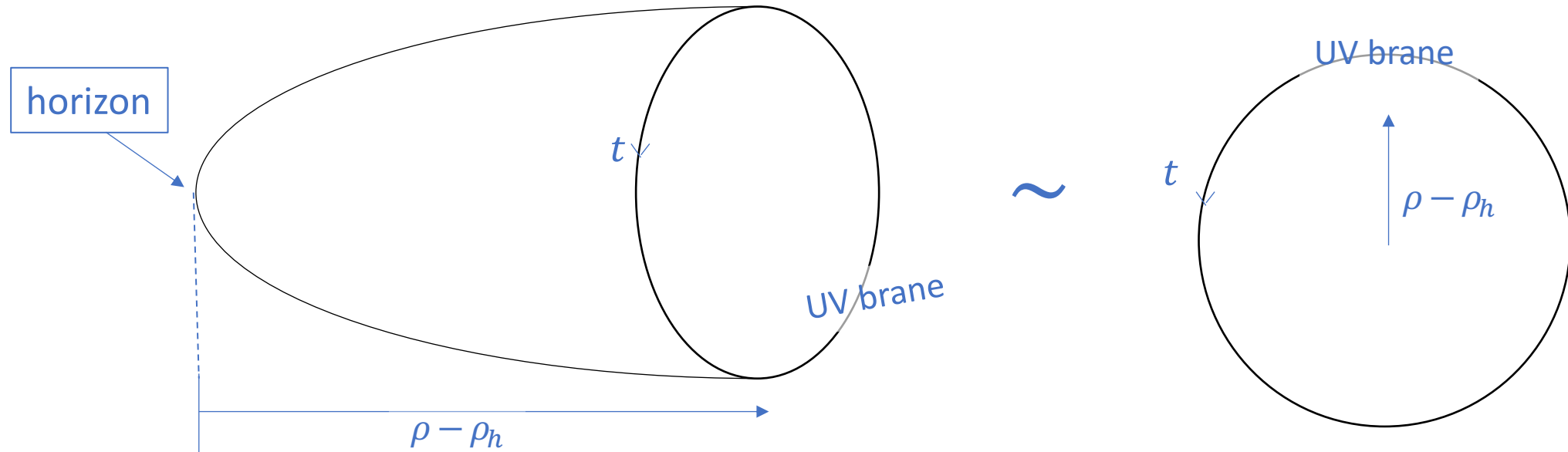
Euclidean AdS-Schwarzschild



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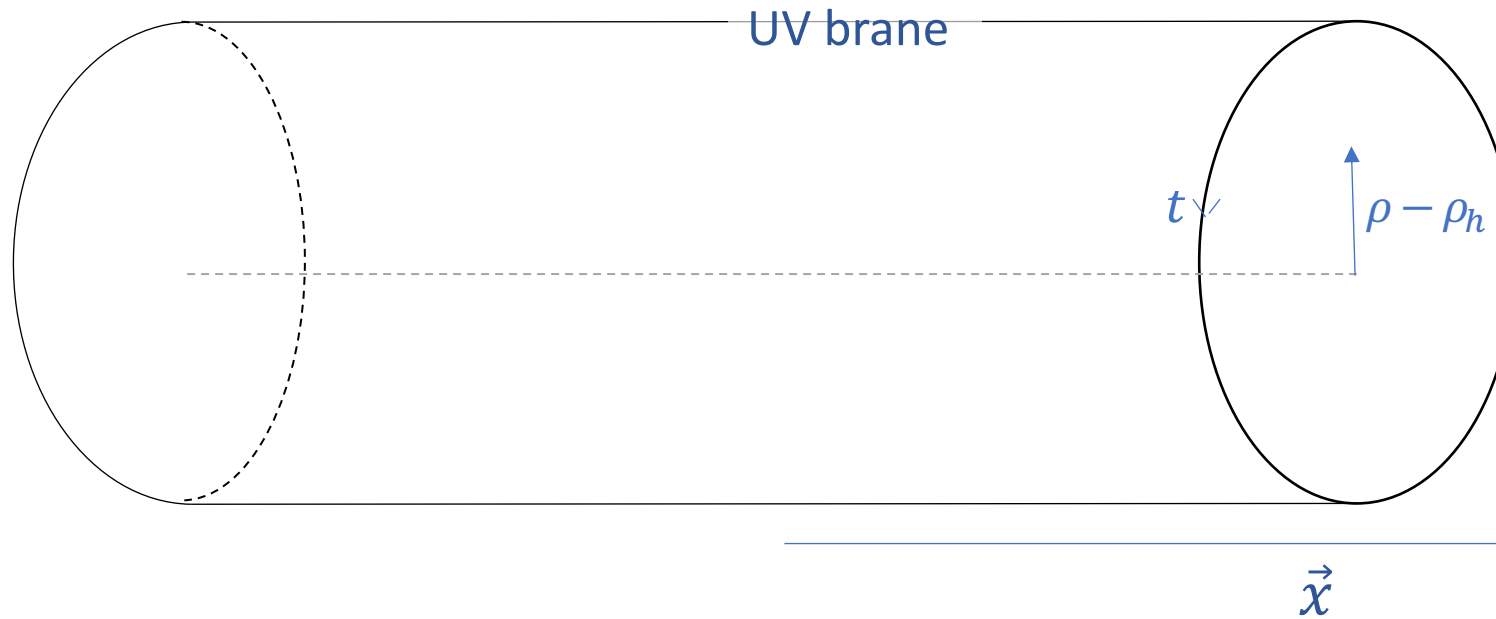
$$t \sim t + \pi/\rho_h$$

Topology of Euclidean AdS-S



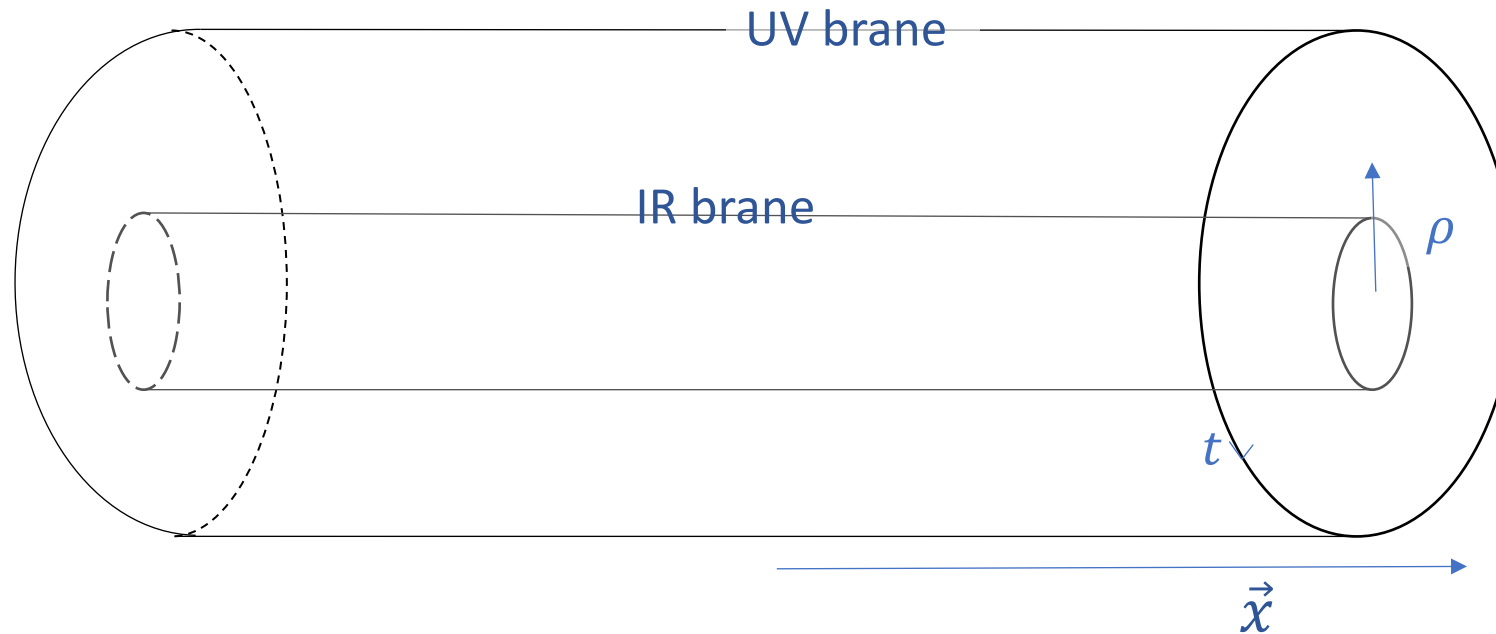
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Topology of AdS-S



$$ds^2 = \left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right) dt^2 + \frac{d\rho^2}{\left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right)} + \rho^2 \sum_i dx_i^2$$

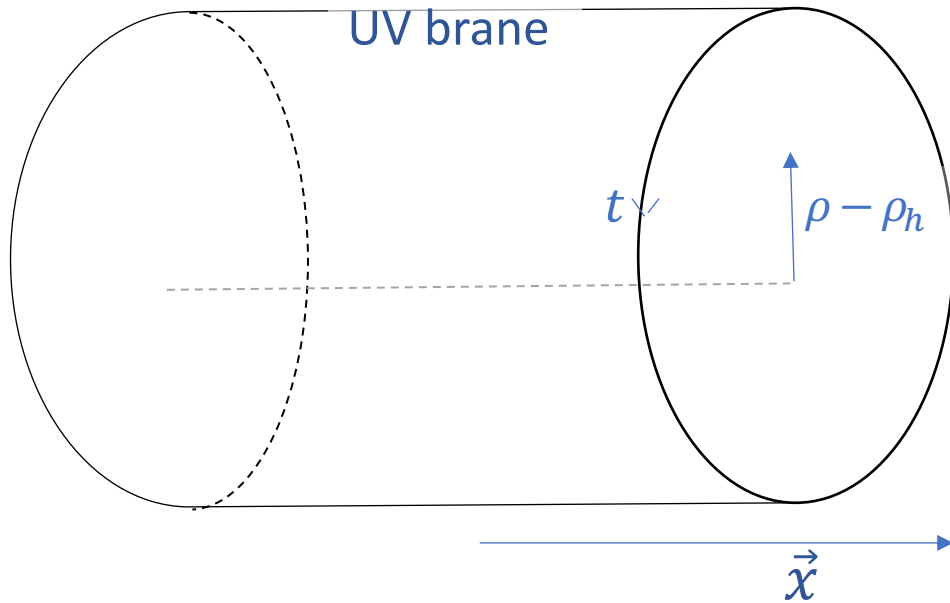
Topology of RS1



$$ds^2 = \rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + \rho^2 \sum_i dx_i^2$$
$$(\rho_{IR} < \rho < \rho_{UV})$$

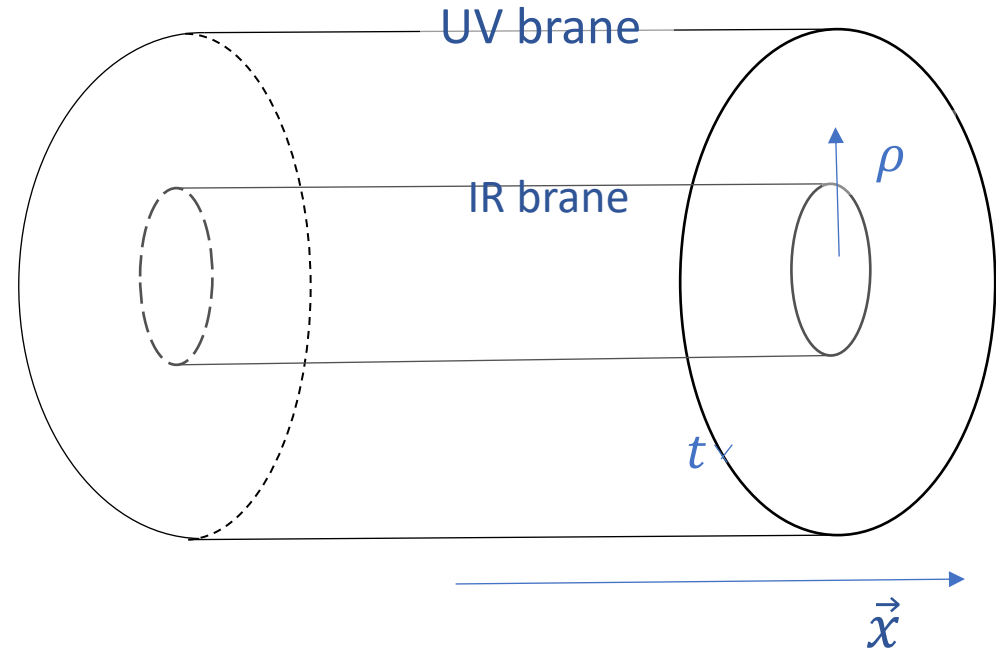
The 5D bounce

AdS-Schwarzschild



$$ds^2 = \left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right) dt^2 + \frac{d\rho^2}{\left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right)} + \rho^2 \sum_i dx_i^2$$

RS 1

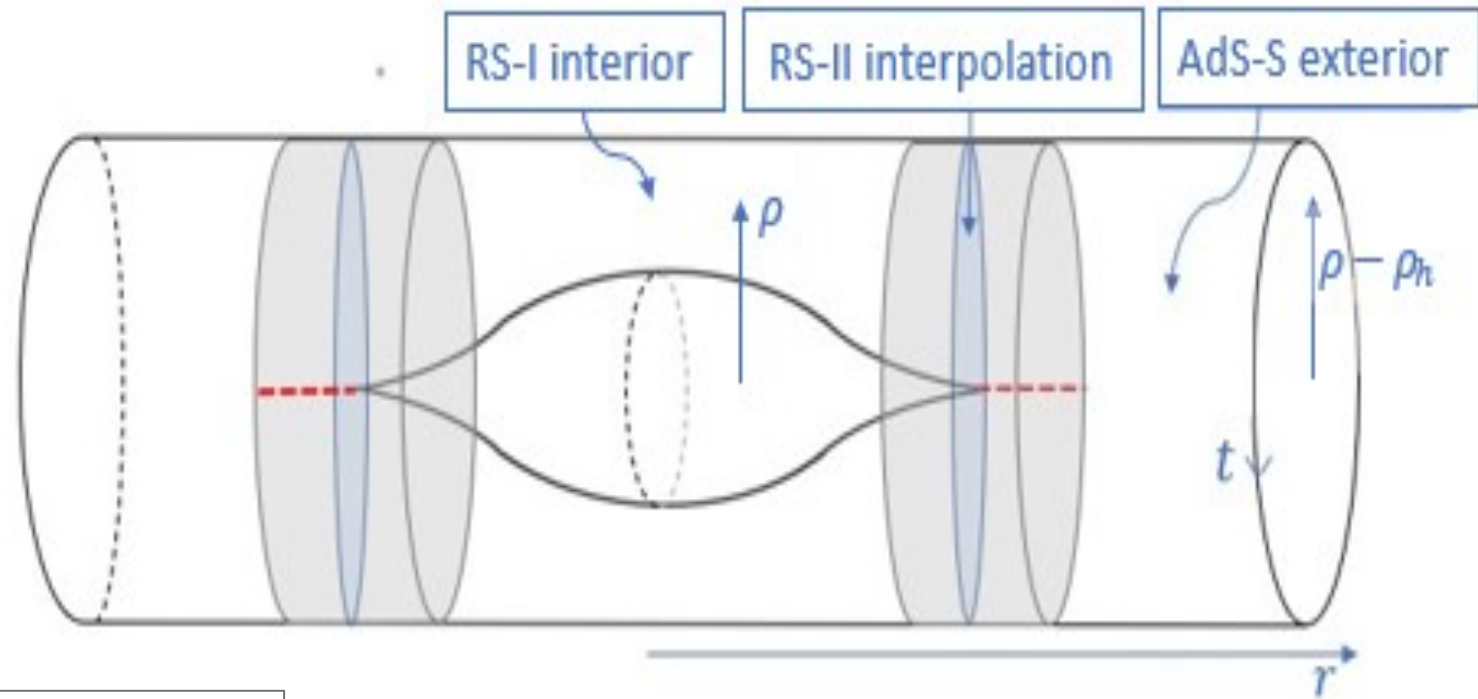


$$ds^2 = \rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + \rho^2 \sum_i dx_i^2$$

$(\rho_{IR} < \rho < \rho_{UV})$

The 5D bounce

- Connect the two phases through their common RS-II limits?



$$\text{AdS-S: } ds^2 = \left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right) dt^2 + \frac{d\rho^2}{\left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right)} + \rho^2 \sum_i dx_i^2$$

$$\text{RS1: } ds^2 = \rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + \rho^2 \sum_i dx_i^2$$

$$(\rho_{IR} < \rho < \rho_{UV})$$

$$\text{RS2 limit: } ds^2 = \rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + \rho^2 \sum_i dx_i^2$$

