Mass Corrections to the DGLAP Equations

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Outline

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Motivation

Decoupling Theorem and Remedy for Large Logarithms of m/μ appearing in $\overline{\text{MS}}$ scheme

Variable-Flavor-Number-like Schemes

as an unphysical solution

A series of composite (CWZ) schemes characterized by the number of active quarks n_f

1. Since the number of active quarks is varied, imposing *matching conditions* at threshold scales are required
2. Changes in the number of active quarks would lead to *jumps* in the splitting functions and the renormalized coupling.

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Mass-Dependent MOM Scheme

as a physical solution

- Decoupling theorem would automatically be satisfied.
- Large logarithms of m/μ do not appear.

Potential disadvantages of mass-dependent MOM schemes:

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Bare and Renormalized PDFs

Bare PDFs

Bare Gauge-Invariant Parton Distribution Function:

$$f_{(0)\,i/h}(\xi) \equiv \langle P|O_i(\xi P^+)|P\rangle_{\rm con}\ , \tag{1}$$

In terms of bare amputated Green functions of parton in parton:
Quark:

$$f_{(0)i/j}(\xi) = \frac{1}{6}\lim_{p^2\to\overline{m}_j^2}\delta_{ab}(\not{p}+\overline{m}_j)_{\alpha\beta}\Gamma^{ab,\alpha\beta}_{(0)ij}(\xi p^+,p) \tag{2}$$

$$\Gamma^{ab,\alpha\beta}_{(0)ij}(k^+,p) \equiv \langle 0|\mathcal{T}O_i(k^+)\psi^{a,\alpha}_{(0)j}(p)\overline{\psi}^{b,\beta}_{(0)j}(-p)|0\rangle_{\rm amp}\ , \tag{3}$$

Gluon:

$$f_{(0)i/g}(\xi) = \frac{1}{16}\lim_{p^2\to\overline{m}_j^2}\delta_{ab}d_{\mu\nu}(p)\Gamma^{ab,\mu\nu}_{(0)ig}(\xi p^+,p)\ , \tag{4}$$

where

$$d_{\mu\nu}(p) \equiv -g_{\mu\nu} + \frac{p_\mu g^+_\nu + p_\nu g^+_\mu}{p^+}\ . \tag{5}$$

$$\Gamma^{ab,\mu\nu}_{(0)ig}(k^+,p) \equiv \langle 0|\mathcal{T}O_i(k^+)A^{a,\mu}_{(0)}(p)A^{b,\nu}_{(0)}(-p)|0\rangle_{\rm amp}\ . \tag{6}$$

Bare and Renormalized PDFs

Renormalized PDFs

Renormalized Gauge-Invariant Parton Distribution Function:

$$f_{(\mathrm{R})i/h}(\xi,\mu) = Z_{ij}(\xi,\mu) \otimes f_{(0)j/h}(\xi), \tag{7}$$

In terms of amputated Green functions of parton in parton:

Quark:

$$f_{(\mathrm{R})i/j}(\xi,\mu) = \frac{1}{6} \lim_{p^2 \to \overline{m}_j^2} \delta_{ab}(\not{p} + \overline{m}_j)_{\alpha\beta} \Gamma^{ab,\alpha\beta}_{(\mathrm{R})ij}(\xi p^+, p; \mu) \tag{8}$$

Gluon:

$$f_{(\mathrm{R})i/g}(\xi,\mu) = \frac{1}{16} \lim_{p^2 \to \overline{m}_j^2} \delta_{ab} d_{\mu\nu}(p) \Gamma^{ab,\mu\nu}_{(\mathrm{R})gj}(\xi p^+, p; \mu) \,, \tag{9}$$

Renormalized amputated Green functions:

$$\Gamma_{(\mathrm{R})ij}(\xi p^+, p; \mu) = Z_{ik}(\xi,\mu) \otimes \Gamma_{(0)kj}(\xi p^+, p) + \ldots, \tag{10}$$

