Determination of generalized parton distributions

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GPDs?

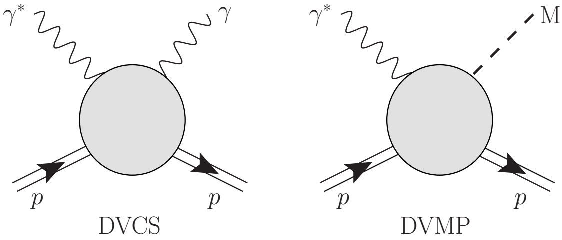
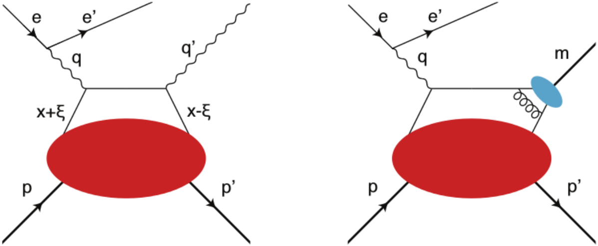


Figure: Deeply Virtual Compton Scattering and Deeply Virtual Meson Production.

Factorization

Hard and soft QCD processes contribute? they may be separable.



(a) Deeply virtual Compton scattering. (b) Deeply virtual meson production.

Universality

Q: Factorisation proof exists? If yes...

The (perturbatively) calculable hard processes are different, but the incalculable soft processes are identical.
The large red blobs in both represent the same GPDs

GPDs

Generalized parton distributions (GPDs), are universal non-perturbative objects entering the description of hard exclusive electroproduction processes

Access to GPDs

- The access to GPDs through DVCS and DVMP is indirect.
- DVCS does not depend directly on GPDs, but on Compton Form Factors (CFFs)
- Fits to DVCS data have been successfully performed since 2008, providing first quantitative experimental information on CFFs. Although these fits do not give a final word on the GPD studies[1602.02763].
- Data: Jefferson Lab (JLab) - COMPASS(CERN) - Electron-Ion Collider (EIC)

GPD fitting

GPD phenomenology is much harder (than PDF ph.)

- GPDs depend on more variables $(x, \xi, t; \mu)$
-
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- They are subject to many constraints(Polynomiality - Positivity - ...).
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GPD fitting

GPD phenomenology is much harder (than PDF ph.)

- GPDs depend on more variables $(x, \xi, t; \mu)$
- They are subject to many constraints(Polynomiality - Positivity - ...).
- GPD parametrizations: Overlap, Covariant Scattering Matrix Approach, Double Distributions, Conformal moments,....

Definition of GPDs

M. Diehl / Physics Reports 388 (2003) 41–277

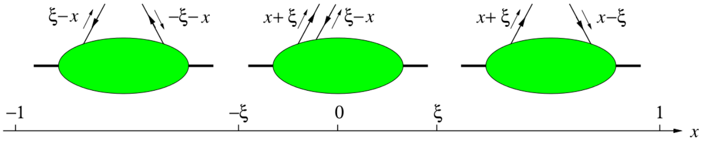

Fig. 5. The parton interpretation of GPDs in the three x-intervals $[-1, -\xi]$, $[-\xi, \xi]$, and $[\xi, 1]$.

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}, \qquad P = \frac{p + p'}{2}, \qquad \Delta = p' - p, \quad t = \Delta^2$$

Definition of GPDs

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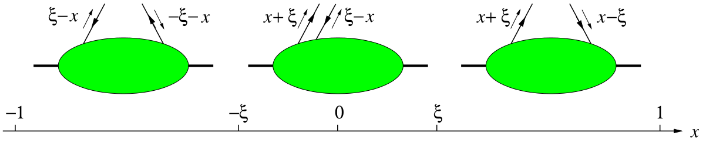


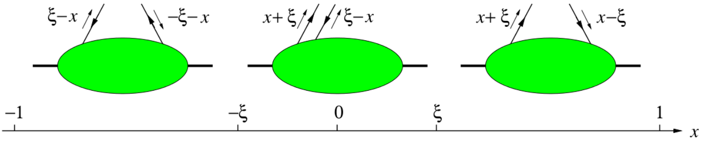
Fig. 5. The parton interpretation of GPDs in the three x-intervals $[-1,-\xi]$, $[-\xi,\xi]$, and $[\xi,1]$.