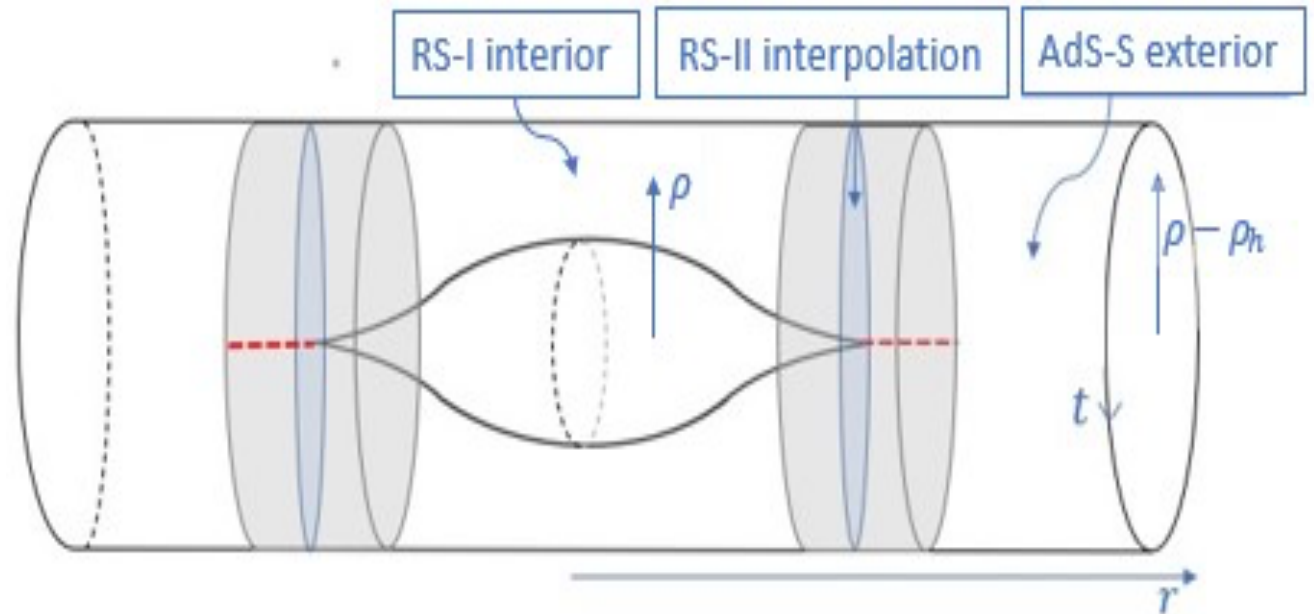
$$(\rho < \rho_{UV})$$

Creminelli, Nicolis & Rattazzi 2002

The 5D bounce

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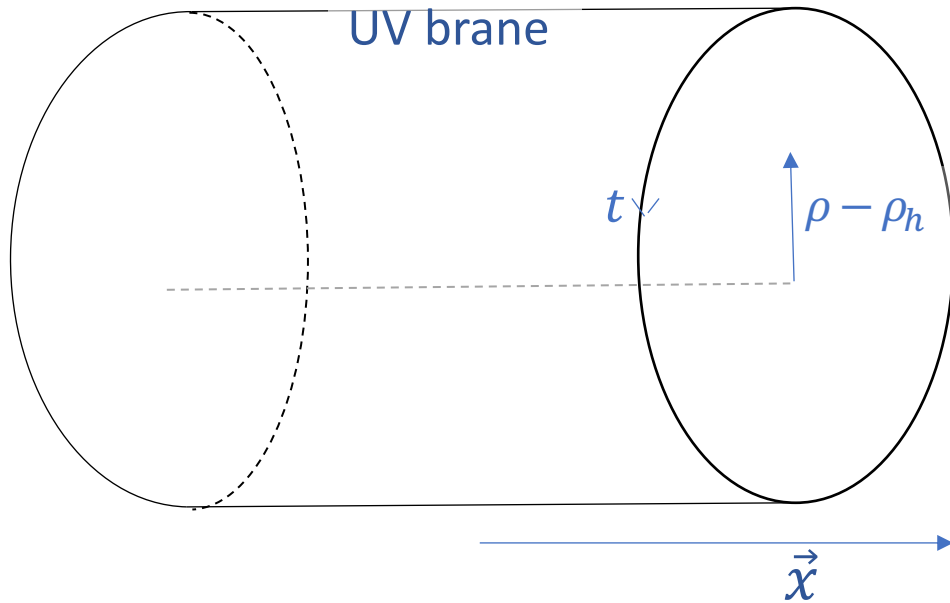
Not fully in 5D EFT control



Creminelli, Nicolis & Rattazzi 2002

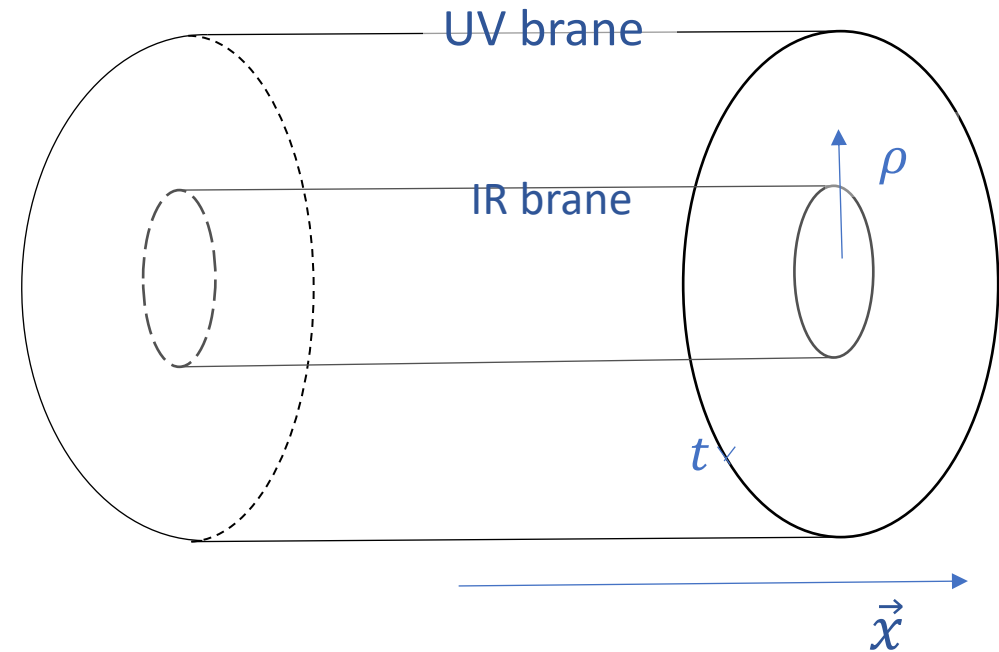
Is there a smooth bounce configuration?

AdS-Schwarzschild



$$ds^2 = \left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right) dt^2 + \frac{d\rho^2}{\left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right)} + \rho^2 \sum_i dx_i^2$$

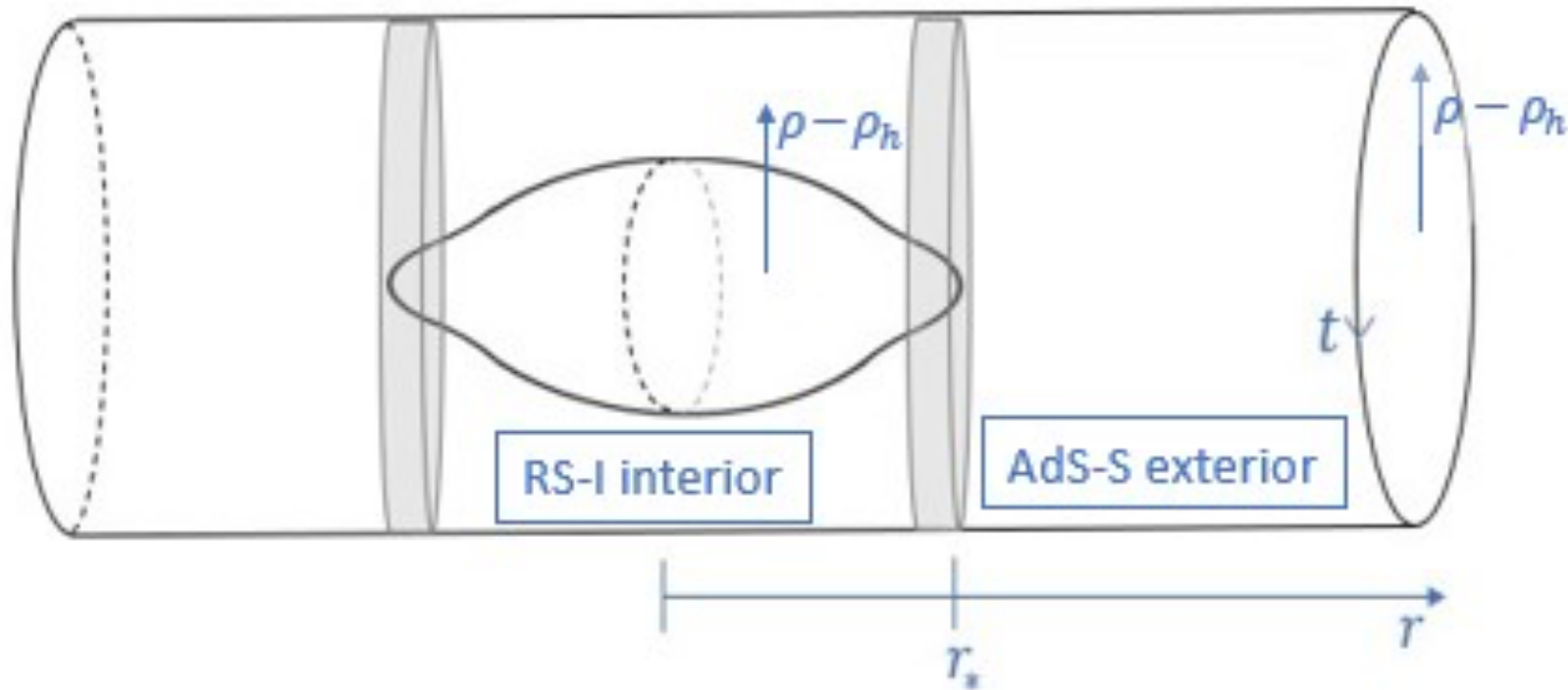
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$$ds^2 = \rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + \rho^2 \sum_i dx_i^2$$

$(\rho_{IR} < \rho < \rho_{UV})$

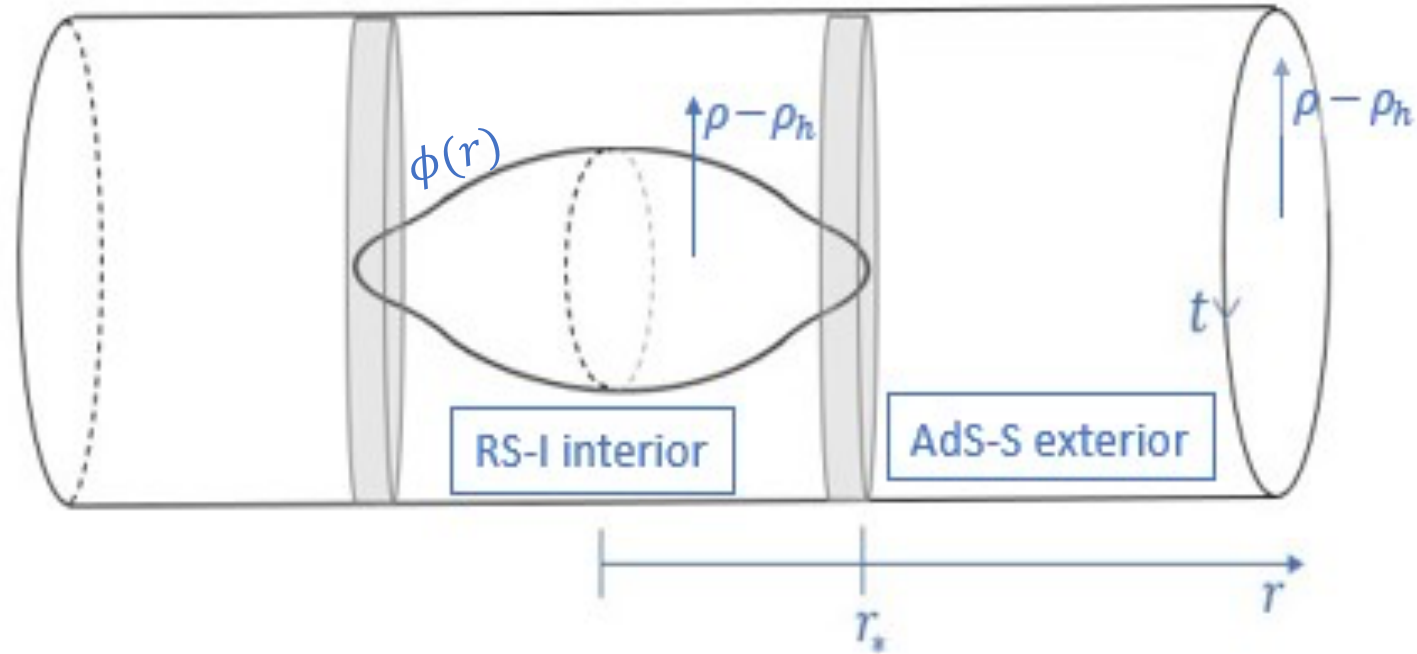
The 5D bounce- smoothness



- ✓ IR-brane can be smoothly sealed at the horizon
- ✓ Smooth, finite curvature, and can be described in 5D EFT

Agashe, Du, M.E., Kumar, Sundrum 2020

Near the horizon



$$ds^2 = \left(\rho^2 - \frac{\rho_h^4}{\rho^2}\right) dt^2 + \frac{d\rho^2}{\left(\rho^2 - \frac{\rho_h^4}{\rho^2}\right)} + \rho^2(dr^2 + r^2 d\Theta^2)$$

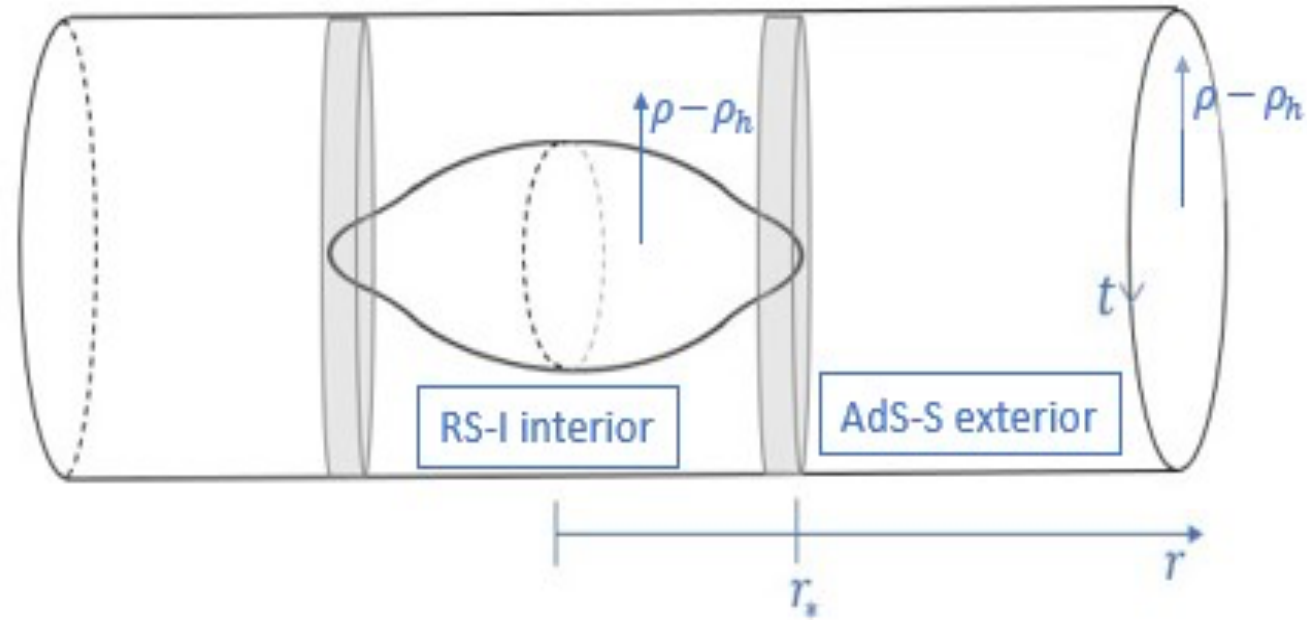
$$\phi(r) < \rho < \rho_{UV}$$

Agashe, Du, M.E., Kumar, Sundrum 2020

Near the horizon

$$ds^2 = \left(\rho^2 - \frac{\rho_h^4}{\rho^2}\right) dt^2 + \frac{d\rho^2}{\left(\rho^2 - \frac{\rho_h^4}{\rho^2}\right)} + \rho^2(dr^2 + r^2 d\Theta^2)$$

$$\phi(r) < \rho < \rho_{UV}$$



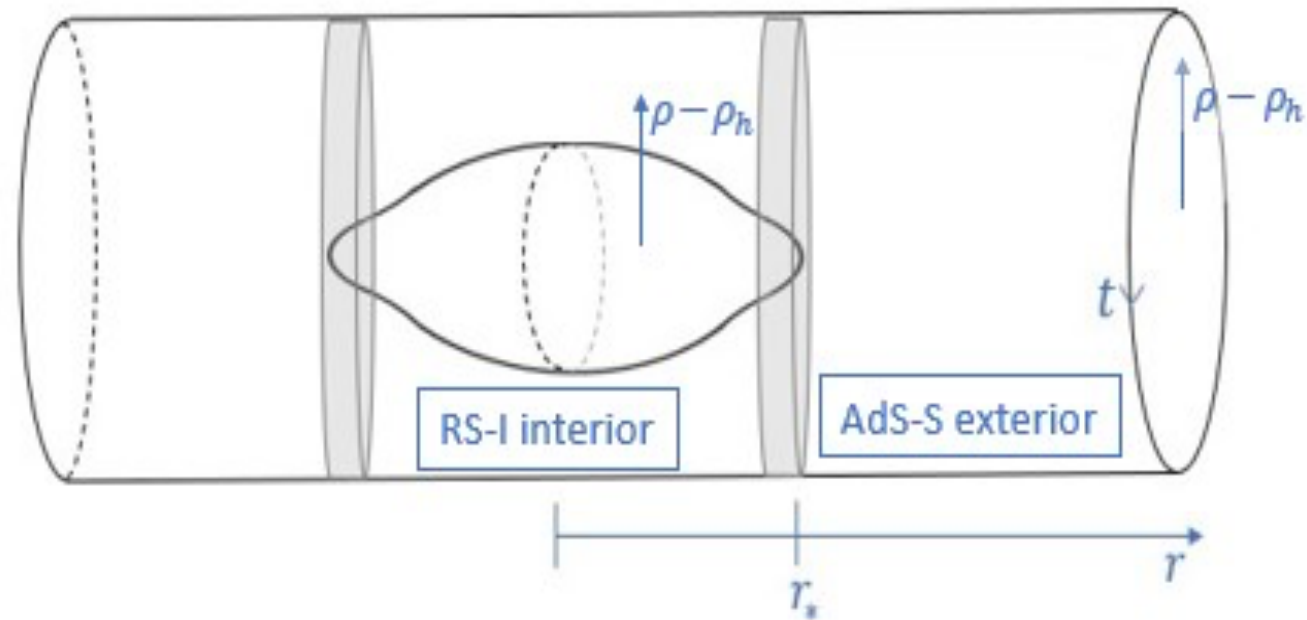
- Condition for smoothness of brane $\left(\frac{d\phi}{dr}\right)^2 \gg \rho_h(\phi - \rho_h)$
- Change coordinate $y = \sqrt{\frac{\phi(r) - \rho_h}{\rho_h}}$: $ds_{ind}^2 \supset 4\rho_h^2 y^2 dt^2 + dy^2$

Agashe, Du, M.E., Kumar, Sundrum 2020

Near the horizon-smoothness

$$ds^2 = \left(\rho^2 - \frac{\rho_h^4}{\rho^2}\right) dt^2 + \frac{d\rho^2}{\left(\rho^2 - \frac{\rho_h^4}{\rho^2}\right)} + \rho^2(dr^2 + r^2 d\Theta^2)$$

$$\phi(r) < \rho < \rho_{UV}$$



- Condition for smoothness of brane $\left(\frac{d\phi}{dr}\right)^2 \gg \rho_h(\phi - \rho_h)$
- Change coordinate $y = \sqrt{\frac{\phi(r) - \rho_h}{\rho_h}}$: $ds_{ind}^2 \supset 4\rho_h^2 y^2 dt^2 + dy^2$ $t \sim t + \pi/\rho_h$

✓ Metric of a disc

Agashe, Du, M.E., Kumar, Sundrum 2020

The 5D bounce solution?