DGLAP Equations and Splitting Functions

From renormalization group equations:

$$\mu^2 \frac{\mathrm{d}\mu^2}{\mu^2} f_{(\mathrm{R})i/h}(\xi, \mu) = P_{ij}(\xi, \mu) \otimes f_{(\mathrm{R})j/h}(\xi). \tag{11}$$

where (in matrix form)

$$P(\mu) = \mu^2 \frac{\mathrm{d}\mu^2}{\mu^2} \ln Z(\mu) \tag{12}$$

At one-loop order:

$$P_{ij}^{[1]}(\xi, \mu) = -\mu^2 \frac{\mathrm{d}\mu^2}{\mu^2} \ln Z_{ij}^{[1]}(\xi, \mu) \tag{13}$$

New Mass-Dependent MOM scheme

Definition

$$\Gamma^{(\gamma)}_{(\mathrm{R})}(p^2;\mu) = \overline{\Gamma}^{(\gamma)}(p^2) - \overline{\Gamma}^{(\gamma)}_{\mathrm{UV}}(p^2 = -\mu^2), \qquad (14)$$

Using once subtraction dispersion relation

$$f(x) - f(c) = \int_{x_{\min}}^{\infty} \mathrm{d}s \frac{x - c}{(s - x - \mathrm{i}\epsilon)(s - c)} \frac{\mathrm{Im} f(s)}{\pi}, \qquad (15)$$

we have:

$$\Gamma^{(\gamma)}_{(\mathrm{R})}(p^2;\mu) = \int_{p^2_{\min}}^{\infty} \mathrm{d}s \frac{s + \mu^2}{(s - p^2 - \mathrm{i}\epsilon)(s + \mu^2)} \frac{\mathrm{UIm}\overline{\Gamma}^{(\gamma)}(s)}{\pi} + \overline{\Gamma}^{(\gamma)}_{\mathrm{IR}}(p^2). \qquad (16)$$

UIm: the particular term of the imaginary part that generates the UV divergence.

Renormalized PDFs

New Mass-Dependent MOM scheme

$$\mathrm{Im}\,\Gamma^{(\gamma)} = \frac{1}{2}\sum_{\mathrm{cut}} \Gamma^{(\gamma)}_{\mathrm{cut}}\,. \tag{17}$$

$$\mathrm{UIm}\,\Gamma^{(\gamma)} = \frac{1}{2}\sum_{\mathrm{cut}} \left(\Gamma^{(\gamma)}_{\mathrm{cut}}\right)_{\mathrm{UV}} \tag{18}$$

Requiring just UIm of the diagrams, our prescription is followed by two simplifications:

1. Just UV limit of the cut diagrams should be calculated instead of their total value.
2. There is no need for a regulator since UIms are finite.

What we are supposed do is to calculate UIm of the amputated Green functions to renormalize them by Eq. (16).

New Mass-Dependent MOM scheme

Cut diagrams for the case of quark in quark

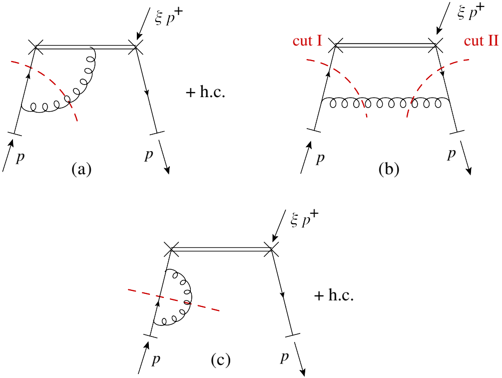


Figure: Cut diagrams of quark in quark amputated Green function at one-loop order.

