

The effect of the magnetic field in the heavy-ion collision

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Part I: Concepts and Motivations

Structure of Matter

One of the main goals of physics is to study the structure of matter in different scales.

What are the fundamental degrees of freedom?

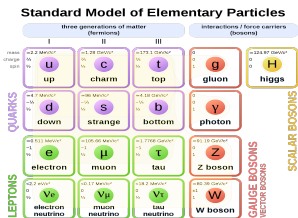
How these degrees of freedom arrange themselves in various ways?

Fire, Earth, Air, Water



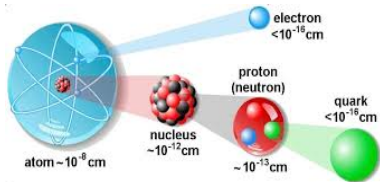
To

Standard Model of Physics

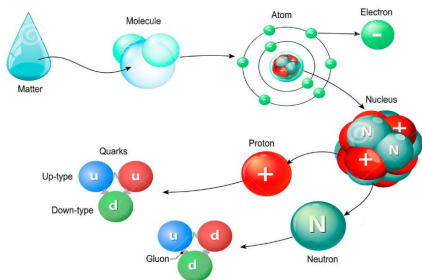
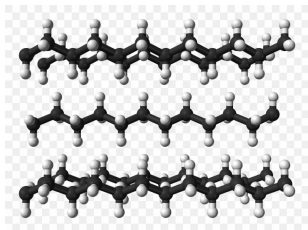


Structure of Matter

The hierarchical structure of matter reflects the existence of various length scales.



The interactions among the relevant degrees of freedom result in a variety of configurations.

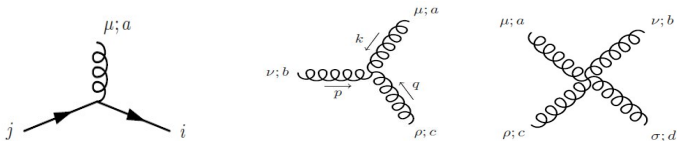


Some Aspects of QCD

At the scales in which the degrees of freedom are made of quarks and gluons, the question is, can strongly interacting matter exhibit different configurations (phases)?

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{\psi} (i\not{\partial} + gT^a A_\mu^a - m) \psi$$

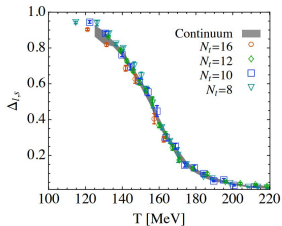
$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c,$$



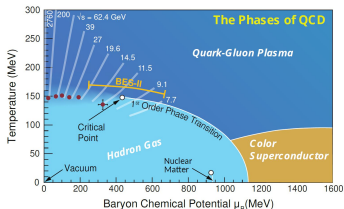
Some Aspects of QCD

At low temperature : $\left\{ \begin{array}{l} \text{The color charge is confined} \\ \text{The Chiral symmetry is broken} \end{array} \right.$

Absence of confinement in the $T \rightarrow \infty$ limit (A. M. Polyakov, Phys. Lett. B **72**, 477 (1978)).



arXiv:1005.3508



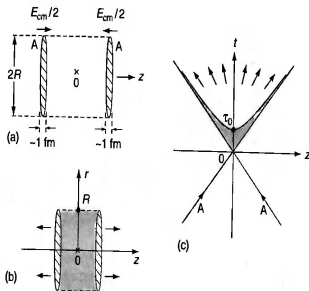
arXiv:1906.00936

Some Aspects of QCD

- The primary purpose is to produce a large system (relative to the size of proton) of matter at $T \approx 200\text{MeV} \approx 2.3 \times 10^{12}\text{K}$
- Such temperatures, more than 100,000 times that at the Sun's core, characterized the universe only a few microseconds after Big Bang.

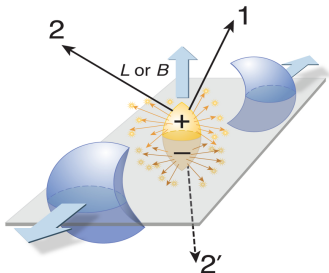
Heavy Ion Collision: Collision process

- 1 An excited system is formed and energy is released in a small volume subjected to a Lorentz contraction. A large number of particles are formed; the mean free path in the resulting system is small compared with its dimension.
- 2 Expansion of the system. The hydrodynamic approach must be used. During the expansion, the λ_{mfp} remains small in comparison to L .
- 3 Freeze-out: λ_{mfp} becomes longer and the interaction becomes weaker.