- In principle one can solve the equations of motion for the metric, Goldberger-Wise field, and the brane(s)
- But difficult in practice!
- An analogous 6D problem has been solved in the thin-wall limit, but that does not address the hierarchies. [Aharony, Minwalla, Wiseman 2006](#)

Bounce ansatz

- A smooth ansatz, bounds S_b in the thin-wall regime:

$$S_{b, \text{thin-wall}}^{\text{ansatz}} \geq S_{b, \text{thin-wall}}^{\text{true}}$$

- Problem of finding the bounce is *not* an action- minimization problem.
- But in the thin-wall regime it can be expressed in terms of a minimization problem for S_1

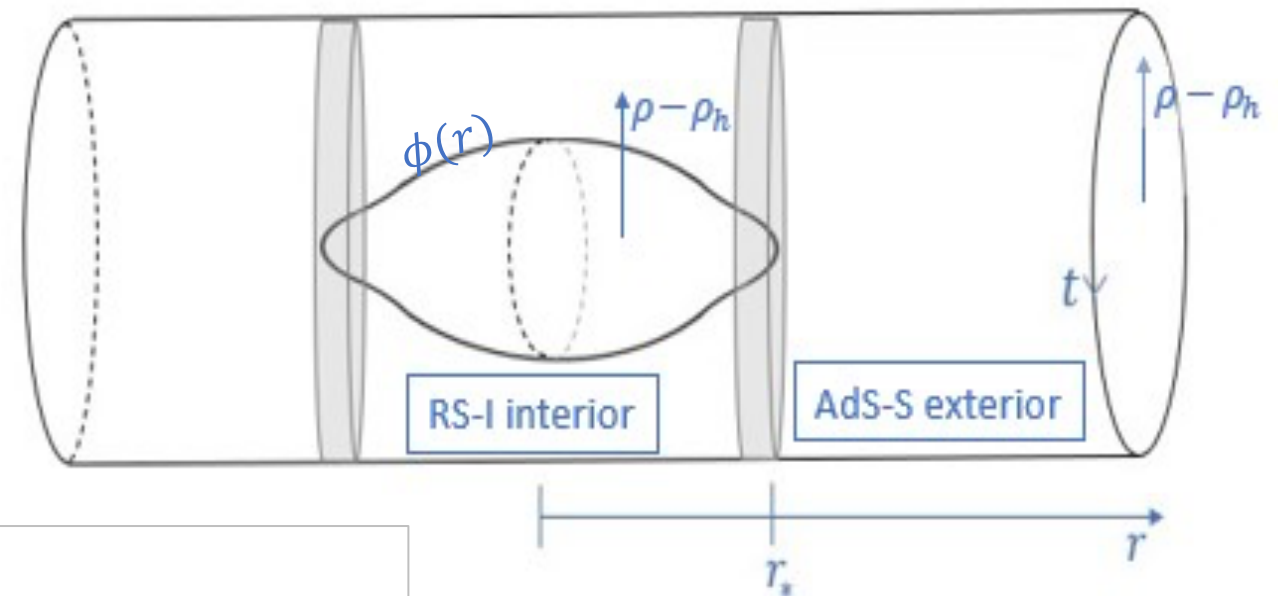
$$S_b = \frac{16\pi}{3} \frac{S_1^3}{(\Delta F)^2 T}$$

- The ansatz provides a reasonable estimate more generally

Bounce ansatz

- Bounds S_b in the thin-wall regime: $S_{b, \text{thin-wall}}^{\text{ansatz}} \geq S_{b, \text{thin-wall}}^{\text{true}}$
- Provides a reasonable estimate more generally

- For $\rho \gg \rho_h$, AdS \approx AdS-S
- For $\rho \sim \rho_h$, bulk geometry \approx AdS-S



$$ds^2 = \left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right) dt^2 + \frac{d\rho^2}{\left(\rho^2 - \frac{\rho_h^4}{\rho^2} \right)} + \rho^2 (dr^2 + r^2 d\Omega^2)$$

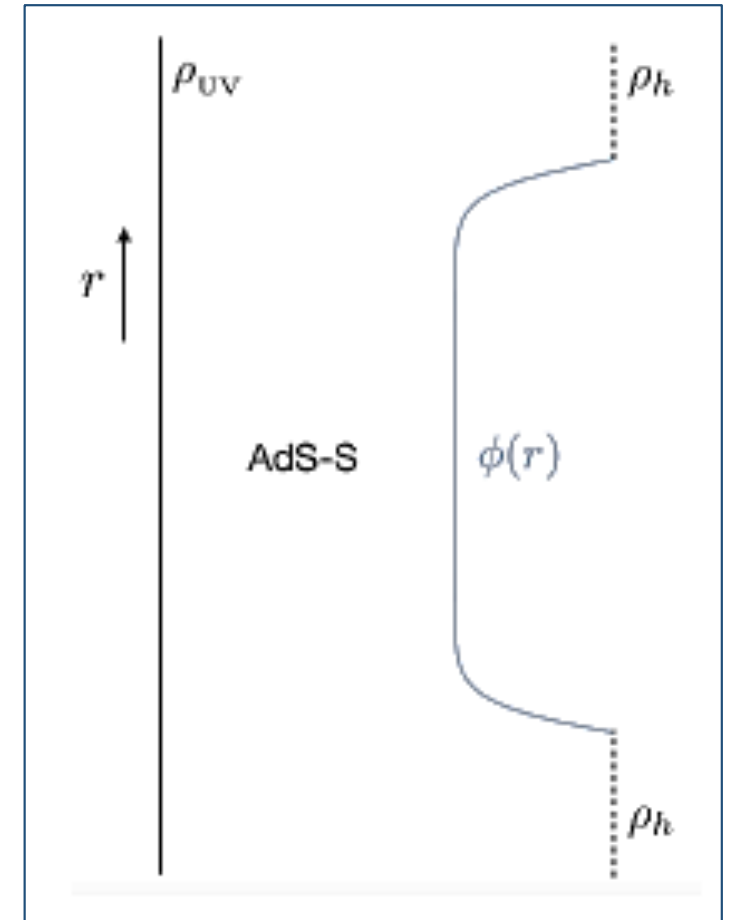
$$\phi(r) < \rho < \rho_{UV}$$

Agashe, Du, M.E., Kumar, Sundrum 2020

Bounce ansatz

$$S_\phi = \frac{4\pi}{T} \int dr r^2 \left[2M_5^3 \left(\frac{2}{\phi^4 - \rho_h^4 + \phi'^2} \left[2\phi(\rho_h^4 - \phi^4) \frac{\phi'}{r} + (6\phi^4 - 2\rho_h^4) \phi'^2 - 2\phi \frac{\phi'^3}{r} + (\phi^4 - \rho_h^4)(4\phi^4 - 2\rho_h^4 - \phi\phi'') \right] + \rho_h^4 - 2\phi^4 - 6\phi^2 (\phi^4 - \rho_h^4 + \phi'^2)^{1/2} \right) + V_{\text{eff}}(\phi) \right]$$

- Optimize in the the smooth ansatz class to get the optimum bound
- For $\phi' \ll \phi^2$ and $\rho_h \ll \phi$, reduces to standard 2 derivative radion action



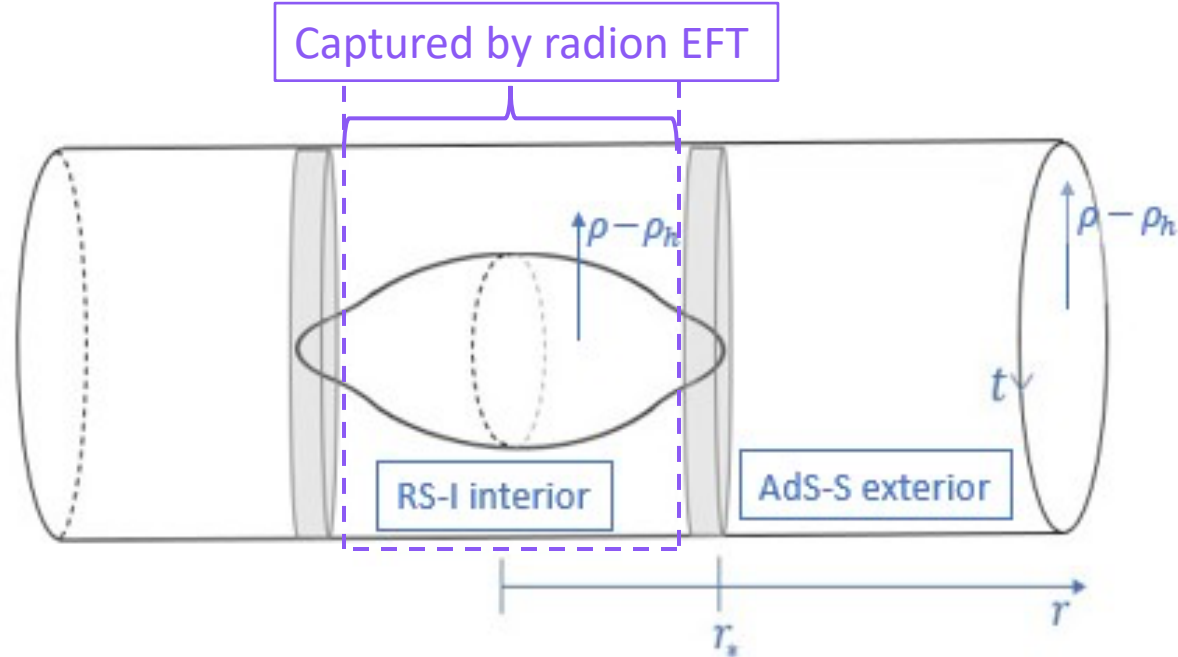
Agashe, Du, M.E., Kumar, Sundrum 2020

Radion dominance approximation

- For small ϵ, λ the region $\varphi \gg \rho_h$ gives the parametrically leading contribution to S_b
- This part can be computed using 2-derivative radion EFT:

$$\mathcal{L}_{radion} = \frac{3N^2}{4\pi^2} \left((\partial\phi)^2 - \lambda\phi^4 \left(1 - \frac{1}{1 + \epsilon/4} \left(\frac{\phi}{\langle\phi\rangle} \right)^\epsilon \right) \right)$$

- Our ansatz and earlier works agree in this region
- The parametrically sub-leading corrections to radion dominance can be quantitatively important



✓ Theory

- Equilibrium description
- Bubble nucleation rate
 - The 5D bounce

Application to cosmology of composite Higgs

- PT in the minimal model
 - (Slow: empty universe or large supercooling and dilution)
- Faster transition rate? Beyond the minimal model
- Supercooled PT

Phenomenology

- Gravitational waves
- Dilution of matter and baryogenesis

Goldberger-Wise Stabilization: minimal model

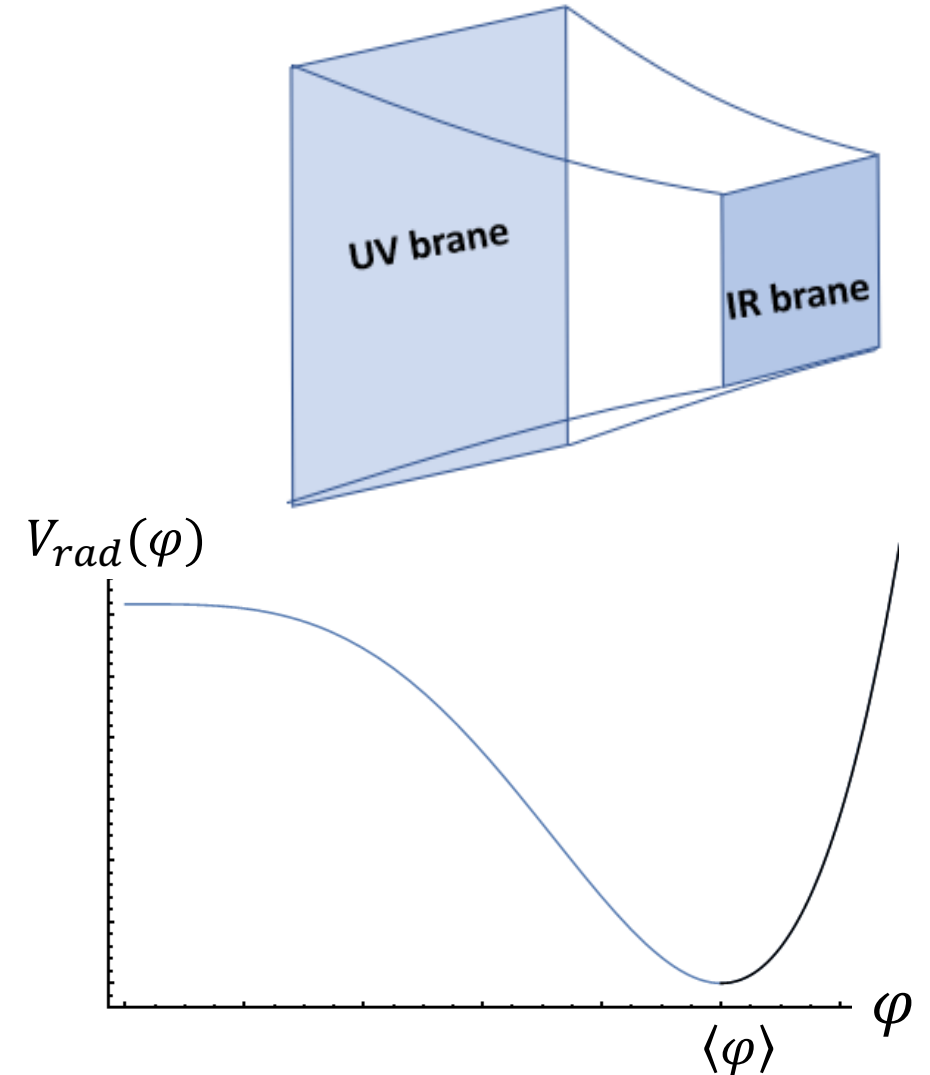
Goldberger & Wise 1999

- With a bulk scalar (Goldberger-Wise) field, minimally with the potential:

$$V_{\text{GW,bulk}}(\Phi) = \frac{1}{2} m^2 \Phi^2$$

$$V_{\text{radion}}(\varphi) = \frac{3N^2}{4\pi^2} \lambda \varphi^4 \left(1 - \omega \left(\frac{\varphi}{\Lambda_{\text{UV}}} \right)^\epsilon \right)$$

Hierarchy is set mainly by $\epsilon \approx \frac{m^2}{4k^2}$: $\ln \frac{M_{\text{Pl}}}{\text{TeV}} \sim \frac{1}{\epsilon}$



Bubble nucleation rate- minimal model

- Bubble nucleation rate:

$$\Gamma \sim T^4 e^{-S_b}$$

- PT completes if $\Gamma \gtrsim H^4$ ($H \sim \frac{T_C^2}{M_{Pl}}$)

$$S_b \lesssim 4 \ln \frac{M_{Pl}}{T_C} \approx 140$$

- For T close to T_C (thin-wall):

$$S_b \approx \frac{N^2}{(\epsilon\lambda)^{3/4}} \frac{4 T_C/T}{(1 - (T/T_C)^4)^2} \gtrsim 10 \frac{N^2}{(\lambda\epsilon)^{3/4}}$$

- Action enhanced by large N and small ϵ
- For $\epsilon \sim \frac{1}{25}$ PT does not complete near T_C

Creminelli, Nicolis & Rattazzi 2002

Beyond the minimal model

- Is it possible to have a prompt/faster phase transition?
- In the minimal model the parameter ϵ that is setting the hierarchy (and hence H) suppresses the rate:

$$S_b \sim 10 \frac{N^2}{\epsilon^{3/4}} > 4 \ln \frac{M_{Pl}}{\text{TeV}} \sim \frac{4}{\epsilon}$$

- Have a small ϵ in the UV, which becomes (effectively) larger in the IR?

$$S_b \sim 10 \frac{N^2}{\epsilon_{IR}^{3/4}} \stackrel{?}{<} 4 \ln \frac{M_{Pl}}{\text{TeV}} \sim \frac{4}{\epsilon_{UV}}$$

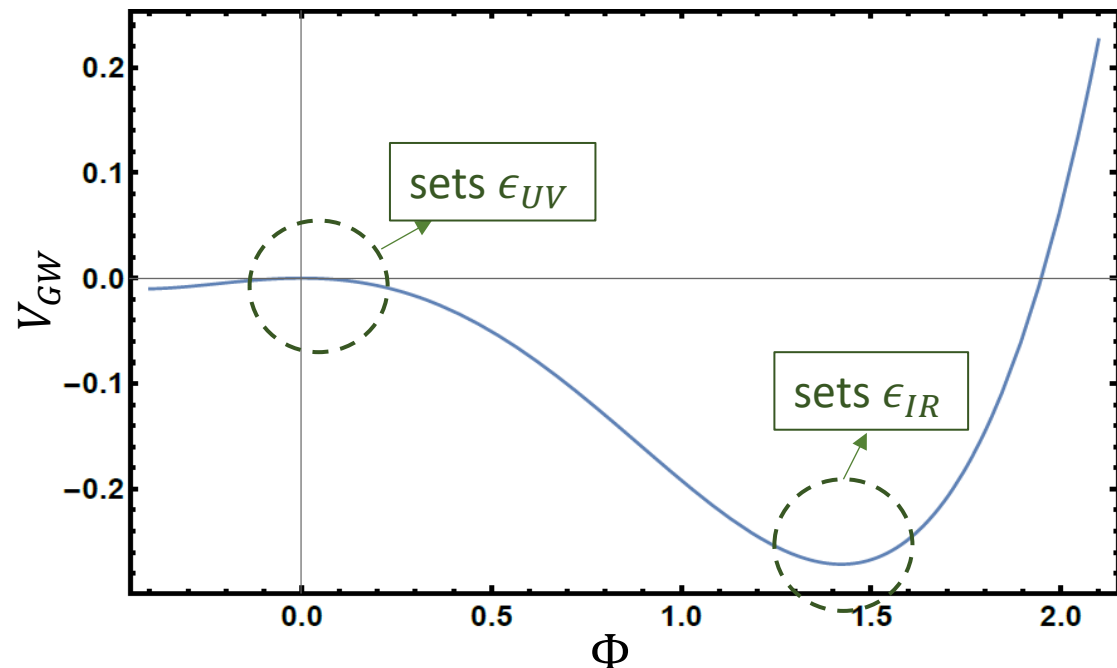
Beyond the minimal model

Agashe, Du, M.E., Kumar, Sundrum 2020

- Goldberger-Wise field with self- interactions:

$$V_{GW}(\Phi) = \frac{1}{2} m^2 \Phi^2 + \frac{1}{3!} \eta \Phi^3 + \frac{1}{4!} \kappa \Phi^4$$