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}, \qquad P = \frac{p+p'}{2}, \qquad \Delta = p' - p, \quad t = \Delta^2$$

$$F^q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\,|p\rangle \Big|_{z^+=0,\,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+u(p) + E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right] (1)$$

Definition of GPDs

M. Diehl / Physics Reports 388 (2003) 41–277



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Definition of GPDs

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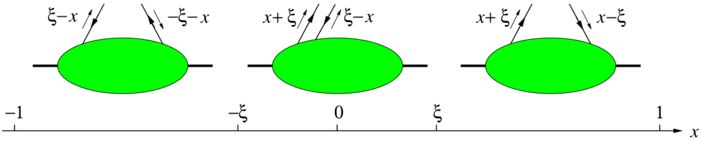

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$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}, \qquad P = \frac{p+p'}{2}, \qquad \Delta = p' - p, \quad t = \Delta^2 \quad (2)$$

$$\widetilde{F}^q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+ \gamma_5\, q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, \mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\, \bar{u}(p')\gamma^+\gamma_5 u(p) + \widetilde{E}^q(x,\xi,t)\, \bar{u}(p')\frac{\gamma_5 \Delta^+}{2m} u(p)\right] (3)$$

Constraints

Forward limit

In the forward kinematic limit, $p = p'$, some GPDs reduce to standard PDFs,

$$F^q(x,0,0) = H^q(x,0,0) = \theta(x)q(x) - \theta(-x)\bar{q}(-x) \;, \tag{4}$$

$$F^g(x,0,0) = H^g(x,0,0) = \theta(x)xg(x) - \theta(-x)xg(-x) \;, \tag{5}$$

$$\widetilde{F}^q(x,0,0) = \widetilde{H}^q(x,0,0) = \theta(x)\Delta q(x) + \theta(-x)\Delta\bar{q}(-x) \;, \tag{6}$$

$$\widetilde{F}^g(x,0,0) = \widetilde{H}^g(x,0,0) = \theta(x)x\Delta g(x) + \theta(-x)x\Delta g(-x) \;. \tag{7}$$

Constraints

Discrete symmetries

- Time reversal and hermiticity imply that GPDs are real and that for all $F = F^q, F^g, \widetilde{F}^q, \widetilde{F}^g$,

$$F(x,\xi,t) = F(x,-\xi,t) \ . \quad (8)$$

- The fact that the gluon is its own antiparticle implies that

$$F^g(x,\xi,t) = F^g(-x,\xi,t) \ , \quad (9)$$

$$\widetilde{F}^g(x,\xi,t) = -\widetilde{F}^g(-x,\xi,t) \ . \quad (10)$$

Constraints

Basic Sum Rules

Sum rules are quite important in the GPD phenomenology. The integrals of GPDs over x are related to the quark contributions to the elastic form factors,

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t), \quad \int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t),$$

$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_A^q(t), \quad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_P^q(t).$$

Many constraints

Ji's Sum Rules

Ji's sum rule [9603249] is another landmark GPD property. The Belinfante energy-momentum tensor $T^{\mu\nu}$ between nucleon states can be parametrized as,

$$\langle p'|T^{\mu\nu}|p\rangle = \bar{u}(p')\left[\frac{1}{2}A(t)\gamma^{(\mu}P^{\nu)} + B(t)P^{(\mu}i\sigma^{\nu)\lambda}\frac{\Delta_\lambda}{4M} + \frac{C(t)}{M}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})\right]u(p)\ , \quad (11)$$

where A, B and C are called gravitational form factors, defined for both the quark and gluon sectors.

Many constraints

Ji's Sum Rules

The derivation of Ji's sum rule starts from the decomposition of the nucleon spin into its quark and gluon contributions

$$\frac{1}{2} = \sum_q J^q + J^g \ , \tag{12}$$

with both terms related to the energy-momentum tensor

$$J^{q,g} = \frac{1}{2}[A^{q,g}(t=0) + B^{q,g}(t=0)] \ . \tag{13}$$

Many constraints

Possibility of solution of proton spin problem

The second Mellin moments of the GPDs H and E from this definition are,

$$\int \mathrm{d}x\, x H^{q,g}(x,\eta,t) = A^{q,g}(t) + 4\eta^2 C^{q,g}(t) \,, \tag{14}$$

$$\int \mathrm{d}x\, x E^{q,g}(x,\eta,t) = B^{q,g}(t) - 4\eta^2 C^{q,g}(t) \,, \tag{15}$$

Many constraints
Polynomiality

x^j moments of quark GPDs are even polynomials in ξ with leading powers given in Table 1 and that x^{j-1} moments of gluon GPDs are polynomials in ξ with leading powers given in Table 2.