New Mass-Dependent MOM scheme

UIm for the case of quark in quark

The total value of diagrams (a) and (a†), indicated by "h.c." in Fig.1(a), are exactly $\mathrm{UIm}\Gamma^{(\mathrm{a}+\mathrm{a}^\dagger)}$, which gives

$$\frac{1}{2}\Gamma^{(\mathrm{a}+\mathrm{a}^\dagger)}_{\mathrm{cut}}(\xi,p) = \frac{g^2}{16\pi}C_{\mathrm{F}}\frac{\gamma^+}{p^+}\left[\frac{2\xi}{1-\xi}\theta(p^2-m^2/\xi)\right]_+ \\ = \mathrm{UIm}\Gamma^{(\mathrm{a}+\mathrm{a}^\dagger)}(\xi,p) \tag{19}$$

New Mass-Dependent MOM scheme

UIm for the case of quark in quark

On the other hand, for the case of the cut diagram in Fig.1(b) we need to extract UIm from the total imaginary part which is given by

$$\frac{1}{2}\left(\Gamma^{(\mathrm{b})}_{\mathrm{cutI}}+\Gamma^{(\mathrm{b})}_{\mathrm{cutII}}\right)(\xi,p)=\frac{-g^2}{16\pi}C_{\mathrm{F}}(1-\xi)\theta(0<\xi<1)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{q}_{\mathbf{T}}^{\mathbf{2}}\,\frac{\mathbf{q}_{\mathbf{T}}^{\mathbf{2}}\frac{\gamma^+}{p^+}+2\xi^2p^+\gamma^- - 4m\xi}{\mathbf{q}_{\mathbf{T}}^{\mathbf{2}}+M(\xi,p^2)}\delta(\mathbf{q}_{\mathbf{T}}^{\mathbf{2}}+M(\xi,p^2)), \qquad (20)$$

where $M(\xi,p^2)\equiv(1-\xi)(m^2-\xi p^2)$.

As mentioned above, UIm can be derived from UV region of the integral, i.e., $\mathbf{q}_{\mathbf{T}}^{\mathbf{2}}\to\infty$. Therefore we have

$$\mathrm{UIm}\;\Gamma^{(\mathrm{b})}(\xi,p)=-\frac{g^2}{16\pi}C_{\mathrm{F}}\frac{\gamma^+}{p^+}(1-\xi)\theta(0<\xi<1)\theta\left(p^2-m^2/\xi\right)\;.$$

New Mass-Dependent MOM scheme

UIm for the case of quark in quark

The contribution of the diagrams (c) and (c†) amount to

$$\frac{1}{2}\Gamma_{\rm cut}^{(c+c^\dagger)}(\xi,p) = \frac{g^2}{16\pi}\int_0^1 d\alpha(1-\alpha) \times \int_0^{+\infty} d\mathbf{q_T^2}\delta\left(\mathbf{q_T^2}+M(\alpha,p^2)\right)\frac{\left(\mathbf{q_T^2}-m^2\right)\frac{\gamma^+}{p^+}+\alpha(\alpha-2)m^2}{\mathbf{q_T^2}-(1-\alpha)^2m^2}\,, \tag{21}$$

which the UV limit of the integral gives the associated UIm as

$$\mathrm{UIm}\Gamma^{(c+c^\dagger)}(\xi,p) = \frac{g^2}{16\pi}C_F\frac{\gamma^+}{p^+}\int_0^1 d\alpha(1-\alpha)\theta\left(p^2-m^2/\xi\right). \tag{22}$$

New Mass-Dependent MOM scheme

UIm for the case of quark in quark

Therefore, by substituting UIm in Eq. (16), the renormalaized Green function of quark in quark amounts to

$$\Gamma^{[1]}_{(\mathrm{R})qq}(\xi, p; \mu) = -\frac{g^2}{16\pi^2} C_{\mathrm{F}} \frac{\gamma^+}{p^+} \left[\frac{1+\xi^2}{1-\xi} \int_{\frac{m^2}{\xi}}^{\infty} \frac{\mathrm{d}s}{s-p^2} \frac{\mu^2+p^2}{\mu^2+s} \right]_+ + \Gamma_{\mathrm{IR}}(\xi, p) \,. \tag{23}$$

Notice the pure plus distribution form of the counterterm resulting in conservation of each flavor number. The renormalized Green function Eq. (23) is identical to subtracting the logarithm part of the bare one at renormalization point $-\mu^2$ as well as pole part of a regulator.

