Equation of state of the strongly interacting matter at finite temperature and finite chemical potential using an exotic PNJL model

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SUBATECH
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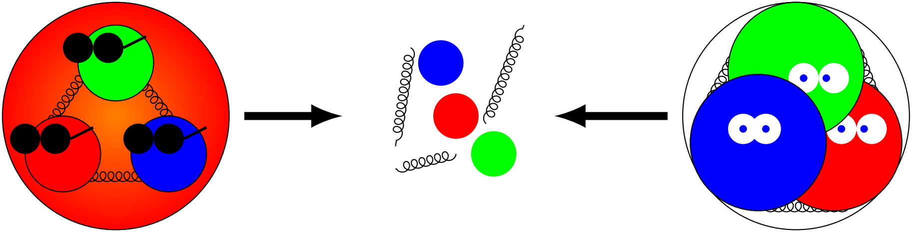
Link : Standard Model → everyday life matter ?

- **Hadronic phase** (proton, neutron..) : Quarks and gluons **confined**
  Experiments see hadronic matter

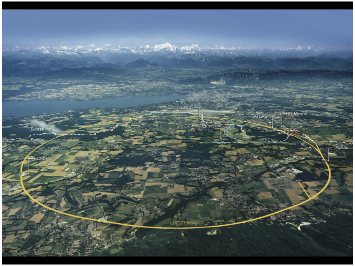
- **Quark-Gluons-Plasma** (QGP) phase : Quarks and gluons **free**

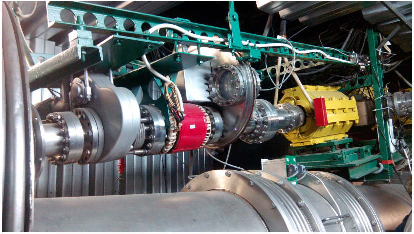


- Experimentally : **heavy ion collisions**
- QGP not directly observable
- But if QGP $\rightarrow$ There must be sign of it ! !
  $\rightarrow$ **probes** in hadronic matter observed by detectors

High E : LHC, RHIC

Lower E : NICA, FAIR/GSI, SPS



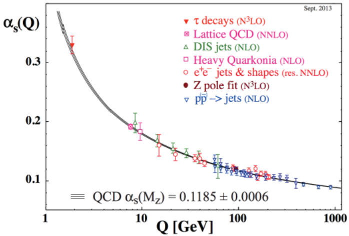


QCD Lagrangian : life is tough

$$\mathcal{L}_{QCD} = i\bar{\psi}^a_k \gamma^\mu \partial_\mu \psi^a_k + g_s \bar{\psi}^a_k \gamma^\mu \lambda^a A^a_\mu \psi^a_k - m_k \bar{\psi}^a_k \psi^a_k - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

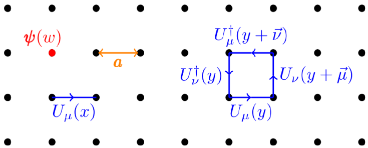
Perturbative approach pQCD

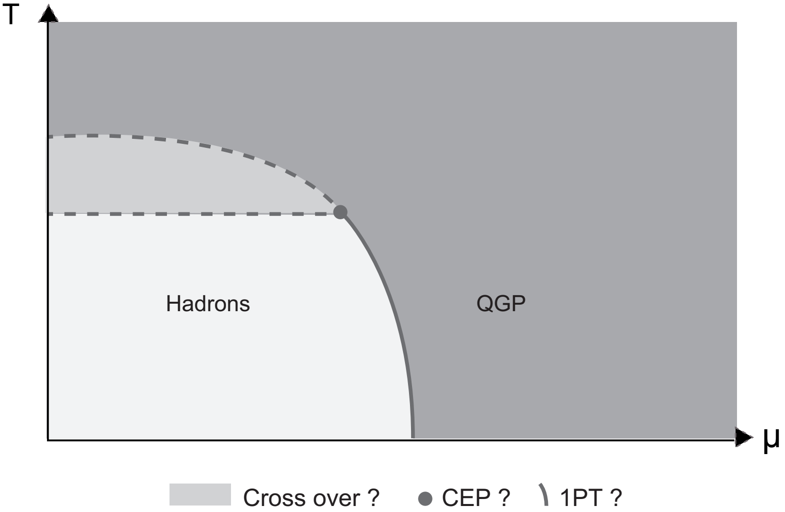
- **Asymptotic freedom** : QCD is **perturbative** at high E / high T, μ.
- Perturbative series as a function of the coupling constant.