Heavy Ion Collision

Y. C. Liu and X. G. Huang, Nucl. Sci. Tech. **31**, no.6, 56 (2020)



- In the heavy-ion collision, an enormous \vec{B} and large \vec{L} is generated.
- Initial magnetic field $|\mathbf{B}| \propto Z_{Au} \gamma \frac{e^2}{4\pi} \frac{v_z^2}{(b/2)^2} \approx 10 m_\pi^2 \approx 10^{19} \text{Gauss}$ for $\sqrt{s} = 200 \text{GeV}$ and $b = 10 \text{fm}$
- Initial \mathbf{L} : $L_y \propto \frac{Ab\sqrt{s}}{2} \propto 10^6 \hbar$

Heavy Ion Collision

In the rest of this talk, I will focus on the \vec{B} :

Questions:

- What are the properties of the magnetic field at the early stage of a collision?
- How does the magnetic field evolve in the pre-equilibrium stage?
- Does the magnetic field survive until the hydrodynamical stage? If so, what is its effect on the expansion?
- What is the possible trace of the \vec{B} on the phase transition?

Magnetic Field at Early Stage of Collision

Lienard-Wiechert potentials:

$$eB(t, \vec{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\vec{v}_n \times \vec{R}_n}{(|\vec{R}_n| - \vec{R}_n \cdot \vec{v}_n)^3} (1 - |\vec{v}_n|^2),$$
$$eE(t, \vec{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\vec{R}_n - |\vec{R}_n| \vec{v}_n}{(|\vec{R}_n| - \vec{R}_n \cdot \vec{v}_n)^3} (1 - |\vec{v}_n|^2).$$

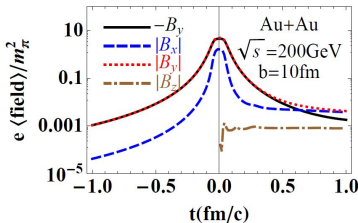
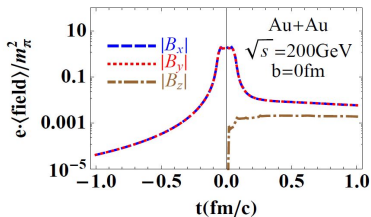
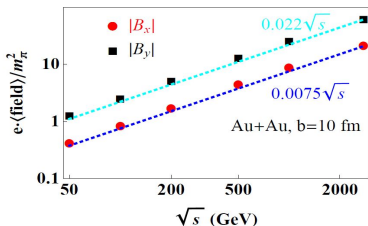
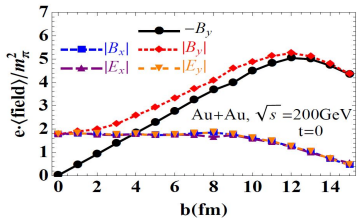
Where $\vec{R}_n = r - r_n$ is the relative position and v_n is the velocity of n-th particle at the retarded time $t_n = t - |\vec{r} - \vec{r}_n|$.

In non-relativistic limit, $v_n \ll 1$, this equations reduce to the Coulombs and Biot-Savart law

$$eE(t, \vec{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\vec{R}_n}{|\vec{R}_n|^3},$$
$$eB(t, \vec{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\vec{v}_n \times \vec{R}_n}{|\vec{R}_n|^3}$$

Magnetic Field at Early Stage of Collision

W. T. Deng and X. G. Huang, PRC (2012)



Magnetic Field at Early Stage of Collision

- Scaling law for electromagnetic fields

$$e \cdot \text{fields} \propto \sqrt{s} f\left(\frac{b}{R_A}\right),$$

Where R_A is the radius of the nucleus.

- Once the spectators are all away from the collision region, The lifetime of the magnetic field due to spectators is

$$t_B \approx \frac{R_A}{\gamma v_z} \approx \frac{2m_N}{\sqrt{s}} R_A,$$

Where for $Au + Au$ collision at $\sqrt{s} = 200 \text{ GeV}$ is,
 $t_B = 0.065 \text{ fm}$.