Cut diagrams for the case of gluon in quark

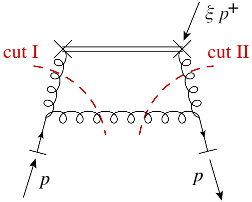


Figure: Cut diagrams of gluon in quark amputated Green function at one-loop order.

Cut diagrams for the case of gluon in quark

The same approach is applied to the other cases. There is one diagram with two cuts for the Green function of gluon in quark, depicted in Fig.2. Evaluated at large transverse momenta, the cut diagrams give

$$\mathrm{UIm}\Gamma^{[1]}_{(0)gq}(\xi, p; \mu) = \frac{g^2}{8\pi} C_{\mathrm{F}} \frac{\gamma^+}{p^+} \frac{2 - 2\xi + \xi^2}{\xi}\, \theta\left(p^2 - \frac{m^2}{\xi(1-\xi)}\right). \tag{24}$$

Notice the transformation $\xi \to 1 - \xi$ under which the counterterm of gluon in quark Green function transforms to counterterm of quark in quark Green function. This manifests the conservation of total momentum.

Cut diagrams for the case of quark in gluon

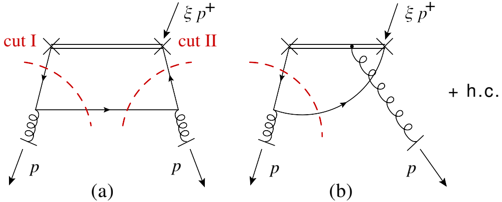


Figure: Cut diagrams of quark in gluon amputated Green function at one-loop order.

Cut diagrams for the case of gluon in quark

In general we have three cut diagrams associated with the case of quark in gluon, illustrated in Fig.3. To get rid of (b) we can project the Green function by $d_{\mu\nu}(p)$, defined in Eq. (5). By so doing, we sum over physical polarizations which results in extracting renormalization factors for unpolarized target gluon:

$$Z^{[1]}_{qg}(\xi,\mu) = \frac{1}{2}\left[d_{\mu\nu}(p)\Gamma^{(a)\mu\nu}_{qg}(\xi,p) - d_{\mu\nu}(p)\Gamma^{(a)\mu\nu}_{(\mathrm{R})qg}(\xi,p;\mu)\right]. \tag{25}$$

Therefore, to find the renormalization factor we just need to obtain UIm of the projected graph (a), which would be

$$\begin{aligned} &\mathrm{UIm}\left[d_{\mu\nu}(p)\Gamma^{(a)\mu\nu}_{qg}(\xi,p)\right] \\ &= -\frac{g^2}{8\pi}T_{\mathrm{R}}\left(2\xi^2 - 2\xi + 1\right)\theta\left(p^2 - \frac{m^2}{\xi(1-\xi)}\right), \end{aligned} \tag{26}$$

where T_{R} is conventional notation for the normalization of the SU(3) group generators.

Outline

Mass-Dependent Splitting Functions

$$P^{[1]}_{qq}(\xi, r) = \frac{g^2}{8\pi^2} C_{\mathrm{F}} \left[\frac{1+\xi^2}{1-\xi} \frac{\xi}{r+\xi} \right]_+ , \tag{27}$$

$$P^{[1]}_{gq}(\xi, r) = \frac{g^2}{8\pi^2} C_{\mathrm{F}} \left[\frac{1+(1-\xi)^2}{\xi} \frac{1-\xi}{r+1-\xi} \right], \tag{28}$$

$$P^{[1]}_{qg}(\xi, r) = \frac{g^2}{8\pi^2} T_{\mathrm{R}} \left[1 - \left(\frac{r}{r+\xi(1-\xi)} \right)^2 \right] \left[\xi^2 + (1-\xi)^2 \right], \tag{29}$$

where $r \equiv m^2/\mu^2$. The conservation of quarks number for each flavor implies that $P_{qq}(\xi)$ should be in a form of a plus distribution, which is automatically satisfied in our scheme. Moreover, the conservation of the total momentum results in the symmetry