- Effective mass: $m_{eff}^2 \sim V''_{GW}(\Phi)$



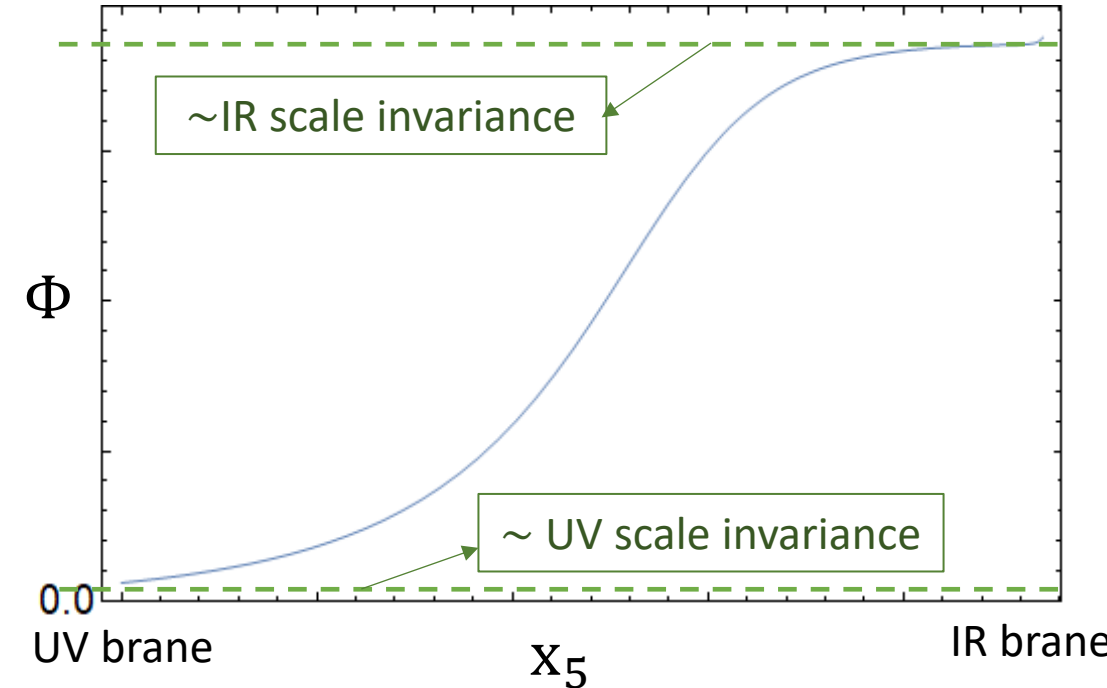
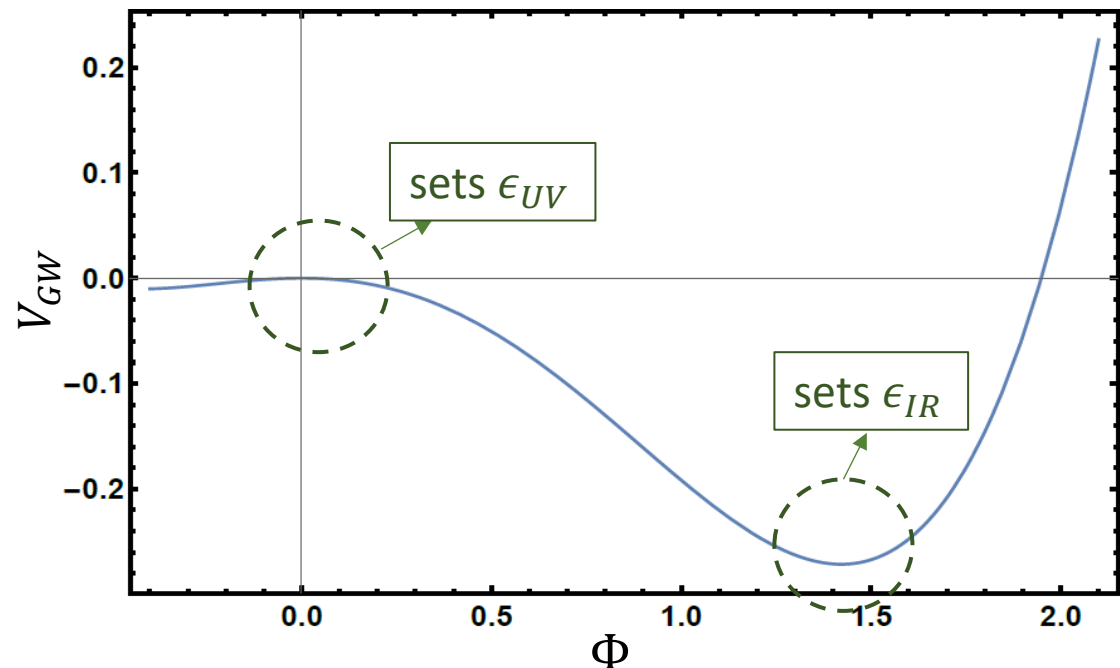
Beyond the minimal model

Agashe, Du, M.E., Kumar, Sundrum 2020

- Goldberger-Wise field with self-interactions:

$$V_{GW}(\Phi) = \frac{1}{2} m^2 \Phi^2 + \frac{1}{3!} \eta \Phi^3 + \frac{1}{4!} \kappa \Phi^4$$

- Effective mass: $m_{eff}^2 \sim V_{GW}''(\Phi)$



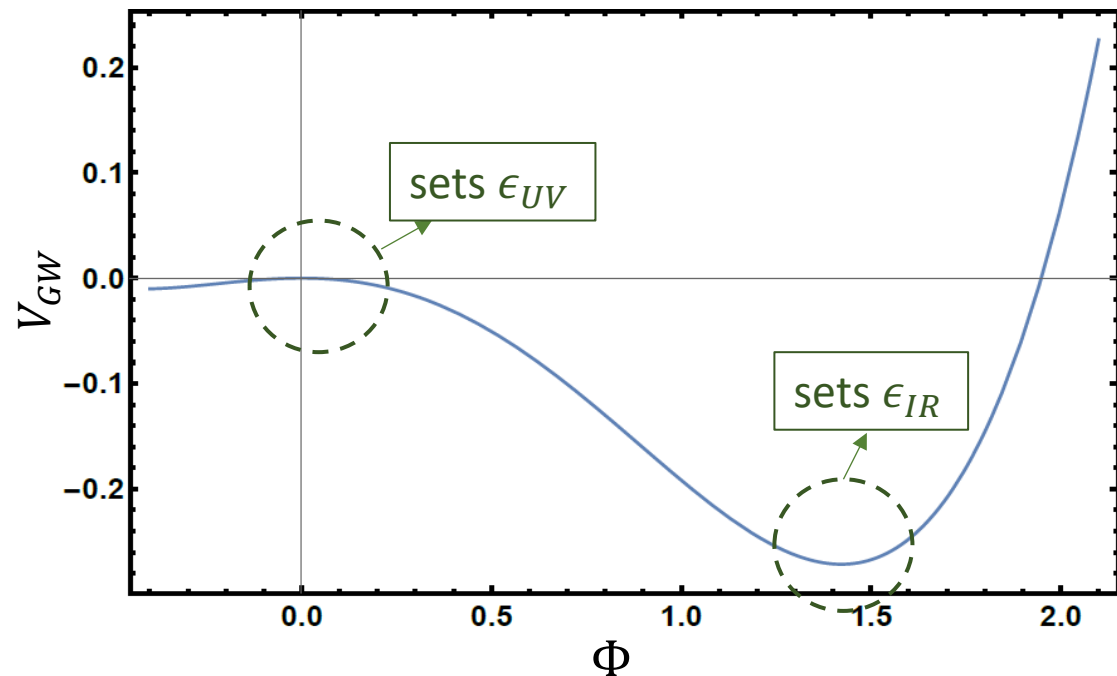
Two fixed points

Agashe, Du, M.E., Kumar, Sundrum
2019, 2020

- Goldberger-Wise field with self-interactions:

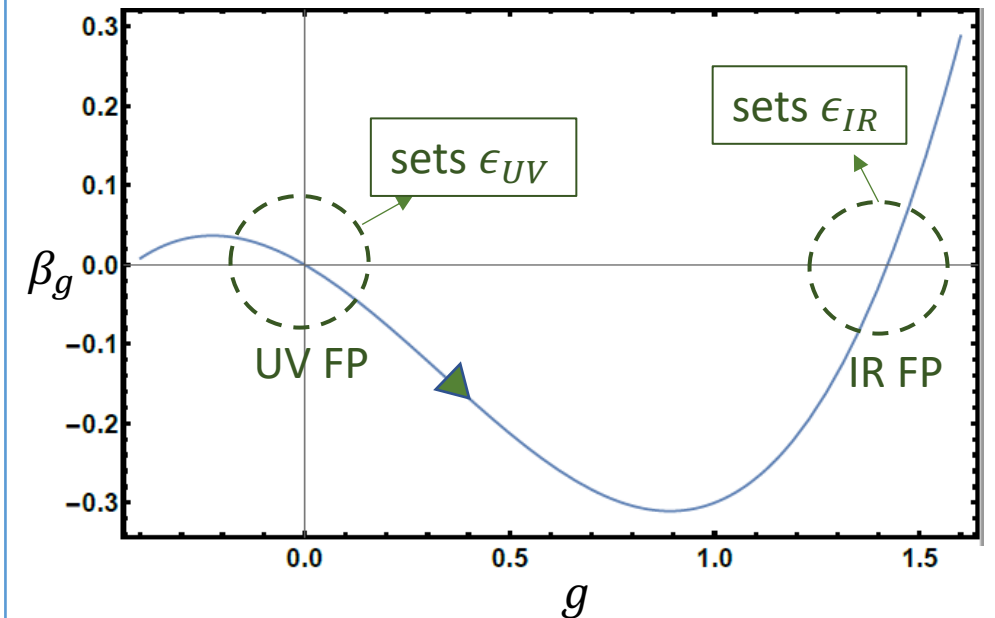
$$V_{GW}(\Phi) = \frac{1}{2} m^2 \Phi^2 + \frac{1}{3!} \eta \Phi^3 + \frac{1}{4!} \kappa \Phi^4$$

- Effective mass: $m_{eff}^2 \sim V_{GW}''(\Phi)$



Dual picture:

- RGE with UV and IR fixed points
- ϵ_{UV} and ϵ_{IR} are the anomalous dimensions corresponding to the UV and the IR fixed points.



Beyond the minimal model

Agashe, Du, M.E., Kumar, Sundrum 2020

$$V_{\text{radion}}(\varphi) \approx \begin{cases} \lambda_{\text{UV}} \varphi^4 \left(1 + \omega \left(\frac{\varphi}{\Lambda_{\text{UV}}} \right)^{-\epsilon_{\text{UV}}} \right) & \varphi > \varphi_{\text{int}} \\ \lambda_{\text{IR}} \varphi^4 \left(1 - \omega_{\text{IR}} \left(\frac{\varphi}{\varphi_{\text{int}}} \right)^{\epsilon_{\text{IR}}} \right) & \varphi < \varphi_{\text{int}} \end{cases}$$

$$\ln \frac{\varphi_{\text{int}}}{\langle \varphi \rangle} \sim \frac{1}{\epsilon_{\text{IR}}} \quad \ln \frac{\Lambda_{\text{UV}}}{\varphi_{\text{int}}} \sim \frac{1}{\epsilon_{\text{UV}}}$$

- ϵ_{UV} controls the radion potential for large φ (in the UV, important for the hierarchies)
- ϵ_{IR} controls the radion potential for small φ (in the IR, important for the PT dynamics)

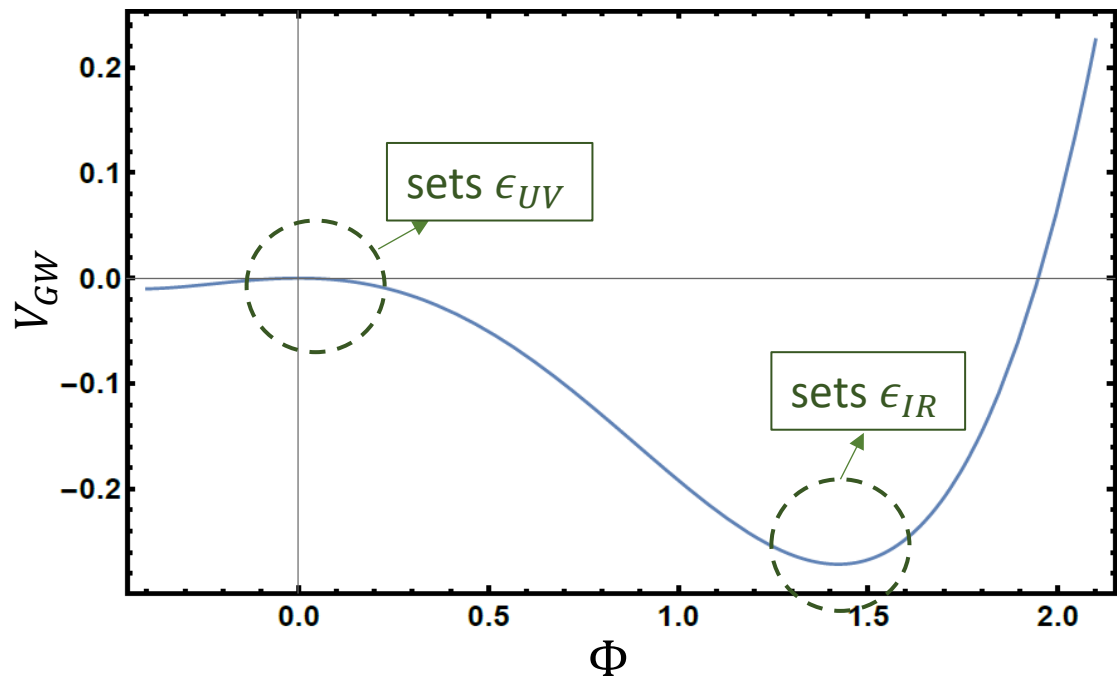
Beyond the minimal model

Agashe, Du, M.E., Kumar, Sundrum 2020

- Goldberger-Wise field with self- interactions:

$$V_{GW}(\Phi) = \frac{1}{2} m^2 \Phi^2 + \frac{1}{3!} \eta \Phi^3 + \frac{1}{4!} \kappa \Phi^4$$

- Effective mass: $m_{eff}^2 \sim V_{GW}''(\Phi)$



Majid Ekhterachian (UMD)

- Nucleation rate enhanced if ϵ_{IR} not too small.
- PT can complete near T_C for parameters :
$$\epsilon_{IR} = \frac{1}{2}, \quad \lambda = 0.5, \quad N \approx 2$$
- Marginally in theoretical control.

Supercooled Phase Transition

- How much supercooling?
- Supercooling and dilution

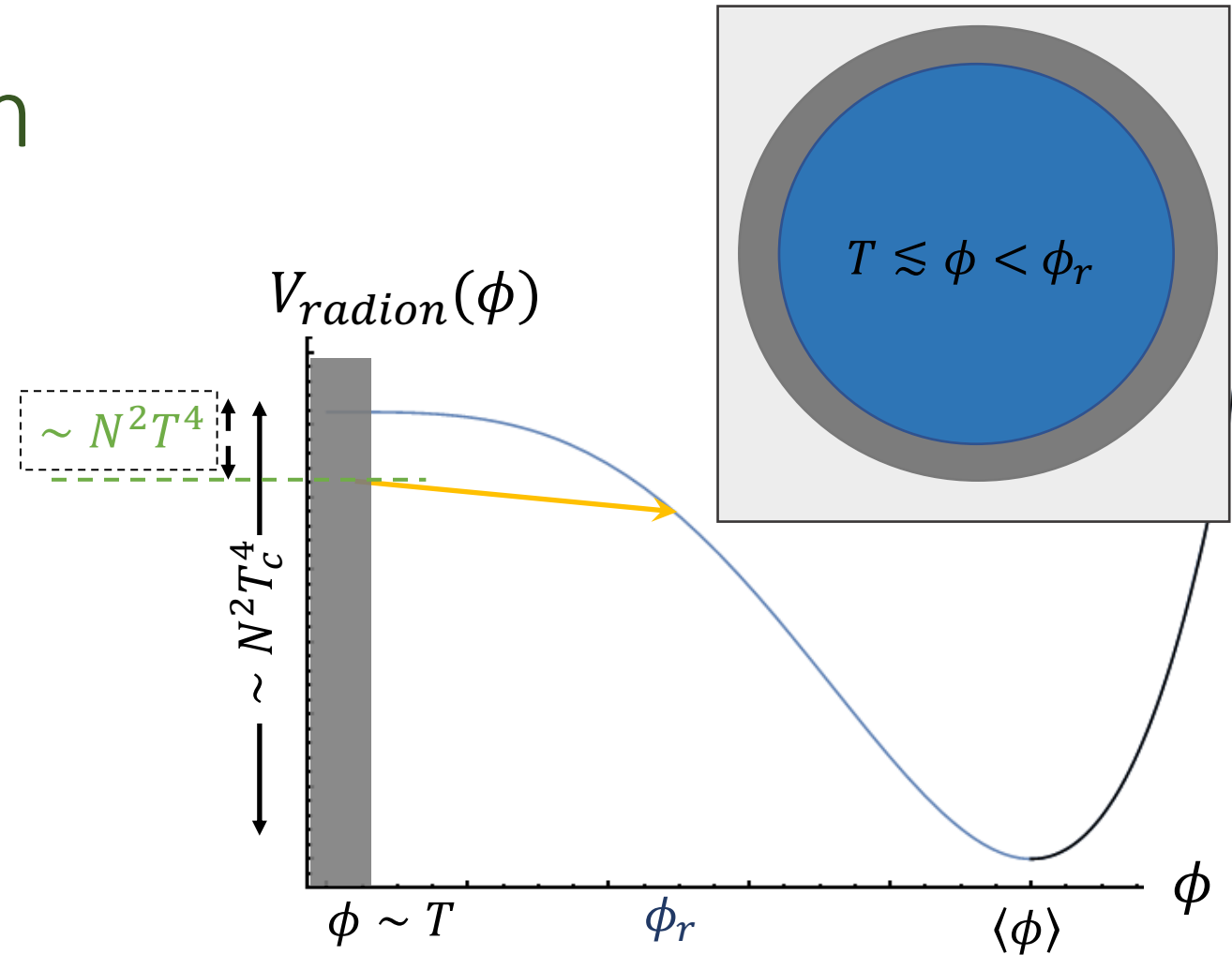
Supercooled transition

- For small T/T_C :

$$\left(\ln \frac{T_C}{T} \gtrsim \frac{1}{\epsilon_{IR}}\right) \quad S_{\text{bounce}} \sim \frac{N^2}{\lambda^{\frac{3}{4}}}$$