- The remnants which move slower than the spectators, essentially slow down the decay of transverse fields.

Magnetic Field at Hydrodynamical Stage

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\sigma \mu} (\nabla^2 \vec{B} - \mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2}),$$
$$\frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{v}}{\partial t} \times \vec{B} = \nabla \times (\vec{v} \times \vec{E}) + \frac{1}{\sigma \mu} (\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2})$$

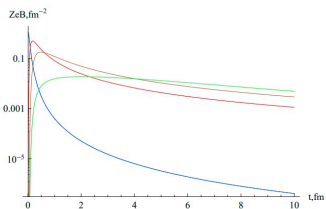
Magnetic Reynolds number is the ratio of convection over diffusion

$$R_m = LU\sigma\mu$$

- If $R_m \gg 1$ we can neglect the diffusion term.
- If $R_m \ll 1$ we can neglect the convection term.

The theoretical result of σ is quite uncertain.

Magnetic Field at Hydrodynamical Stage



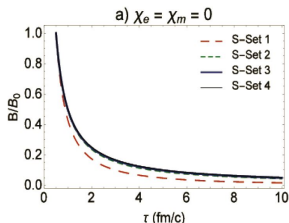
blue: vacuum

Red: $\sigma = 5.8 \text{ MeV}$

Brown: $\sigma = 16 \text{ MeV}$

Green: Expanding

[arXiv:1301.0099](https://arxiv.org/abs/1301.0099)



Set 1: $\sigma = 4 \text{ MeV}$

Set 2: $\sigma = 40 \text{ MeV}$

Set 3: $\sigma = 400 \text{ MeV}$

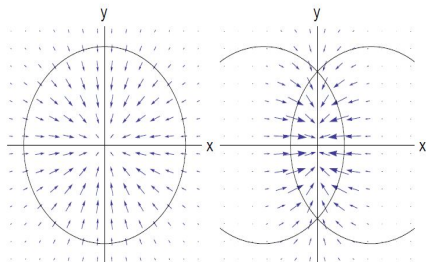
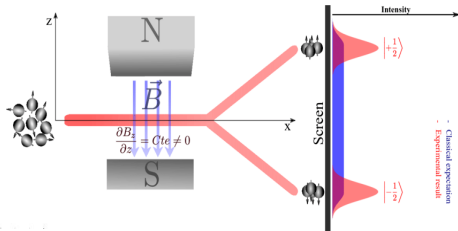
Set 4: $\sigma = 4000 \text{ MeV}$

[arXiv:1705.00536](https://arxiv.org/abs/1705.00536)

Part II: Spin interaction with background Magnetic field

Paramagnetic Squeezing

A non-uniform magnetic field can exert a force on a magnetic moment.



G. S. Bali et.al, PRL(2014)

An external inhomogeneous magnetic field exerts an anisotropic force on the system,

$$\vec{F} = (\vec{M} \cdot \vec{\nabla}) \vec{B}$$

The Energy-Momentum Tensor

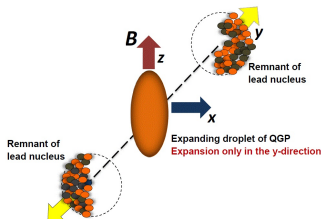
- In MHD the energy-momentum tensor is defined as:

$$T_{(0)}^{\mu\nu} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu} - \Pi^\mu u^\nu + F^\mu{}_\alpha M^{\nu\alpha} + F^\mu{}_\lambda F^{\lambda\nu} + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

With $\Pi^\mu \equiv 2u_\lambda F^{[\mu} M^{\lambda]\nu}$ and $\Delta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu$.

- In limit of vanishing electric field ($\Pi^\mu = 0$) :

$$T_{LRF}^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p_\perp & 0 & 0 \\ 0 & 0 & p_\perp & 0 \\ 0 & 0 & 0 & p_\parallel \end{pmatrix}$$



with $p_\perp = p - BM$ and $p_\parallel = p$.