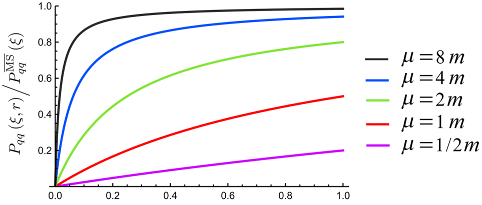
$$P_{gq}(1-\xi) = P_{qq}(\xi), \tag{30}$$

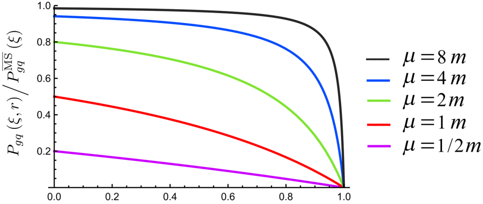
which is also automatically respected in our scheme. In addition to these, the other required symmetry,

$$P_{gg}(1-\xi) = P_{gg}(\xi), \tag{31}$$

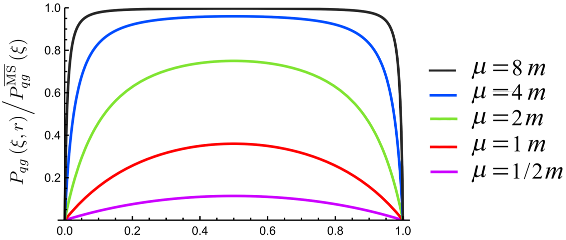
is also respected.

Mass-Dependent Splitting Functions





Mass-Dependent Splitting Functions



Mass-Dependent Splitting Functions

The mass correction can be determined using sum-rule

$$\int_0^1 \mathrm{d}\xi\ \xi P_{gg}(\xi,\mu) + \sum_i^6 \int_0^1 \mathrm{d}\xi\ \xi P_{q_i g}(\xi,\mu) = 0\ , \tag{32}$$

which is resulted from the conservation of the total momentum. That is a replacement of the flavor number n_f in the conventional P_{gg} with the summation

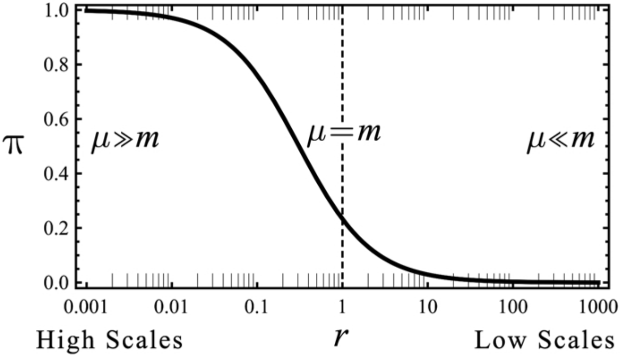
$$n_f \to \sum_{i=1}^{6} \pi(r_i), \tag{33}$$

where

$$\pi\,(r) \equiv \frac{1}{1+4r}\left[1 + r - 6r^2 + \frac{12r^3}{\sqrt{1+4r}} \ln \frac{\sqrt{1+4r}+1}{\sqrt{1+4r}-1}\right]. \tag{34}$$

Note that the flavor number is fixed at 6 in Eq. (33). Having a smooth behavior across the heavy quark thresholds, the function Eq. (34) is analogous with the step function $\theta(\mu - \mu_{\text{threshold}})$ in VFN schemes.

Mass-Dependent Splitting Functions



Outline

Conclusion

1. We introduced a new mass-dependent MOM scheme laking in common disadvantages of MOM schemes. Once subtraction dispersion relation is used to make the required calculations simpler.
2. This proposed scheme automatically respects all the symmetries required for the splitting functions.
3. In addition, required calculations are so simple that the scheme is applicable at higher-order approximations.
4. Although some phenomenology works are needed to test the theory, automatically decoupling heavy quark in low scales is obvious from the mass correction terms in the proposed splitting functions.


THANK YOU FOR YOUR TIME AND ATTENTION