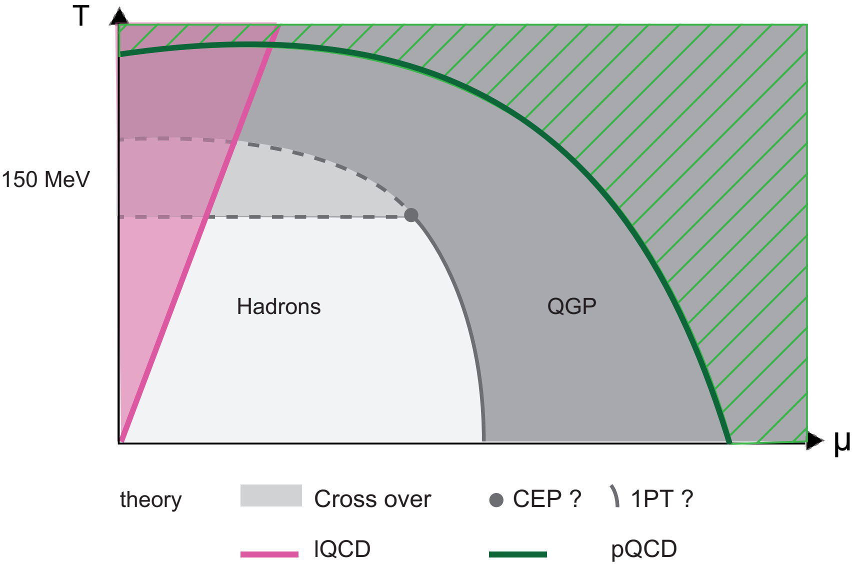
Lattice approach lQCD

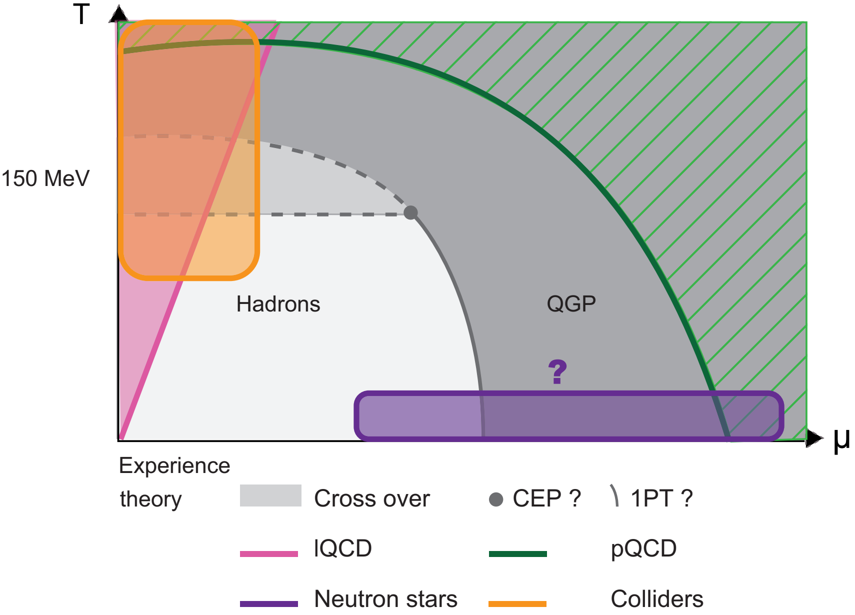
- Space-time **discretized** on a lattice.
- Matter = nodes, gluons = lines connecting nodes
- Does not work at finite μ





Possible phase diagram


T
150 MeV
Hadrons
QGP
μ
theory
Cross over
CEP ?
1PT ?
lQCD
pQCD

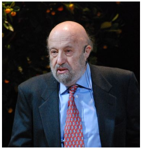

T
150 MeV
Hadrons
QGP
?
μ
Experience
theory
Cross over
CEP ?
1PT ?
lQCD
pQCD
Neutron stars
Colliders

- NICA, FAIR under construction.
- **Lower beam energies** : lower T and higher μ.
  $\rightarrow$ search for **phase transitions** (where, type) and **CEP**.
- Need **predictions** (which **beam energy**, which **observables**, which **behaviour** expected).

$\rightarrow$ **Effective models**

- (P)NJL model
  - Mass of the quarks
  - Mesons
  - Polyakov extension

Nambu



Jona-Lasinio

- Equation of state
  - Equation of state beyond mean field
  - Parametrisation of the Quark-gluon interaction
  - Phase diagram of strongly interacting matter

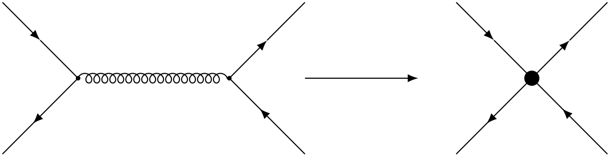
- Transport coefficients

Effective model

- **Approximations** of a more general theory.
- Restricted to a **special domain of energy** $\rightarrow$ low energy and finite μ.

Contact interaction - Fermi theory like

Static approximation : **no gluons** propagating the interaction



Frozen gluons

$$\frac{1}{p^2 - m_g^2} = -\frac{1}{m_g^2}$$

if $p \ll m_g$

Nambu-Jona-Lasinio (NJL) Lagrangian

$$\mathscr{L}_{NJL} = \overline{\psi}_k^a(i\partial\!\!\!/ - m)\psi_k^a + G(\overline{\psi}_k^a\gamma_\mu\psi_k^a)^2$$
$$+ K\left(det\left[(\bar{\psi}_k^a(1+\gamma_5)\psi_k^a\right] + det\left[\bar{\psi}_k^a(1-\gamma_5)\psi_k^a\right]\right)$$

∼ Symmetries of QCD : Good !