GPD	even j	odd j
H^q, E^q	η^j	η^{j+1}
$H^q + E^q$	η^j	η^{j-1}
$\widetilde{H}^q$, $\widetilde{E}^q$	η^j	η^{j-1}

Table: (1) Leading powers of the x^j Mellin moments of the twist-2 chiral-even GPDs in the quark sector.

Many constraints

Polynomiality

x^j moments of quark GPDs are even polynomials in ξ with leading powers given in Table 1 and that x^{j-1} moments of gluon GPDs are polynomials in ξ with leading powers given in Table 2.

GPD	even j	odd j
H^g, E^g	0	η^{j+1}
$H^g + E^g$	0	η^{j-1}
$\widetilde{H}^g$, $\widetilde{E}^g$	η^j	0

Table: (2) Leading powers of the x^{j-1} Mellin moments of the twist-2 chiral-even GPDs in the gluon sector.

Many constraints

Positivity

- Positivity is a strong model-independent constraint.
- Positivity bounds emerge from the definition of the norm on a Hilbert space, and thus are fundamental properties of GPDs(and PDFs).
- Positivity bounds are inequalities between GPDs and the corresponding PDFs at well-defined kinematic configurations

$$|H^q(x,\xi,t)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}\,, \qquad (16)$$

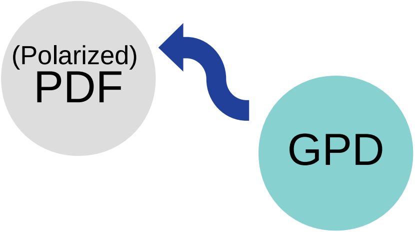
valence GPDs $\mathcal{G}_v = H_v, E_v$ for flavor q are expressed in terms of "quark GPDs" $\mathcal{G}$ as

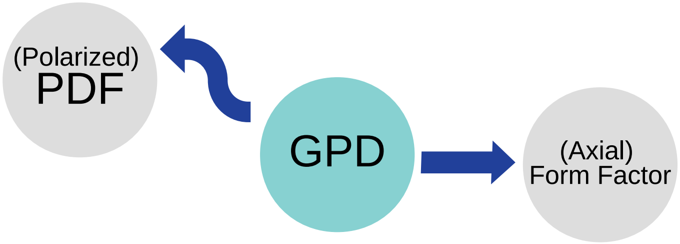
$$\mathcal{G}_v^q(x,t) = \mathcal{G}^q(x,\xi=0,t) + \mathcal{G}^q(-x,\xi=0,t), \tag{17}$$

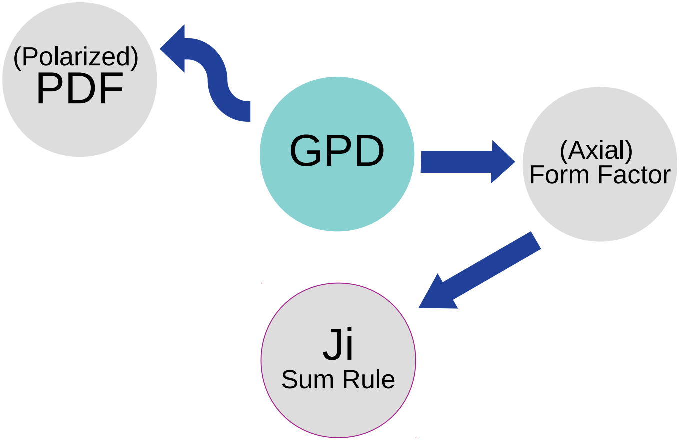
GPD PARAMETRIZATIONS

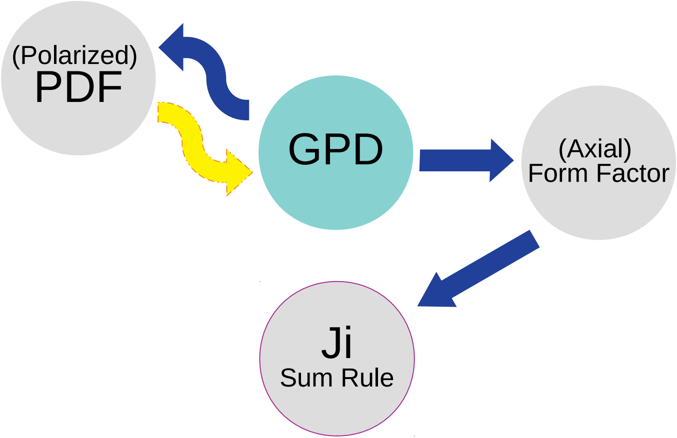
- Overlap
- Covariant Scattering Matrix Approach
- Conformal moments
- Double Distributions
- Exponential ansatz (zero skewness)