No enhancement by small ϵ_{IR}

- Larger $\epsilon_{IR} \rightarrow$ less supercooling.
- A period of inflation: dilution of baryon and DM number densities



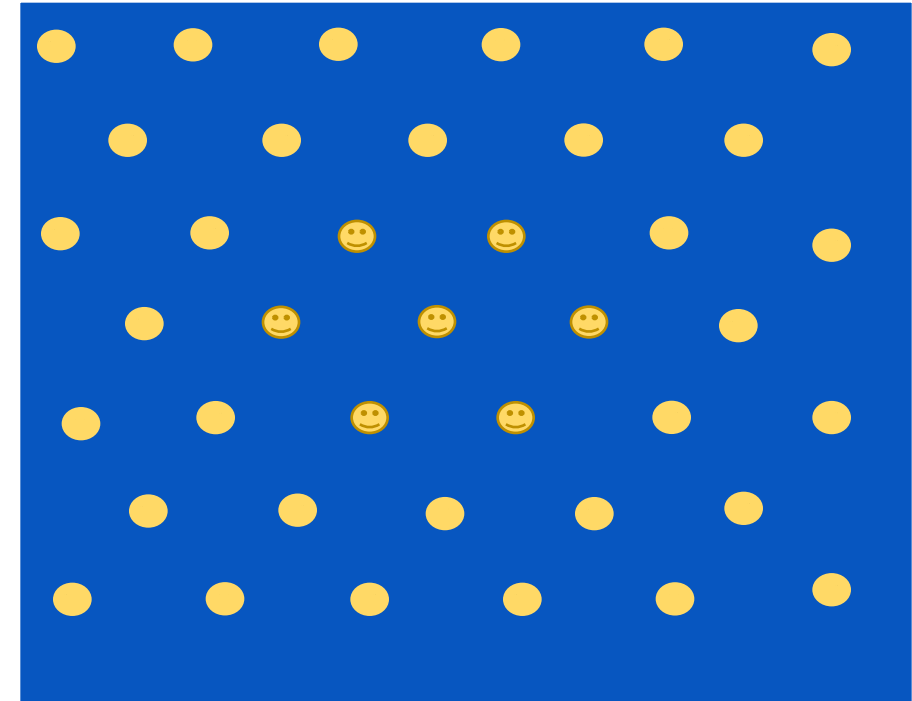
Randall & Servant 2006
Konstandin & Servant 2011

Supercooled PT- dilution

- Before the phase transition

$$\eta_{\text{before}} \equiv \frac{n_{\Delta B}(T_c)}{n_{\gamma}(T_c)}$$

$T \approx T_c$



Supercooled PT- dilution

- Before the phase transition

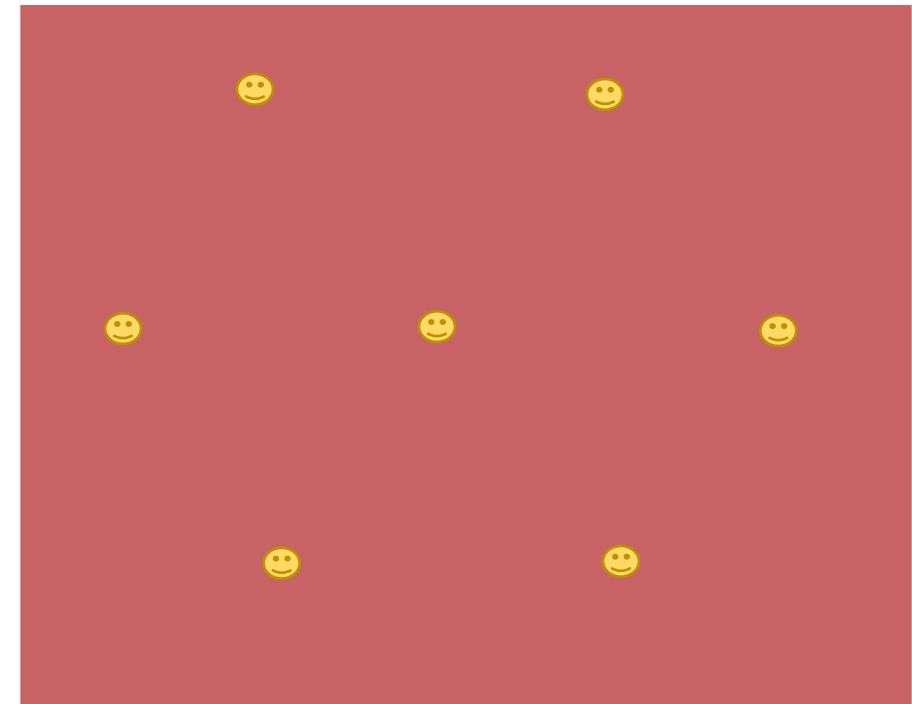
$$\eta_{\text{before}} \equiv \frac{n_{\Delta B}(T_c)}{n_{\gamma}(T_c)}$$

- As supercooling happens:

$$n_{\Delta B} \propto T^3, \quad n_{\gamma} \propto T^3$$

$$\eta(T_n) = \eta_{\text{before}}$$

$$T \ll T_c$$



Supercooled PT- dilution

- Before the phase transition:

$$\eta_{\text{before}} \equiv \frac{n_{\Delta B}(T_c)}{n_\gamma(T_c)}$$

- As supercooling happens:

$$n_{\Delta B} \propto T^3, \quad n_\gamma \propto T^3$$

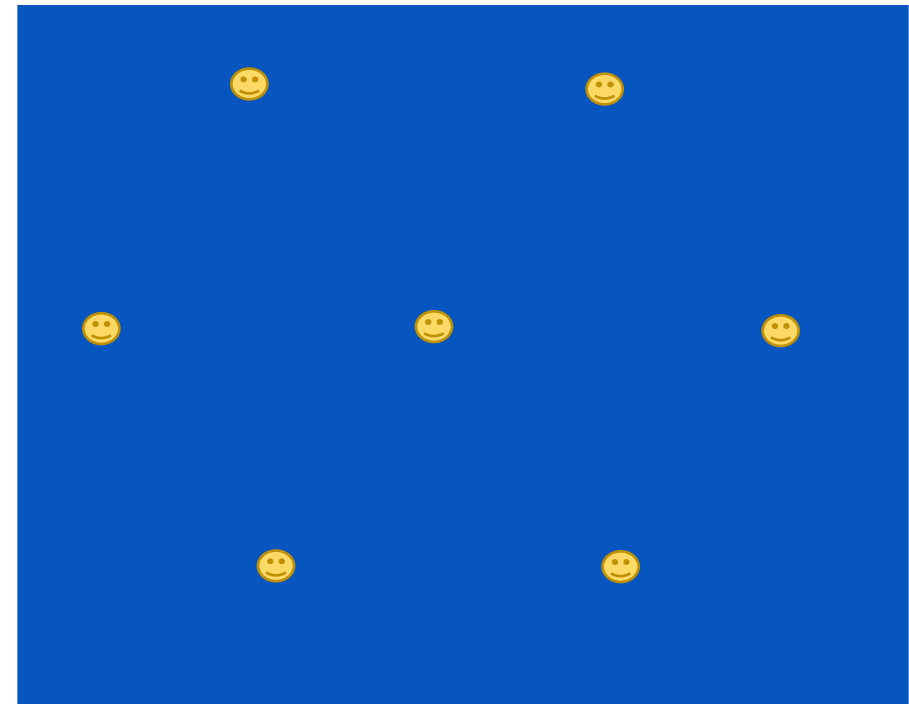
$$\eta(T_n) = \eta_{\text{before}}$$

- After the PT completes, universe is reheated to $T \sim T_c$

$$n_{\Delta B, \text{after}} \approx n_{\Delta B}(T_n), \quad n_\gamma \sim T_c^3$$

$$\eta_{\text{after}} \sim \eta_{\text{before}} (T_n/T_c)^3$$

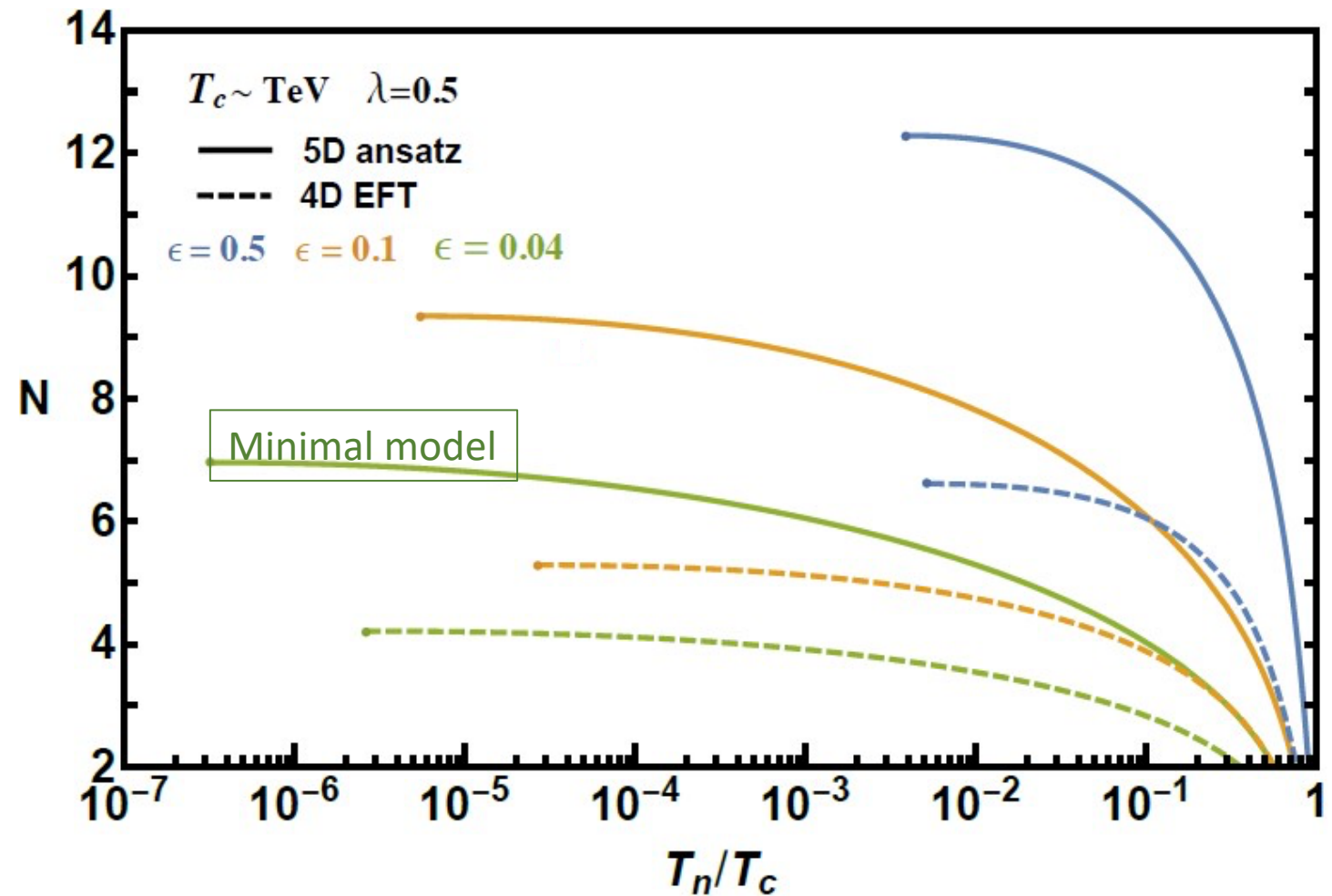
$$T \sim T_c$$



Results

Larger ϵ_{IR} :

- ✓ Less supercooling
- ✓ Less dilution of preexisting matter
- ✓ Larger N allowed to complete the PT



Gravitational wave signal

Stochastic gravitational wave background

- Strength and the spectrum of gravitational waves from PT depend on β_{GW} :

$$f_{\text{peak}} \sim 0.03 \text{ mHz} \frac{T_c}{\text{TeV}} \frac{\beta_{\text{GW}}}{H}$$

$$\Omega_{\text{GW}} \sim 10^{-5} \left(\frac{H}{\beta_{\text{GW}}} \right)^2$$

Caprini et al, 2015

- $1/\beta$ is the duration of the PT:

$$\frac{\beta_{\text{GW}}}{H} = - \frac{T}{\Gamma} \frac{d\Gamma}{dT} \Big|_{T_n}$$

Turner, Weinberg & Widrow 1992
Kosowsky & Turner 1992
Kosowsky, Turner & Watkins 1992

- For generic PTs (not the models considered here):

$$\Gamma \sim T^4 e^{-S_b}$$

$$\frac{\beta_{\text{GW}}}{H} \approx \frac{dS_b}{d \ln T} \Big|_{T_n} \sim S_b(T_n) \sim 100$$

- Even larger if PT completes close to T_c : $\frac{\beta_{\text{GW}}}{H} \sim S_b(T_n)/(1 - T_n/T_c)$

Stochastic gravitational wave background

- Strength and the spectrum of gravitational waves from PT depend on β_{GW} :

- $1/\beta$ is the duration of the PT: $\frac{\beta_{GW}}{H} = -\frac{T}{\Gamma} \frac{d\Gamma}{dT} \Big|_{T_n}$

Turner, Weinberg & Widrow 1992
 Kosowsky & Turner 1992
 Kosowsky, Turner & Watkins 1992

- For generic PTs (not the models considered here):

$$\frac{\beta_{GW}}{H} \approx \frac{dS_b}{d \ln T} \Big|_{T_n} \sim S_b(T_n) \sim 100$$

For composite Higgs models:

- S_b independent of T in the supercooled limit (result of 4D scale invariance):

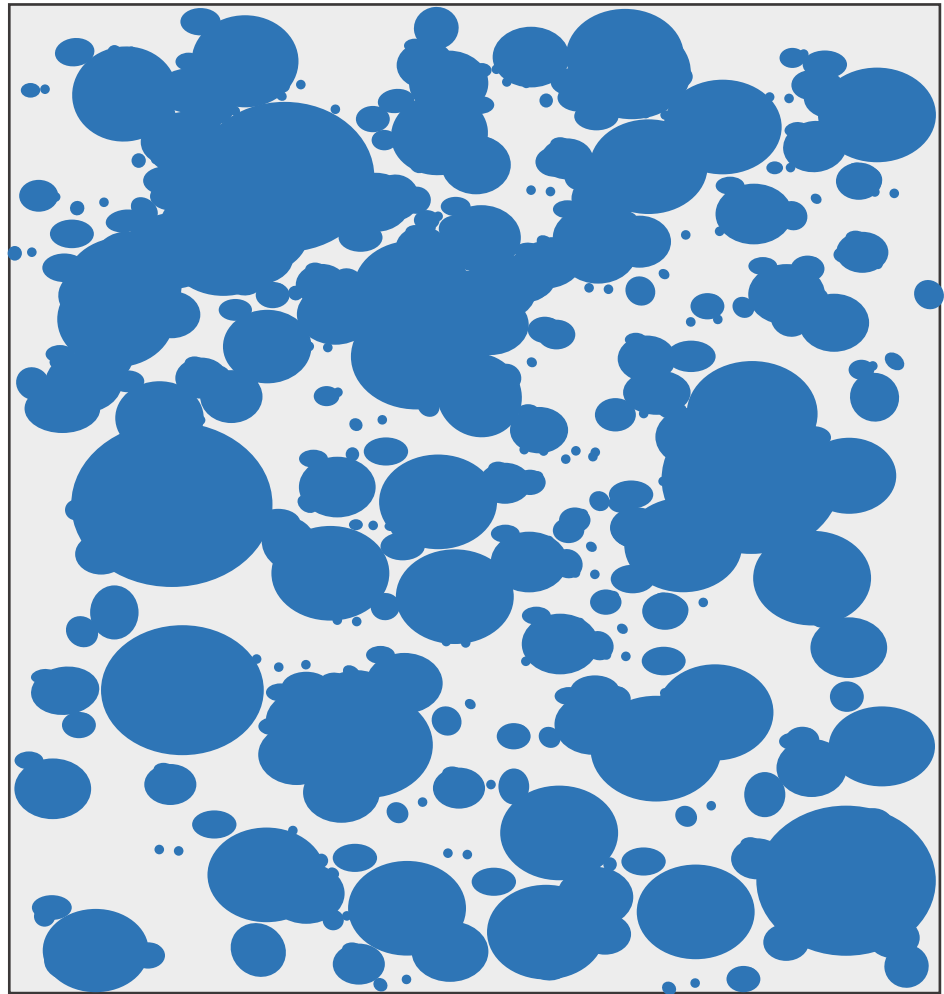
- $\frac{\beta}{H} \approx -4 + 3 \epsilon_{IR} \left(\frac{T_n}{\lambda^{1/4} \langle \phi \rangle} \right)^{\epsilon_{IR}} \ln \left(\frac{M_P}{T_C} \right)$