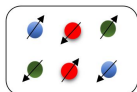
Wigner Function Approach



Classical Transport

- Boltzmann transport equation for f (distribution of particle):

$$p^\mu \partial_\mu^x f + F^\mu \partial_\mu^p f = C[f]$$



Quantum Transport

- In order to extend the phase space and describe the transport phenomenon related to spin, we need the spin distribution function.
- Quantum analogue of classical 1PD is the Wigner function.
- $f(x, \vec{p}) \rightarrow W(x, p)$

Wigner Function Approach

- In quantum mechanics, momentum and position are not simultaneously observable.
- Constructing the phase space by choosing the appropriate momentum and position, in analogy to the two-body system:

$$\begin{aligned}\hat{X} &\equiv \frac{1}{2} (\hat{X}_1 + \hat{X}_2), & \hat{Q} &\equiv \hat{P}_1 + \hat{P}_2, \\ \hat{Y} &\equiv (\hat{X}_1 - \hat{X}_2), & \hat{P} &\equiv \frac{1}{2} (\hat{P}_1 - \hat{P}_2),\end{aligned}$$

which leads to the

$$\begin{aligned}[\hat{Y}, \hat{P}] &= [\hat{X}, \hat{Q}] = i\hbar, \\ [\hat{X}, \hat{P}] &= [\hat{Y}, \hat{Q}] = 0\end{aligned}$$

Wigner Function Approach

E. P. Wigner, Phys. Rev. **40**, 749 (1932)

- Wigner suggested the following expression for probability in phase space

$$P = \left(\frac{1}{2\pi\hbar} \right)^n \int_{-\infty}^{\infty} dy_1 \cdots dy_n \psi^*(x_1 + y_1, \cdots, x_n + y_n) \psi(x_1 - y_1, \cdots, x_n - y_n) e^{2i(p_1 y_1 + \cdots + p_n y_n)/\hbar}$$

- It gives the probability in coordinate space when integrated with respect to p

$$\int dp_1 dp_2 \cdots dp_n P(x_1, \cdots, x_n; p_1, \cdots, p_n) = |\psi(x_1, \cdots, x_n)|^2$$

Wigner Function Approach

- It also gives the probability in momentum space when integrated with respect to x

$$\int dx_1 dx_2 \cdots dx_n P(x_1, \cdots, x_n; p_1, \cdots, p_n) = \left| \tilde{\psi}(p_1, \cdots, p_n) \right|^2$$

- $P(x_1, \cdots, x_n; p_1, \cdots, p_n)$ can not be interpreted as probability for coordinates and momenta because it may acquire negative values.

Wigner Function Approach

D. Vasak, M. Gyulassy and H. T. Elze, *Annals Phys*(1987)

- The Wigner function is defined from the Wigner operator $\mathcal{W}_{\alpha\beta}(r, k) = \langle : \hat{\mathcal{W}}_{\alpha\beta}(r, k) : \rangle$ Where

$$\hat{\mathcal{W}}_{\alpha\beta}(r, k) = \int d^4 r' e^{-ik \cdot r'} \bar{\psi}_\beta \left(r + \frac{r'}{2} \right) \mathcal{U} \left(r + \frac{r'}{2}, r - \frac{r'}{2} \right) \psi_\alpha \left(r - \frac{r'}{2} \right)$$

Where

$$\mathcal{U} \left(r + \frac{r'}{2}, r - \frac{r'}{2} \right) = \mathcal{P} \exp \left(ier'^\mu \int_0^1 ds A_\mu \left(r - \frac{r'}{2} + sr' \right) \right)$$

Wigner Function Approach

N. Sadooghi and S.M.A.T., PRD(2020)

Quantization of Dirac field in a uniform magnetic field

$$\begin{aligned}\Psi_\alpha(x) &= \frac{1}{\sqrt{V}} \sum_{n,s} \int \frac{dp_y dp_z}{(2\pi)^2} \frac{1}{\sqrt{2E_n}} \left(e^{-i(E_n t - p_y y - p_z z)} a_{p_y p_z, s}^{(n)} \right. \\ &\quad \left. \times \Psi_{\alpha\rho}^n(\xi) u_{s,\rho}(\tilde{p}) + e^{i(E_n t - p_y y - p_z z)} b_{p_y p_z, s}^{(n)\dagger} \Phi_{\alpha\rho}^n(\bar{\xi}) v_{s,\rho}(\tilde{p}) \right)\end{aligned}$$