GPD


(Polarized)
PDF
GPD


(Polarized)
PDF
GPD
(Axial)
Form Factor


(Polarized)
PDF
GPD
(Axial)
Form Factor
Ji
Sum Rule


(Polarized)
PDF
GPD
(Axial)
Form Factor
Ji
Sum Rule

Model

The valence GPDs H_v^q, for example, can be related to ordinary valence PDFs as

$$H_v^q(x,t,\mu^2) = q_v(x,\mu^2)\exp[tf_q(x)], \tag{18}$$

The profile functions $f_q(x)$ can have the simple form

$$f_q(x) = \alpha'(1-x)\log\frac{1}{x}. \tag{19}$$

Model

Parametrization scan

The behavior of profile function $f(x)$ can be well characterized by the forms

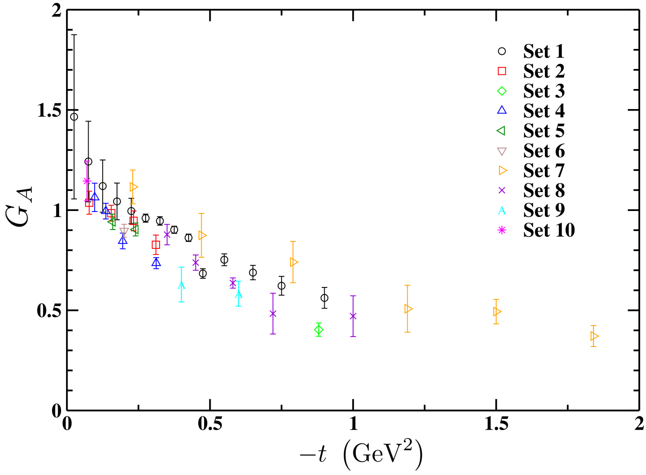
$$f_q(x) = \alpha'(1-x)^2 \log\frac{1}{x} + B_q(1-x)^2 + A_q x(1-x), \qquad (20)$$

and

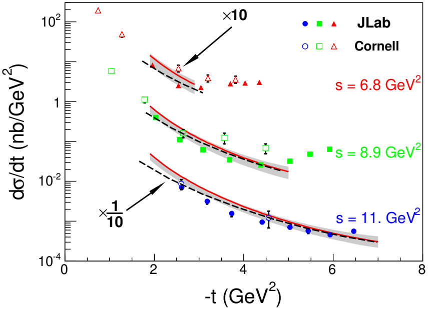
$$f_q(x) = \alpha'(1-x)^3 \log\frac{1}{x} + B_q(1-x)^3 + A_q x(1-x)^2. \qquad (21)$$

Axial FF (G_A)

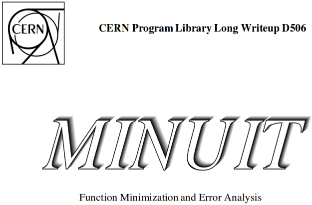
Reduced data set



WACS data
[Danagoulian et al. 07]

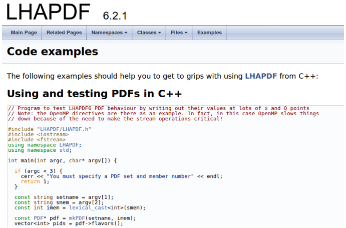


Analysis


CERN Program Library Long Writeup D506

MINUIT

Function Minimization and Error Analysis


LHAPDF 6.2.1

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Code examples

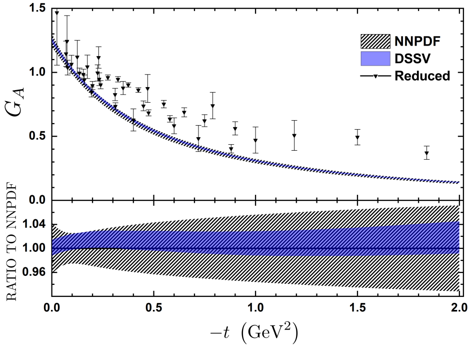
The following examples should help you to get to grips with using **LHAPDF** from C++:

Using and testing PDFs in C++

```
// Program to test LHAPDF6 PDF behaviour by writing out their values at lots of x and Q points
// Note: the OpenMP directives are there as an example. In fact, in this case OpenMP slows things
// down because of the need to make the stream operations critical!

#include "LHAPDF/LHAPDF.h"
#include <iostream>
#include <fstream>
using namespace LHAPDF;
using namespace std;

int main(int argc, char* argv[]) {

  if (argc < 3) {
    cerr << "You must specify a PDF set and member number" << endl;
    return 1;
  }

  const string setname = argv[1];
  const string smem = argv[2];
  const int imem = lexical_cast<int>(smem);

  const PDF* pdf = mkPDF(setname, imem);
  vector<int> pids = pdf->flavors();
```

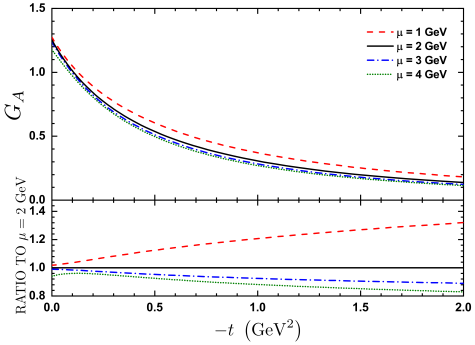
PRD100

PDF uncertainty study



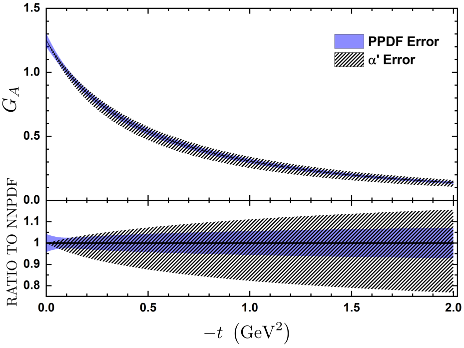
PRD100

Factorization scale uncertainty study



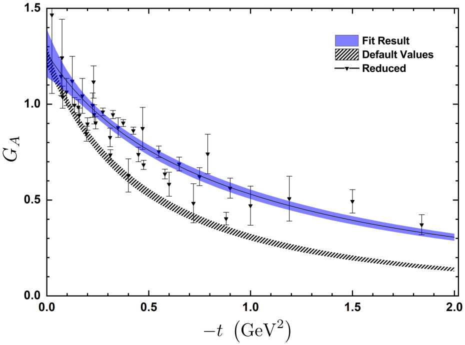
PRD100

Model uncertainty study