- Temperature dependence controlled by ϵ_{IR}

- Small β_{GW} , for small ϵ_{IR}

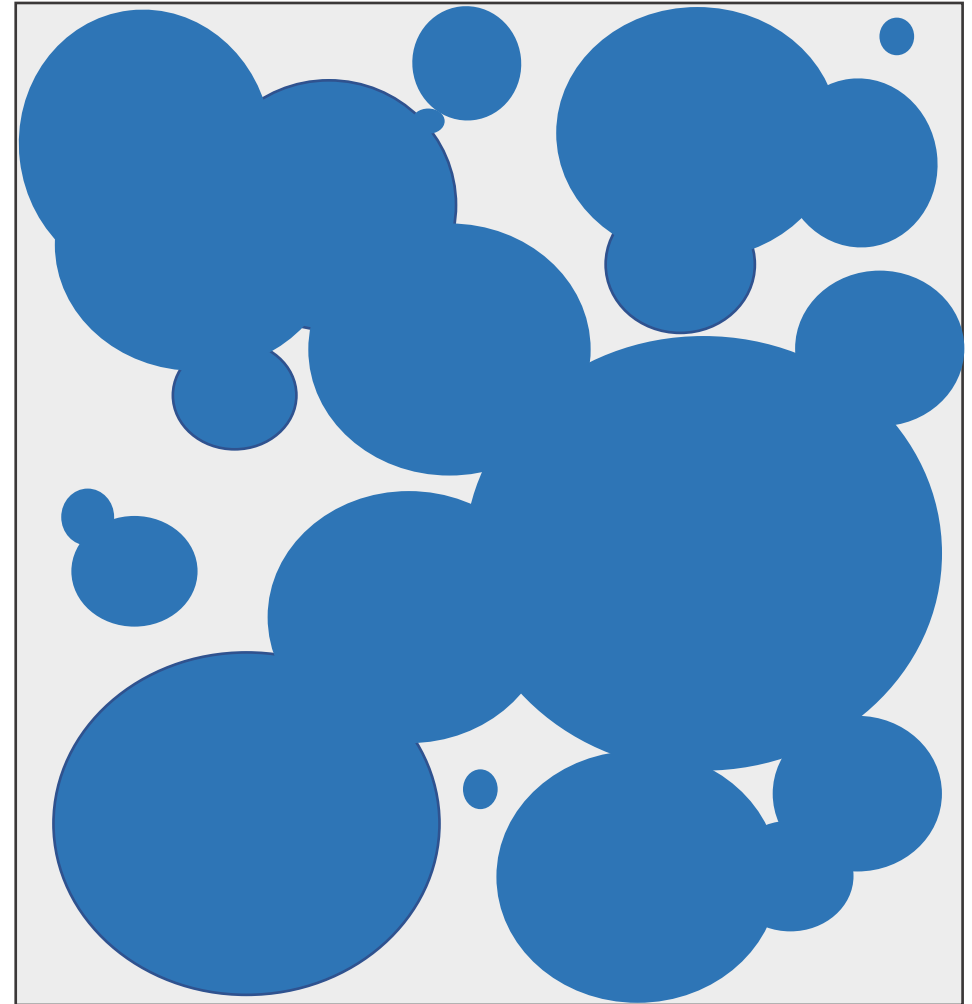
Agashe, Du, M.E., Kumar, Sundrum 2019
 Konstandin & Servant 2011

Larger β_{GW}



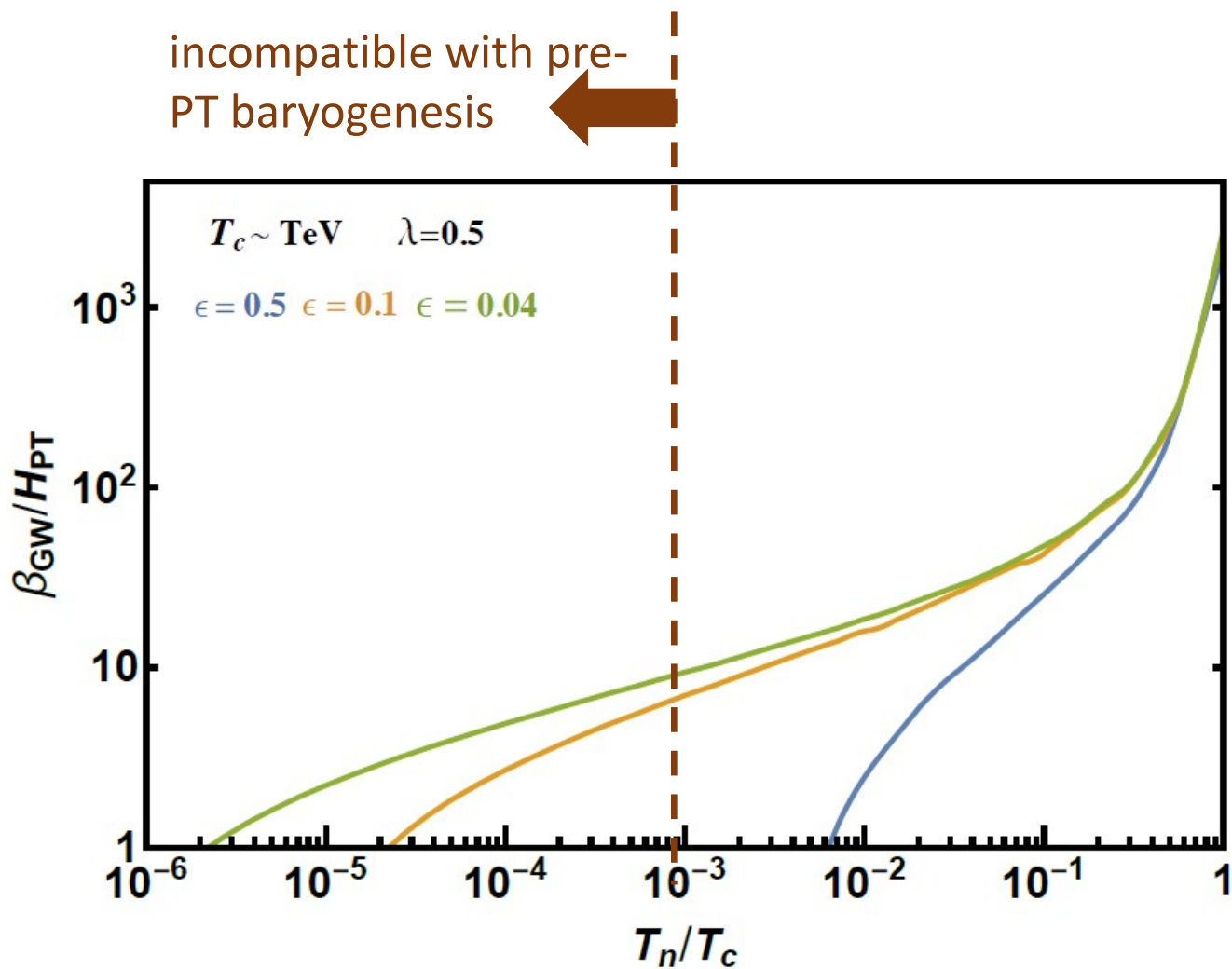
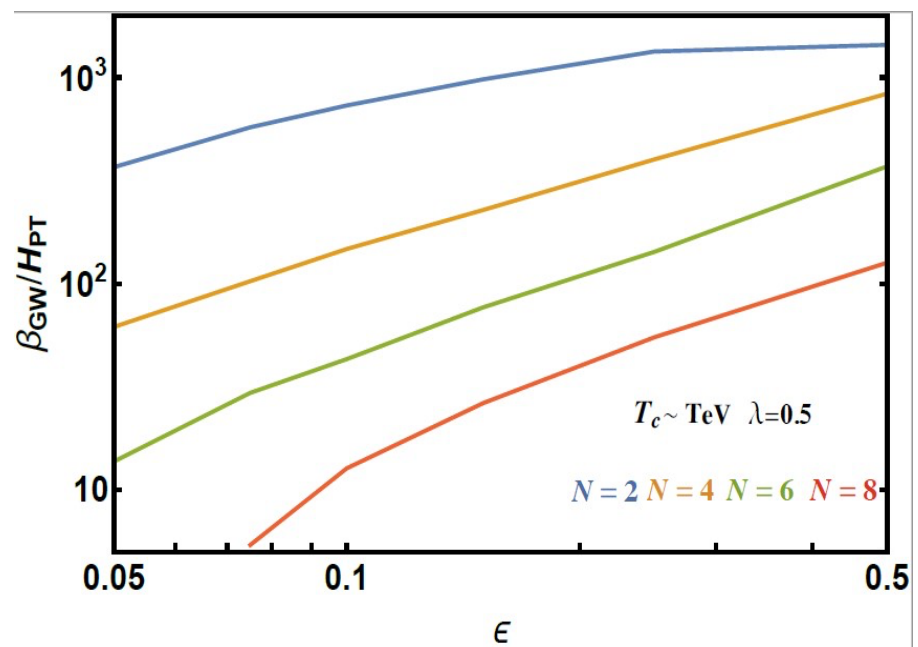
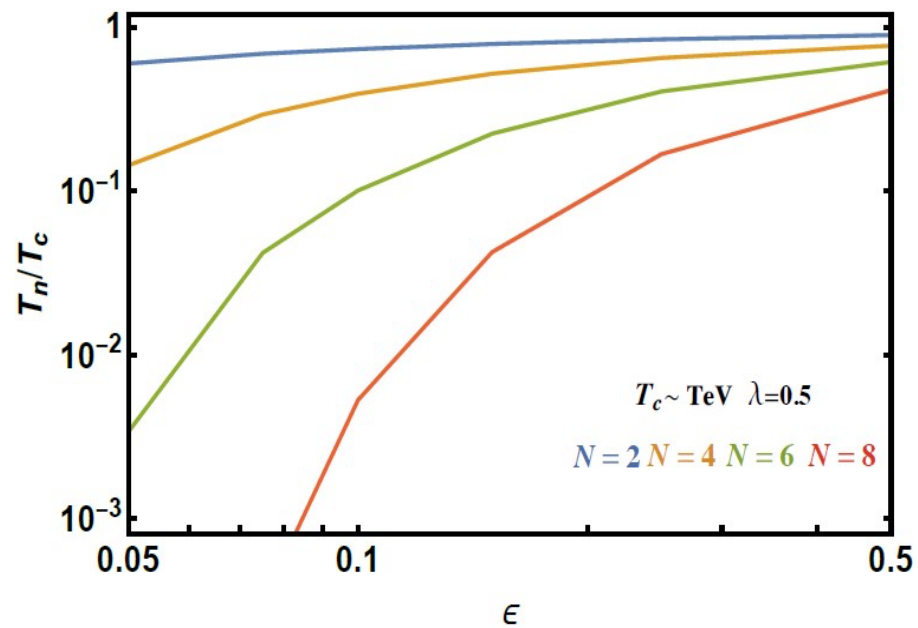
$$\frac{\beta_{\text{GW}}}{H} \sim 20$$

Smaller β_{GW}

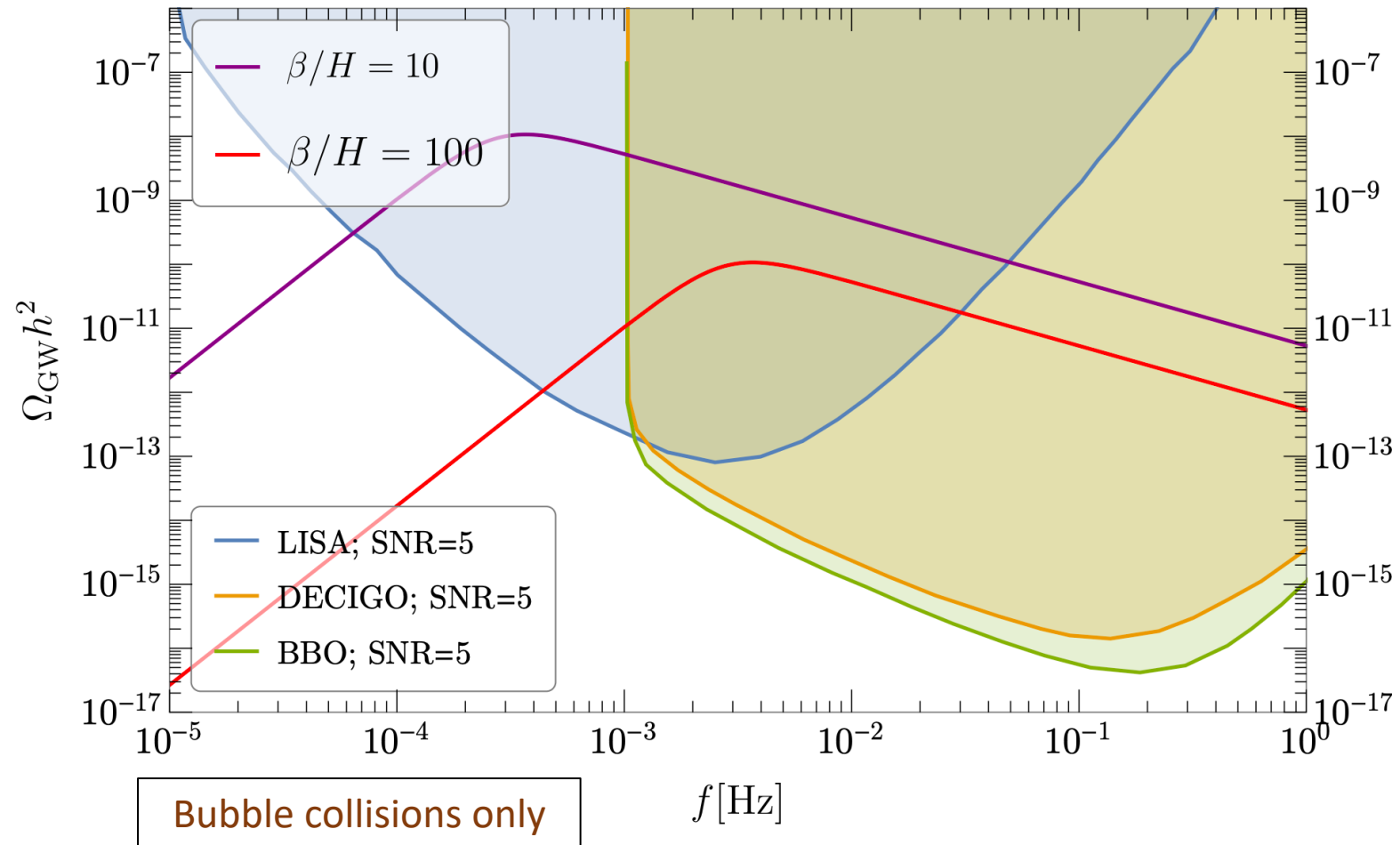


$$\frac{\beta_{\text{GW}}}{H} \sim \text{few}$$

H^{-1}



Stochastic gravitational wave background



More phenomenology

- Small β_{GW} can result in stronger gravitational wave signal
- Strong gravitational waves in composite Higgs models can allow for observable gravitational wave anisotropies Geller, Hook, Sundrum, Tsai 2019
- Dilaton/radion mass $\propto \sqrt{\epsilon_{IR}}$, correlated with the gravitational wave signal.
- Fast PT opens the possibility of baryogenesis in composite Higgs models at scales above the PT temperature. Agashe, Du, M.E., Fong, Hong, Vecchi 2019 & ongoing work

Summary

- Confinement PT of composite Higgs models can be studied in the scenario of **spontaneous confinement** and using **holography**.
- The **holographic dual** 5D formulation (RS) allows for a controlled description of the PT dynamics within 5D EFT.
- **Slow PT** in the **minimal model**, leading to empty universe or large supercooling and dilution of (dark) matter.
- **Beyond the minimal model**: separate “critical exponents” controlling the hierarchies (ϵ_{UV}) and PT dynamics (ϵ_{IR})
- PT can complete **without large supercooling**, compatible with genesis of matter abundances before PT.
- **Gravitational wave** signal and **radion mass** sensitive to ϵ_{IR} .

Thank you!

Extra Slides

Spontaneous Confinement- Dilaton EFT

A large N CFT:

$$\mathcal{L} = \mathcal{L}_{CFT}$$

Dilaton Lagrangian:

$$\mathcal{L}_{dilaton} = \frac{N^2}{16\pi^2} \left((\partial\phi)^2 - \lambda\phi^4 \right)$$

- Large N and small $\lambda \lesssim 1$
- Spontaneous confinement, $\langle\phi\rangle \neq 0$, not stable

Spontaneous Confinement- Dilaton EFT

But with small explicit breaking of scale invariance:

$$\mathcal{L}(\Lambda_{UV}) = \mathcal{L}_{CFT} + \frac{1}{\Lambda_{UV}^\epsilon} \mathcal{O}$$

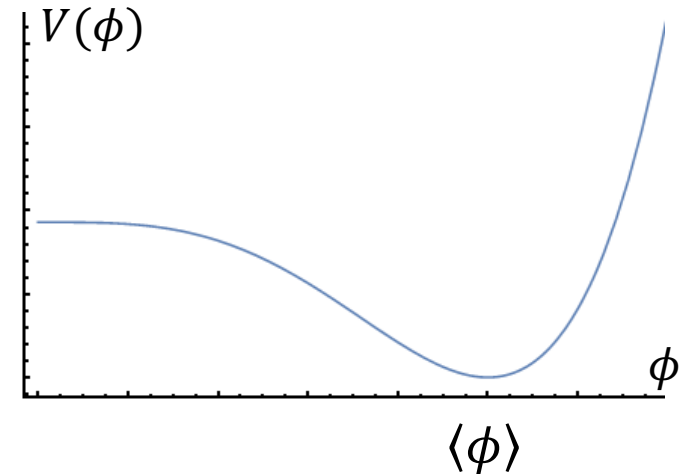
Deform the CFT
 $[O] = 4 + \epsilon$

Dilaton Lagrangian:

$$\mathcal{L}_{dilaton} = \frac{N^2}{16\pi^2} \left((\partial\phi)^2 - \lambda\phi^4 \left(1 - \omega \left(\frac{\phi}{\Lambda_{UV}} \right)^\epsilon \right) \right)$$

- ϵ parameterizes **explicit breaking** of scale invariance and sets the hierarchy

$$\ln \frac{M_{Pl}}{\text{TeV}} \sim \ln \frac{\Lambda_{UV}}{\langle\phi\rangle} \sim \frac{1}{\epsilon} \sim 30$$



The (de)confinement PT

- Critical Temperature: $\frac{T_C}{\langle\varphi\rangle} \sim (\epsilon\lambda)^{\frac{1}{4}} \ll 1$

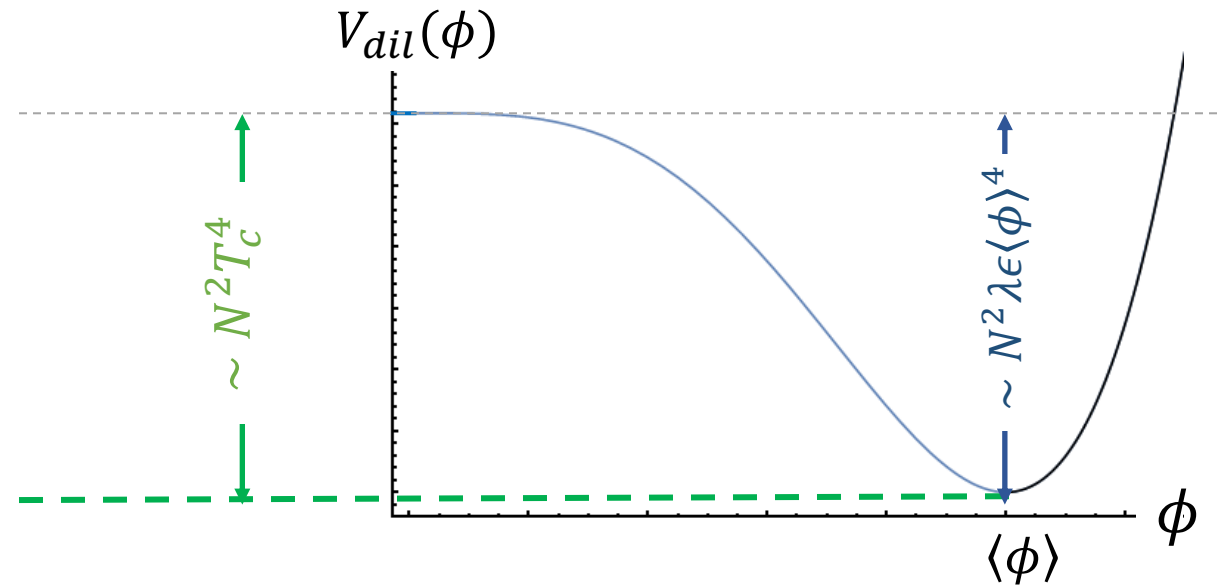
Justifies radion EFT

- Typical composites not excited

$$m_{\text{comp}} \sim \langle\varphi\rangle \gg T$$

- PT is first order, for small ϵ or λ .