With $\Pi_n = 1 - \delta_{n,0}$, $\xi = \sqrt{2eB} \left(x - \frac{p_y}{eB} \right)$, $\bar{\xi} = \sqrt{2eB} \left(x + \frac{p_y}{eB} \right)$,

$$\Psi^n(\xi) = \mathcal{P}_+ f_+^n(\xi) + \Pi_n \mathcal{P}_- f_-^n(\xi)$$

$$\Phi^n(\bar{\xi}) = \mathcal{P}_+ f_+^n(\bar{\xi}) + \Pi_n \mathcal{P}_- f_-^n(\bar{\xi})$$

$$f_+^n = N_n 2^{-n/2} e^{-\xi^2/4} H_n\left(\frac{\xi}{\sqrt{2}}\right)$$

$$f_-^n = N_{n-1} 2^{-(n-1)/2} e^{-\xi^2/4} H_{n-1}\left(\frac{\xi}{\sqrt{2}}\right)$$

$$\tilde{p}_{q,\kappa}^\mu = (E_n^q, 0, -\kappa s_q \sqrt{2n|qeB|}, p_z)$$

Wigner Function Approach

N. Sadooghi and S.M.A.T., PRD(2020)

Energy-momentum tensor can be derived from the Wigner function as follows

$$T^{\mu\nu} = \left(\frac{\delta_\lambda^\mu \delta_\sigma^\nu + \delta_\sigma^\mu \delta_\lambda^\nu}{2} - \eta^{\mu\nu} \eta_{\lambda\sigma} \right) \int \frac{d^4 k}{(2\pi)^4} k^\sigma \text{Tr}(\gamma^\lambda \mathcal{W}(r, k))$$

which leads to

$$T^{00} = \frac{eB}{\pi^2} \sum_{n=0}^{\infty} \alpha_n \int_0^{\infty} \frac{dk_z}{E_n} (E_n^2 - m^2) f(E_n),$$

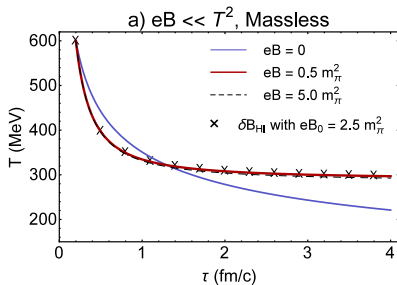
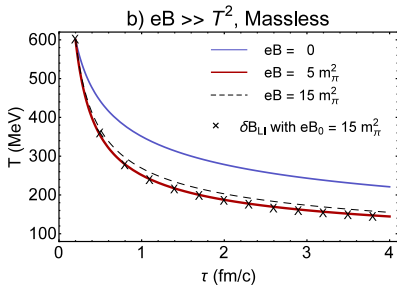
$$T^{11} = T^{22} = \frac{eB}{\pi^2} \sum_{n=0}^{\infty} \alpha_n \int_0^{\infty} \frac{dk_z}{E_n} (neB + m^2) f(E_n),$$

$$T^{33} = \frac{eB}{\pi^2} \sum_{n=0}^{\infty} \alpha_n \int_0^{\infty} \frac{dk_z}{E_n} (k_z^2 + m^2) f(E_n),$$

where $\alpha_n = 2 - \delta_{n,0}$ is the spin degeneracy factor.

Wigner Function Approach

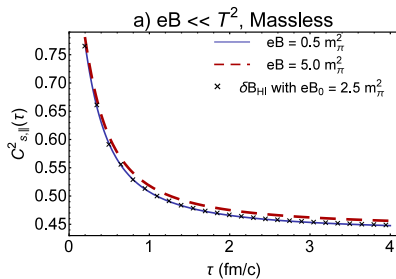
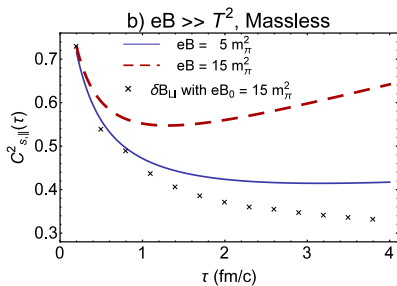
N. Sadooghi and S.M.A.T., PRD(2020)



$$\frac{d\epsilon}{d\tau} + \frac{1}{\tau}(\epsilon + p_{\parallel} - BM) = 0$$

Wigner Function Approach

N. Sadooghi and S.M.A.T., PRD(2020)



Thank you for your attention!