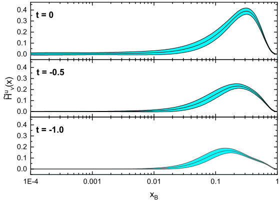
PRD100

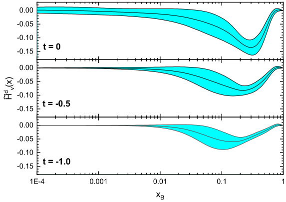
Simple ansatz fit

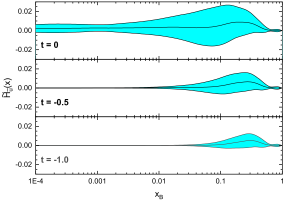


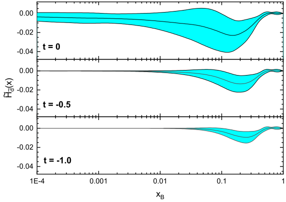
PRD100

Final (pol.) GPSs



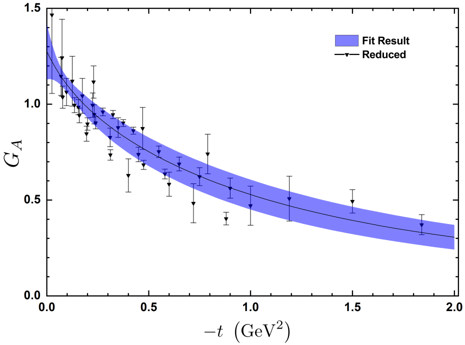






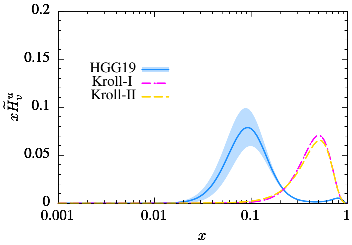
PRD100

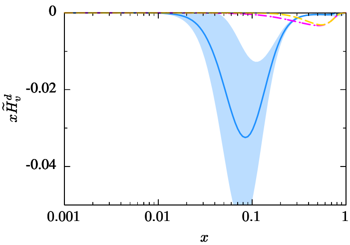
Best ansatz fit



PRD102

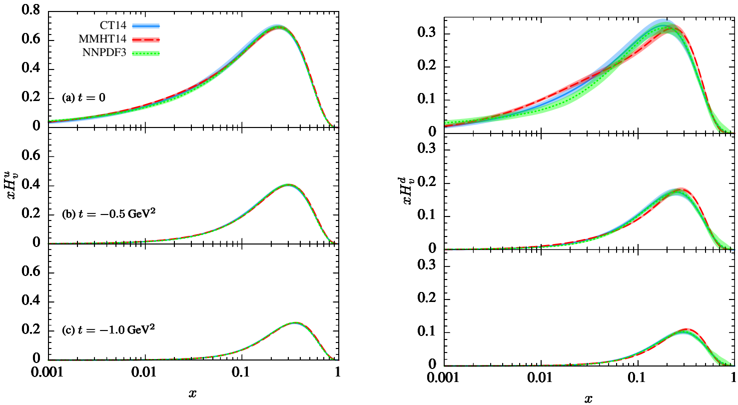
Motivation





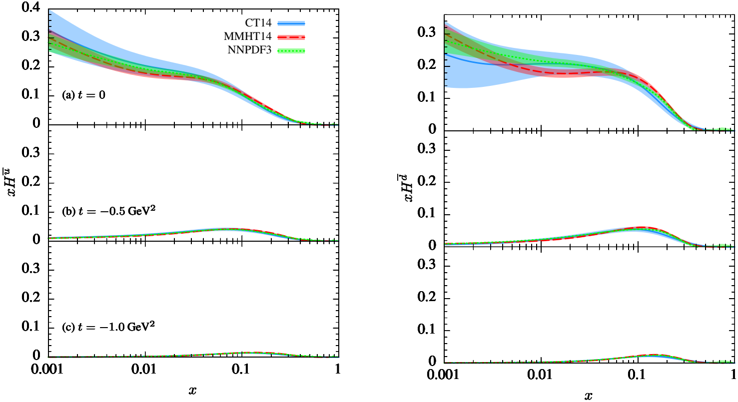
PRD102

PDF study -Valence quarks



PRD102

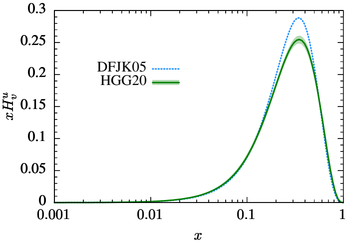
PDF study- Sea quarks

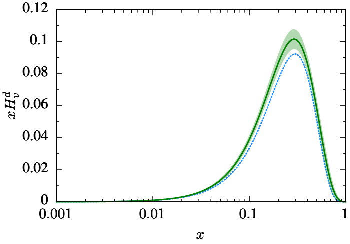


PRD102

Unpolarized GPD results - Valence

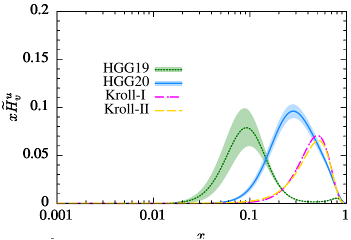
Different observables: Dirac & Pauli FFs vs. AFF & WACS

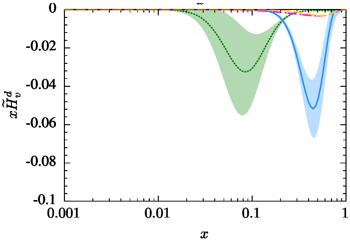




PRD102

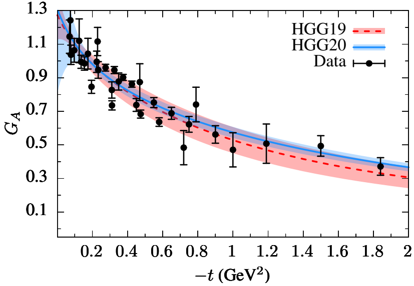
Pol. GPD results - Comparison





PRD102
Best ansatz fit

Result of simultaneous analysis of G_A and WACS data.