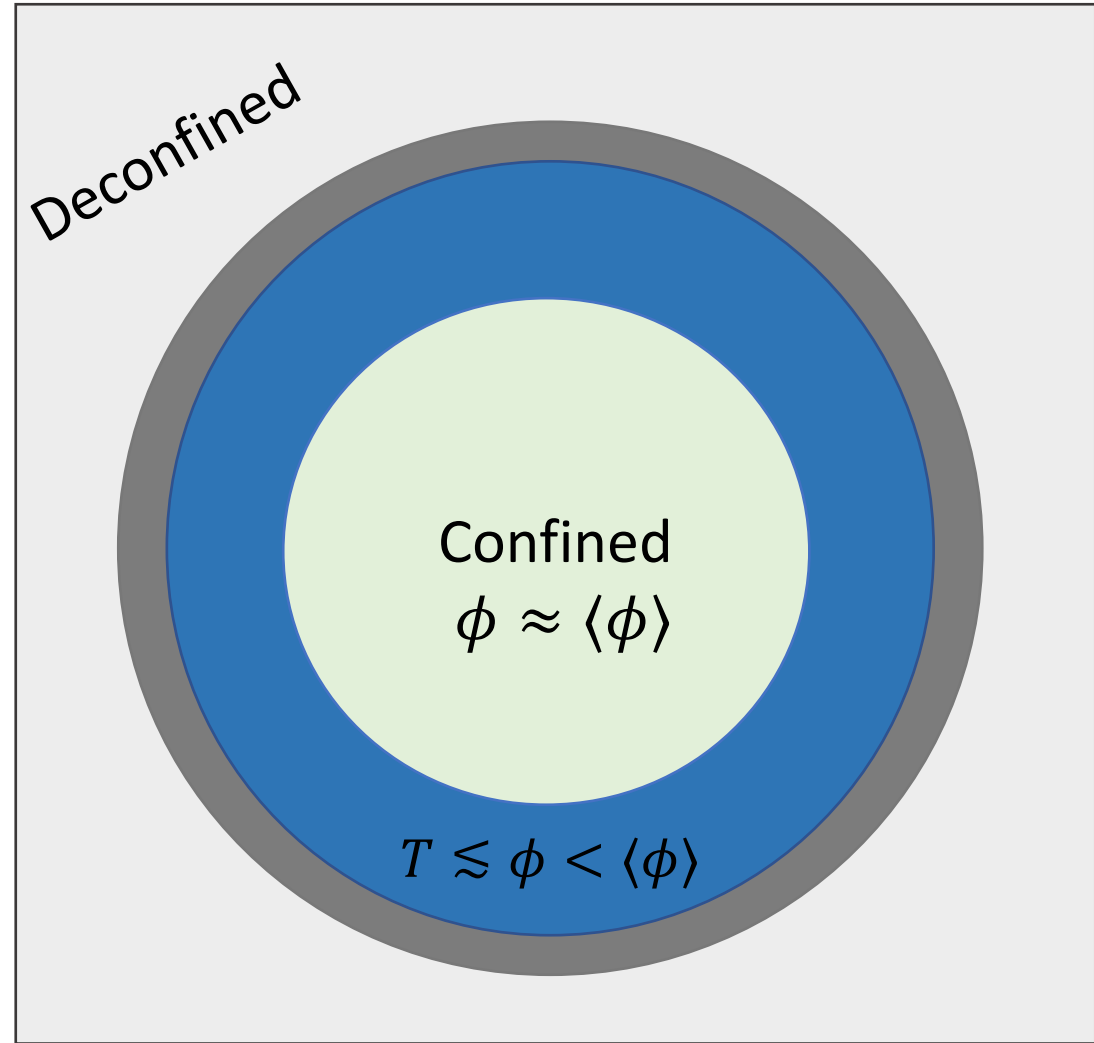
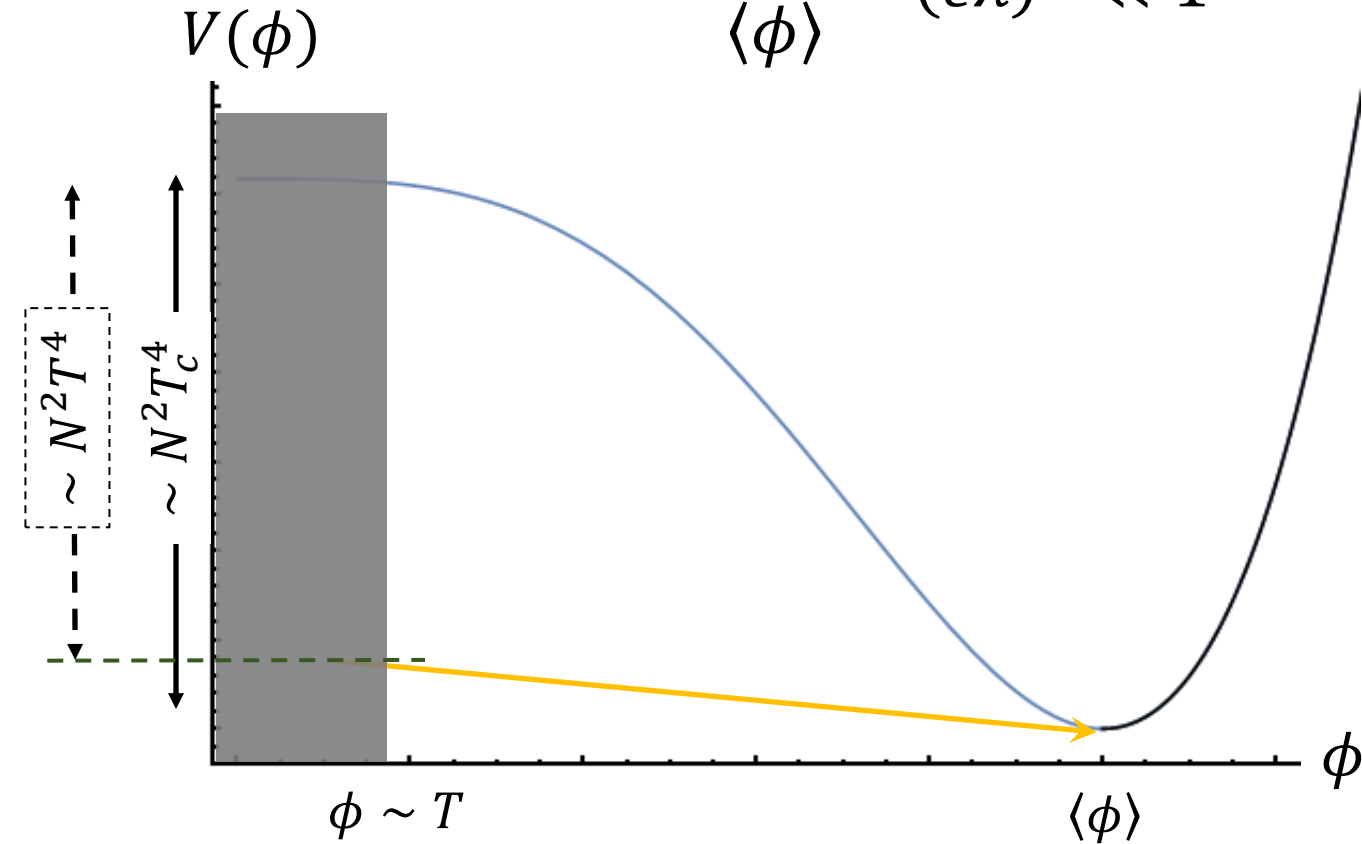
1st order \Rightarrow bubble nucleation



Creminelli, Nicolis & Rattazzi 2002

Bubble Nucleation

$$\frac{T_c}{\langle \phi \rangle} \sim (\epsilon \lambda)^{\frac{1}{4}} \ll 1$$



Bubble Nucleation

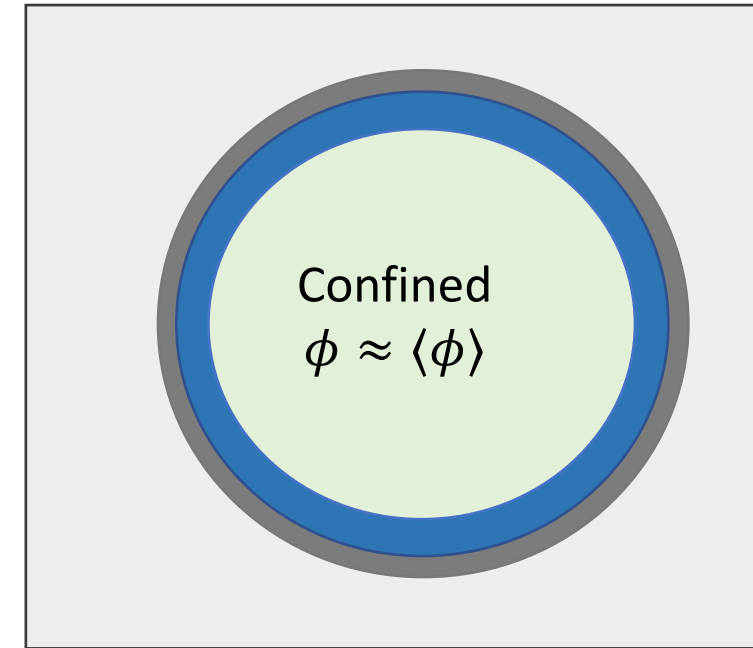
- For T close to T_c (thin-wall):

$$S_{\text{bounce}} = \frac{16\pi}{3} \frac{S_1^3}{(\Delta F)^2 T}$$

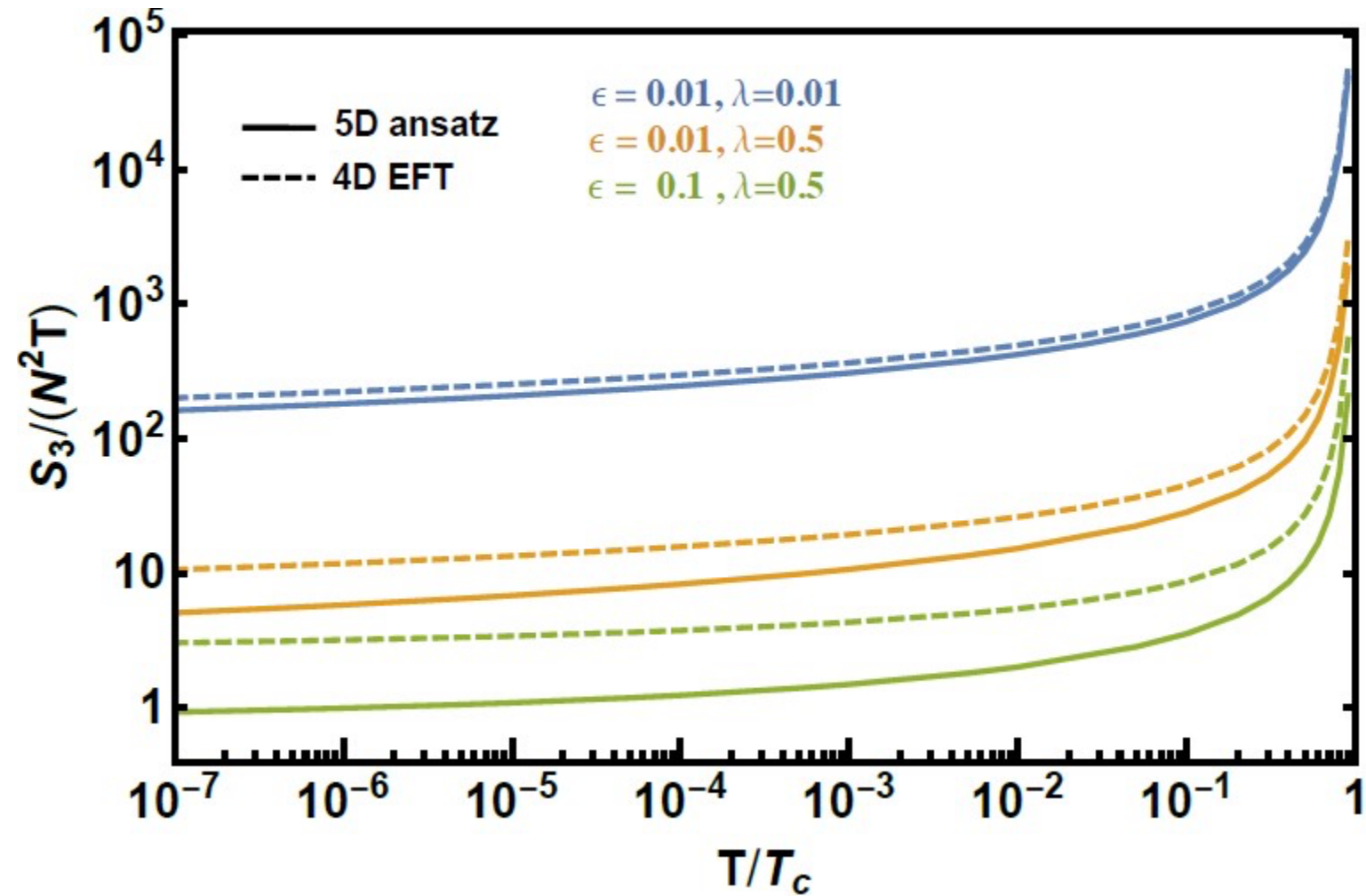
- S_1 : tension of the wall
- Contribution of $\varphi \gg T$ region enhanced by small ϵ, λ :

$$S_1 \sim \frac{N^2}{(\epsilon\lambda)^{1/4}} T_c^3$$

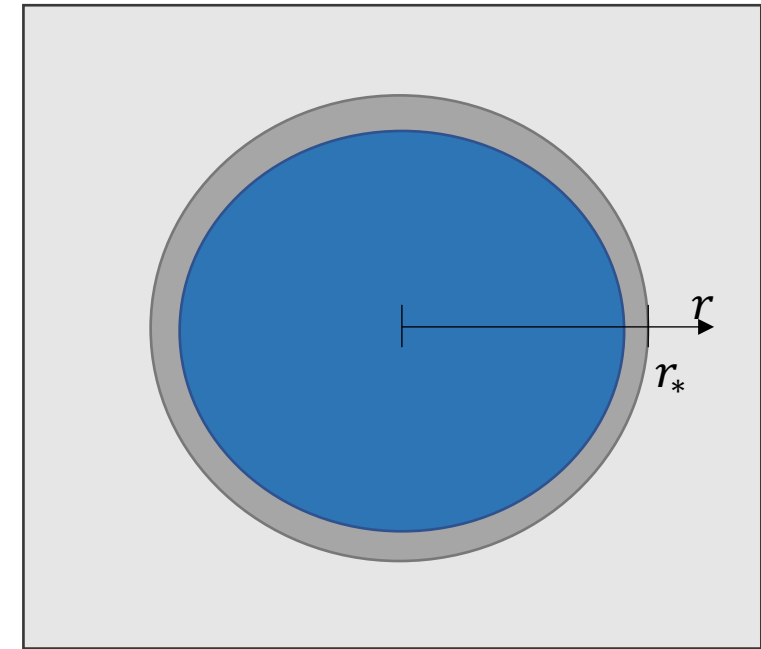
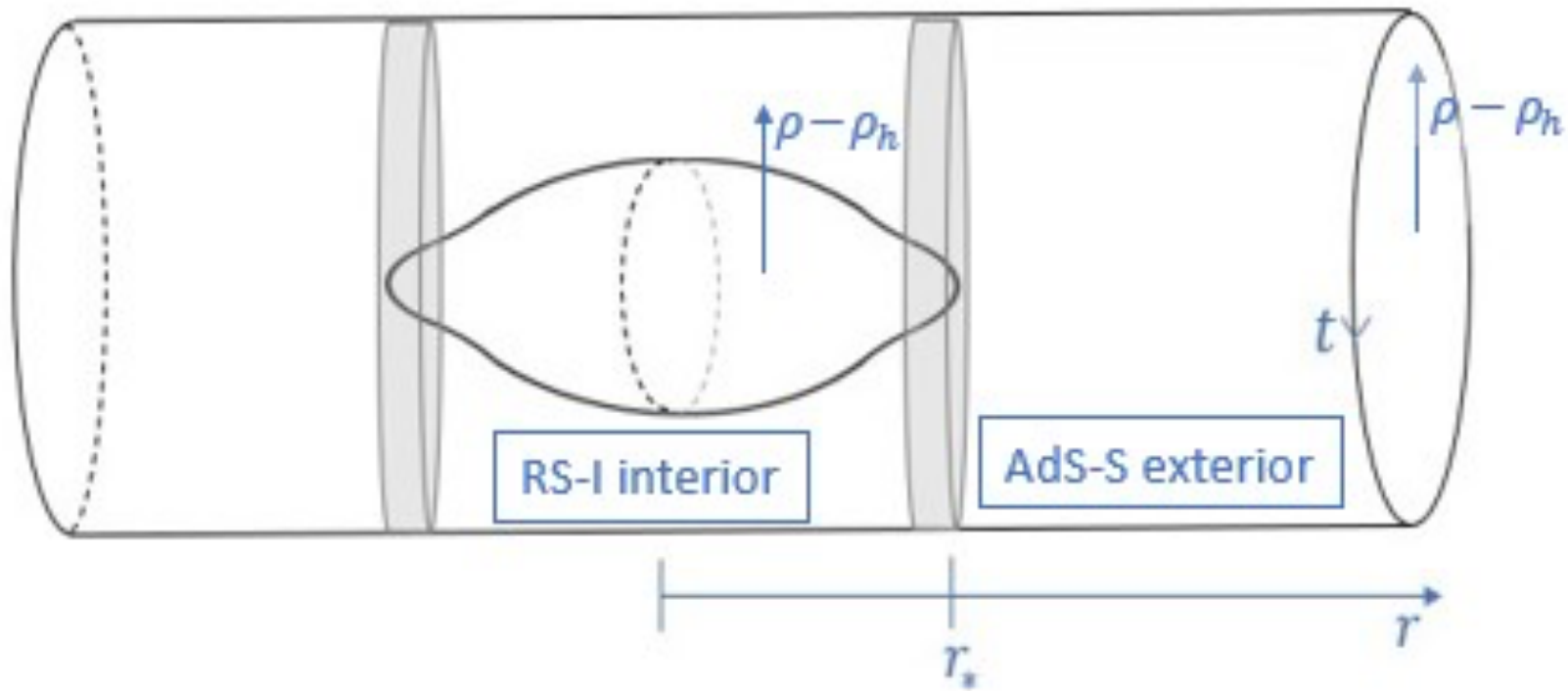
- Similar enhancement not expected for $\varphi \sim T$ region