PRD102

Best ansatz fit

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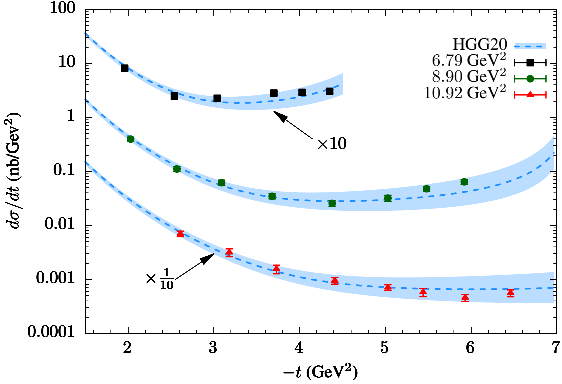
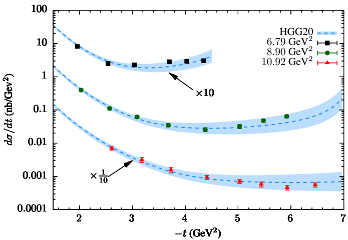
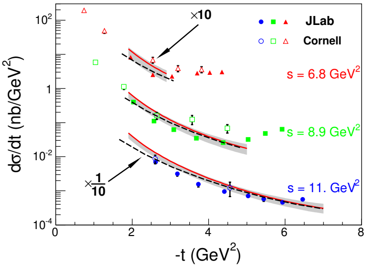


PRD102

Best ansatz fit

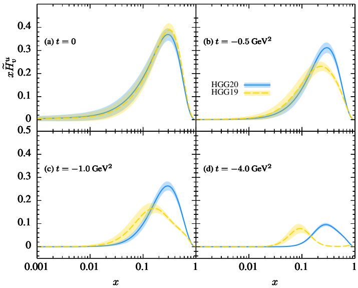


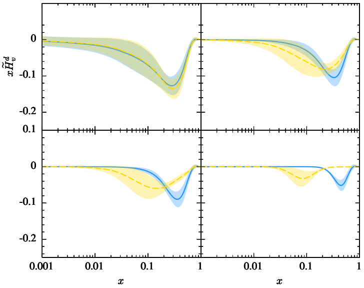


Reserve Slides

PRD102

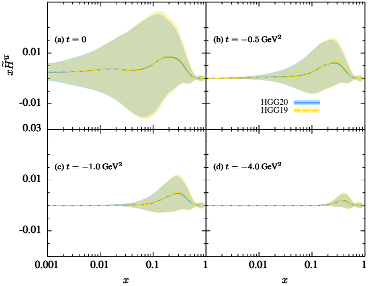
Pol. GPD results - Valence

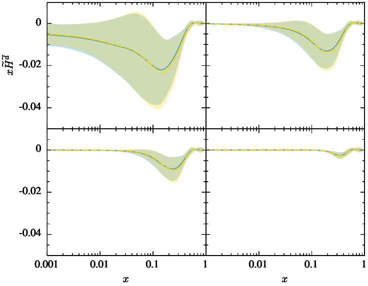




PRD102

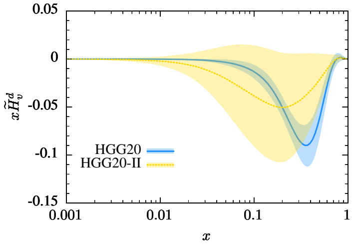
Pol. GPD results - Sea





PRD102

Alternative ansatz for unpol. GPDs



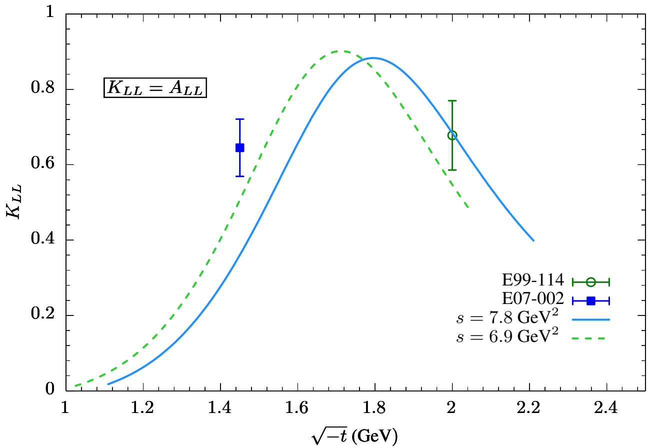
The helicity correlation parameters A_{LL} and K_{LL} which are related to the WACS cross section as follows,

$$A_{LL} = \frac{\mathrm{d}\sigma(++)\ -\ \mathrm{d}\sigma(+-)}{\mathrm{d}\sigma(++)\ +\ \mathrm{d}\sigma(+-)},$$
$$K_{LL} = \frac{\mathrm{d}\sigma(++)\ -\ \mathrm{d}\sigma(+-)}{\mathrm{d}\sigma(++)\ +\ \mathrm{d}\sigma(+-)}. \qquad (22)$$

Massless quarks $\rightarrowtail A_{LL} = K_{LL}$.