The 5D bounce



The 5D bounce- smoothness



- ✓ IR-brane can be smoothly sealed at the horizon
- ✓ Bounce is smooth and can be described in 5D EFT

Agashe, Du, M.E., Kumar, Sundrum 2020

RS1 and Goldberger-Wise mechanism

- In RS1 model, hierarchies are related to the position of the IR brane:

$$\frac{\text{TeV}}{M_{\text{Pl}}} \sim e^{-kX_5}$$

- IR-brane stabilized using a bulk scalar (Goldberger-Wise) field, minimally with the potential:

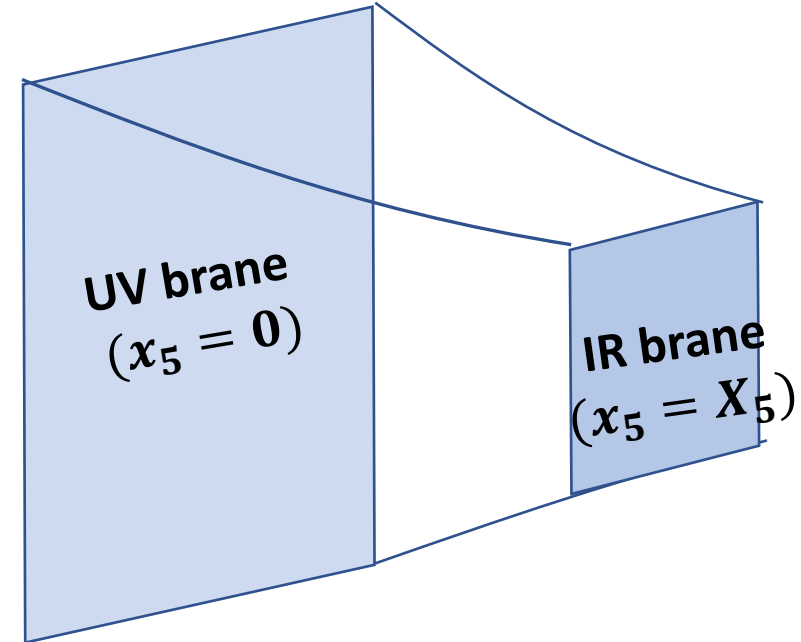
$$V_{\text{GW}}(\Phi) = \frac{1}{2} m^2 \Phi^2$$

- Generates a potential for the radion, $\varphi = k e^{-kX_5}$, the field corresponding to the position of the IR brane.

$$V_{\text{radion}}(\varphi) = \frac{3N^2}{4\pi^2} \lambda \varphi^4 \left(1 - \omega \left(\frac{\varphi}{\Lambda_{\text{UV}}} \right)^\epsilon \right)$$

- Hierarchy is set mainly by $\epsilon \approx \frac{m^2}{4k^2}$: $\ln \frac{M_{\text{Pl}}}{\text{TeV}} \sim \frac{1}{\epsilon}$

Randall & Sundrum 1999



$$ds^2 = e^{-2kx_5} \eta_{\mu\nu} dx^\mu dx^\nu + dx_5^2$$

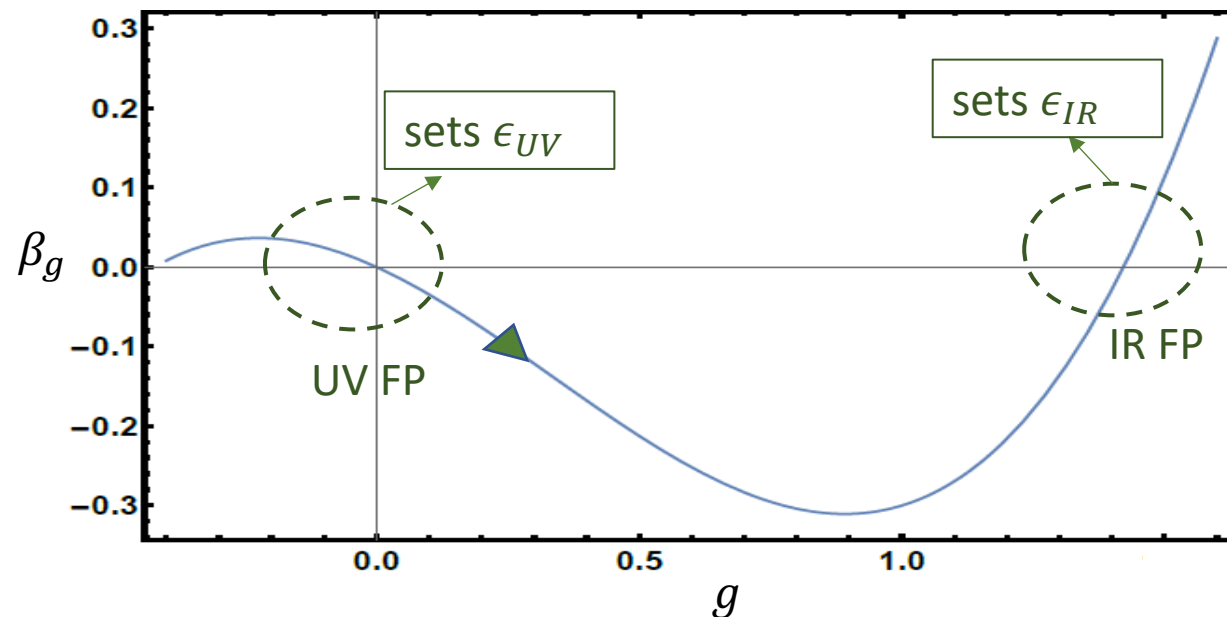
$$0 < x_5 < X_5$$

Goldberger & Wise 1999

Separate fixed points- faster PT

- RGE with UV and IR fixed points
- ϵ_{UV} and ϵ_{IR} are the anomalous dimensions corresponding to the UV and the IR fixed points.

Agashe, Du, M.E., Kumar, Sundrum 2019



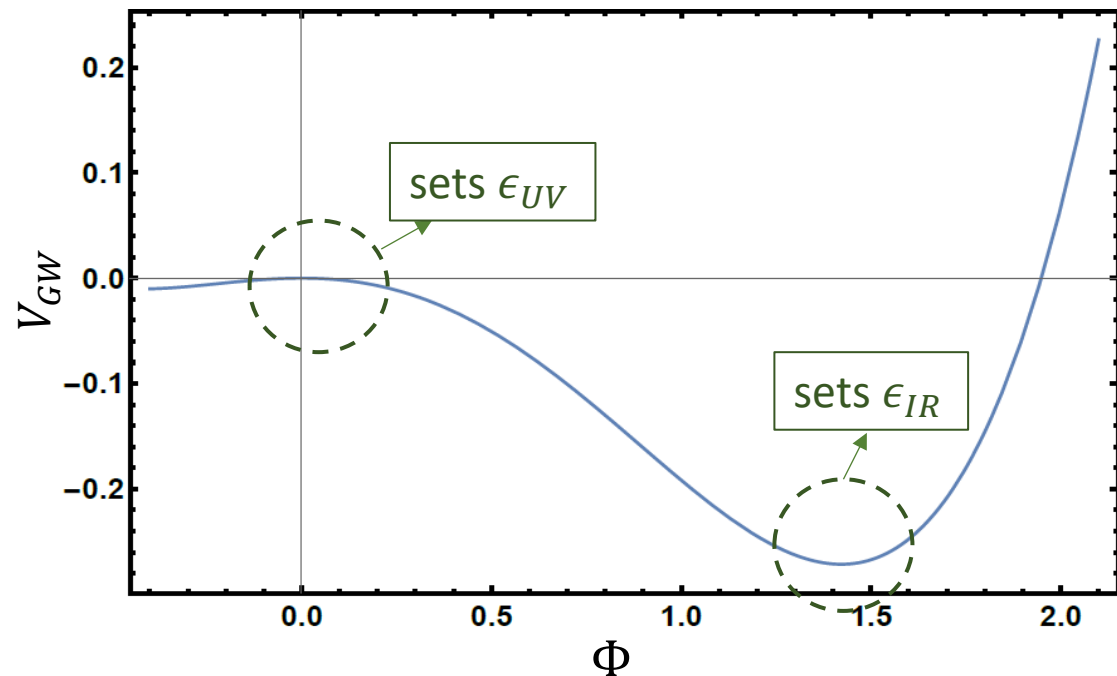
Two fixed points

Agashe, Du, M.E., Kumar, Sundrum
2019, 2020

- Goldberger-Wise field with self-interactions:

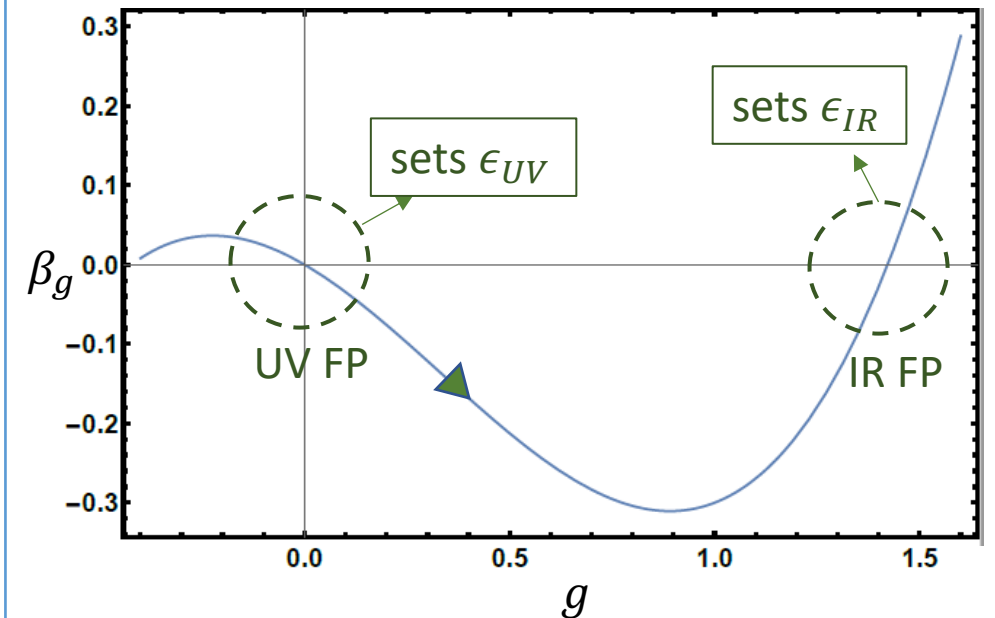
$$V_{GW}(\Phi) = \frac{1}{2} m^2 \Phi^2 + \frac{1}{3!} \eta \Phi^3 + \frac{1}{4!} \kappa \Phi^4$$

- Effective mass: $m_{eff}^2 \sim V_{GW}''(\Phi)$



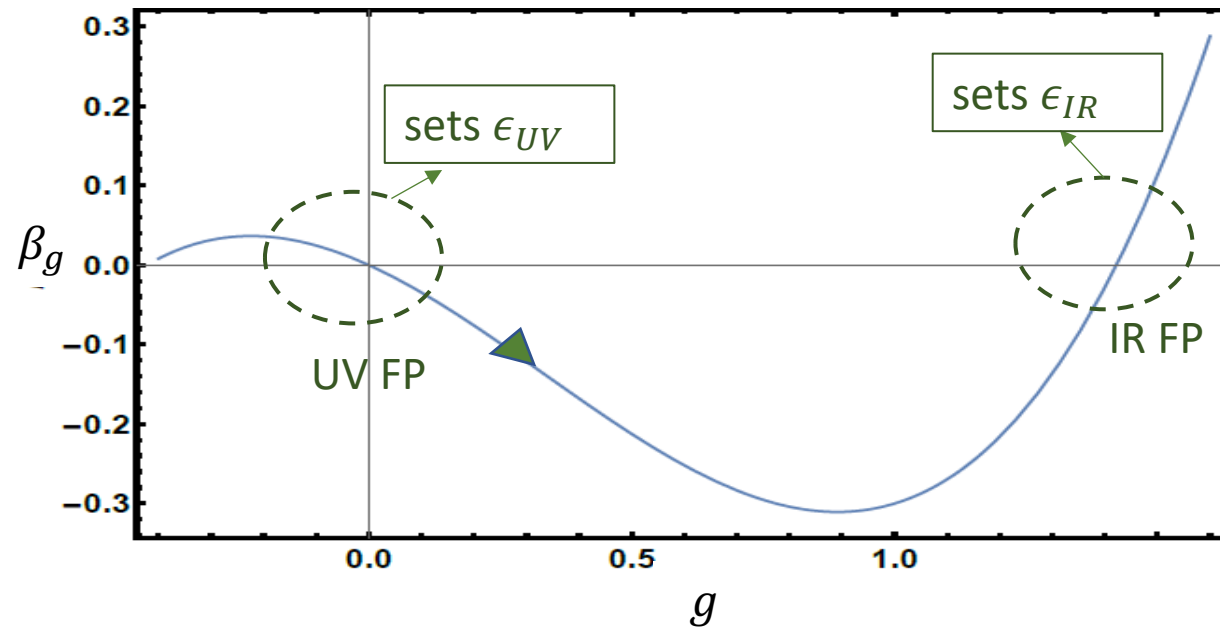
Dual picture:

- RGE with UV and IR fixed points
- ϵ_{UV} and ϵ_{IR} are the anomalous dimensions corresponding to the UV and the IR fixed points.



Separate fixed points- faster PT

- RGE with UV and IR fixed points
- ϵ_{UV} and ϵ_{IR} are the anomalous dimensions corresponding to the UV and the IR fixed points.



Agashe, Du, M.E., Kumar, Sundrum 2019

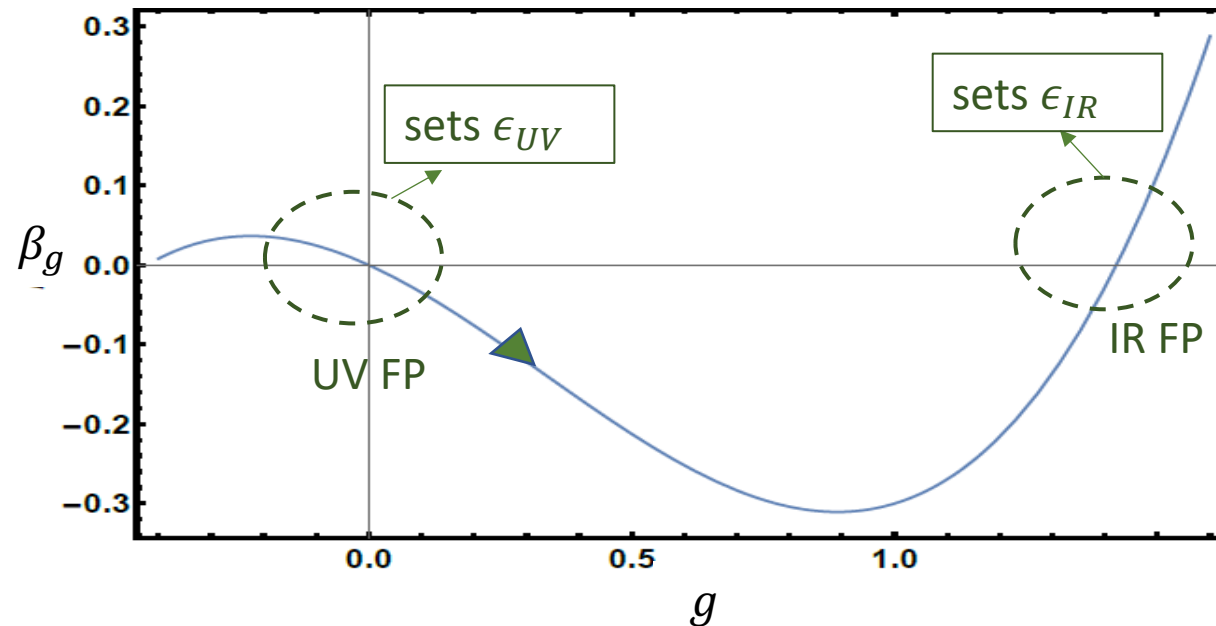
- Nucleation rate enhanced if ϵ_{IR} not too small.
- PT can complete near T_c for parameters :

$$\epsilon_{IR} = \frac{1}{2}, \quad \lambda = 0.5, \quad N \approx 2$$

- Marginally in theoretical control.

Separate fixed points- faster PT

- RGE with UV and IR fixed points
- ϵ_{UV} and ϵ_{IR} are the anomalous dimensions corresponding to the UV and the IR fixed points.



Agashe, Du, M.E., Kumar, Sundrum 2019

Holographic dual: Self-interacting
Goldberger-Wise