PRD102

Helicity correlation



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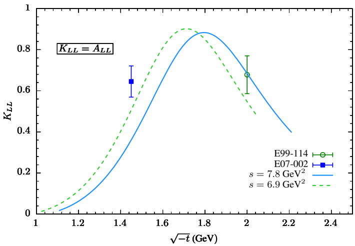
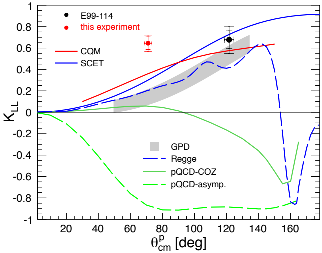
Helicity correlation





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Proton tomography $q_v(x,\vec{b})$

the impact parameter dependent parton distribution related to H can be defined as follows

$$q_v(x,\vec{b}) = \int \frac{\mathrm{d}^2\vec{\Delta}}{(2\pi)^2}\, \mathrm{e}^{-\mathrm{i}\vec{b}.\vec{\Delta}}\, H_v^q(x, t=-\vec{\Delta}^2). \tag{23}$$

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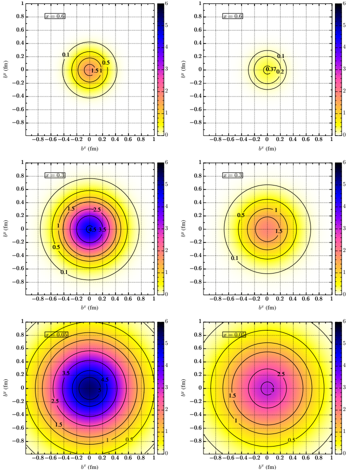
Proton tomography $q_v(x, \vec{b})$



Proton tomography $q_v^X(x,\vec{b})$

In a transversly polarized proton (specifically x-axis)

$$q_v^X(x,\vec{b}) = q_v(x,\vec{b}) - \frac{b^y}{m}\,\frac{\partial}{\partial \vec{b}^2}\,e_v^q(x,\vec{b}), \tag{24}$$

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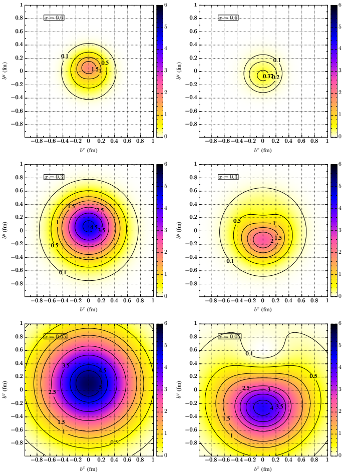
Proton tomography $q_v^X(x, \vec{b})$



Shift in $\Delta q_v(x, \vec{b})$

Note that, in this case, the corresponding shift for the distance between the struck quark and the spectator system is as follows

$$s_q(x) = \frac{\langle b^y \rangle_x^q}{1-x}, \tag{25}$$

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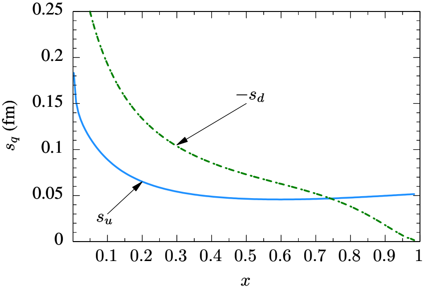
Shift in $\Delta q_v(x,\vec{b})$



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Pol. proton tomography $\Delta q_v(x, \vec{b})$

The impact parameter distribution of the longitudinally polarized quarks in a longitudinally polarized nucleon, $\Delta q_v(x, \vec{b})$ are defined as follows,

$$\Delta q_v(x, \vec{b}) = \int \frac{\mathrm{d}^2 \vec{\Delta}}{(2\pi)^2} \, \mathrm{e}^{-\mathrm{i}\vec{b}.\vec{\Delta}} \, \widetilde{H}_v^q(x, t = -\vec{\Delta}^2), \qquad (26)$$

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Pol. proton tomography $\Delta q_v(x, \vec{b})$