- ✓ Chiral symmetry $SU_L(3) \otimes SU_R(3)$: $mq \sim 0$
- ✓ Color symmetry $SU_c(3)$ (but global)
- ✓ Flavour symmetry $SU_f(3)$ $m_u = m_d = m_s$

Problems

- × No center symmetry Z(3) :**No Confinement**
- $U_A(1)$ broken by 6-fermion interaction term.
- × **Non renormalisable** : cutoff Λ(loops)

Free parameters

$m_q^0 = 0.0055\, GeV$

$m_s^0 = 0.134\, GeV$

$\Lambda = 0.569\, GeV$

$G = \frac{2.3}{\Lambda^2}\, GeV^{-2}$

$K = \frac{11}{\Lambda^5}\, GeV^{-5}$

Fixed by **experimental vacuum data**.

Mass of the quarks :

- **Grand potential** Ω in infinite volume calculated from **partition function** : $Z = Tr[\exp(-\beta\Omega)]$

- Partition function from NJL Lagrangian

$$Z = \int \mathscr{D}_{\psi}\mathscr{D}_{\bar{\psi}}\mathscr{D}_{A_{\mu}} \exp(i \int d^4x \mathscr{L}_{NJL})$$

- $\Omega_{NJL} = -2N_C(\int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} E_p + T \int_0^{\infty} (\ln[1 + \exp(-\beta(E_p - \mu))] + \ln[1 + \exp(-\beta(E_p + \mu))]) + 2G \sum_k < \bar{\psi}_k\psi_k >^2 - 4K \prod_k < \bar{\psi}_k\psi_k >$

Free part, Mean field corrections.

- Mass at thermodynamical equilibrium, $Z \rightarrow 1$:

$$\frac{d\Omega}{dm} = 0 \rightarrow \text{Gap equations.}$$

- Gap equations : mass = **Higgs** mechanism + medium interaction **chiral condensate** $\langle\bar{\psi}\psi\rangle$ (mean field)

$$M_k = m_k - 4G < \overline{\psi}_k \psi_k > (M_k) + 2K < \overline{\psi}_{k'} \psi_{k'} > (M'_k) < \overline{\psi}_{k''} \psi_{k''} > (M''_k)$$

- **Isospin** symmetry $m_u = m_d \rightarrow$ 2 unknowns M_u and M_s, 2 equations .
- $k \neq k' \neq k'' \rightarrow$ **flavour mixing**
- Need $\langle\overline{\psi}_k \psi_k\rangle(M_k) \sim$ one-fermion loop A_{NJL}.

$$A_{NJL}(\Lambda) = -4\int_0^{\Lambda} \frac{p^2 dp}{\sqrt{p^2+m^2}} \left(f\left(\frac{E-\mu}{T}\right) - f\left(\frac{-E-\mu}{T}\right)\right) \quad (1)$$

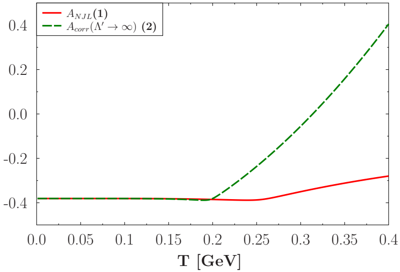
- Interaction non-renormalisable = one-fermion (A) loop is divergent.
- Need of the cutoff Λ

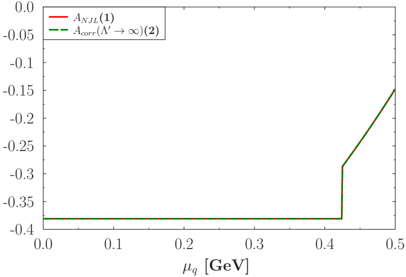
Separate the integral into a **divergent** one (vaccum part) and **convergent** one (thermal part).

$$A_{corr}(\Lambda,\Lambda') = -4\Big(\int_0^{\Lambda} \frac{p^2 dp}{\sqrt{p^2+m^2}} \quad (2)$$

$$-\int_0^{\Lambda'}\Big(\frac{p^2}{\sqrt{p^2+m^2}} f\left(\frac{E-\mu}{T}\right) + \frac{p^2}{\sqrt{p^2+m^2}} f\left(\frac{E+\mu}{T}\right)\Big)dp\Big) \quad (3)$$

- $A_{corr}(\Lambda,\Lambda) = A_{NJL}(\Lambda)$
- $\Lambda' \to \infty$ includes **large momenta** contributions

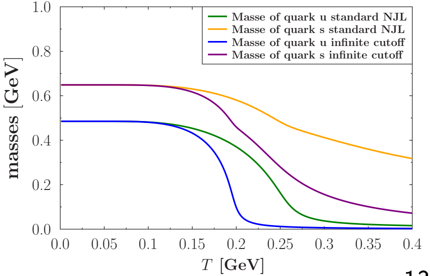




Large T, **large momenta contributions important** : free gas

Chiral symmetry broken by chiral condensate at low T.

Only Higgs mass at large T : chiral symmetry almost restored



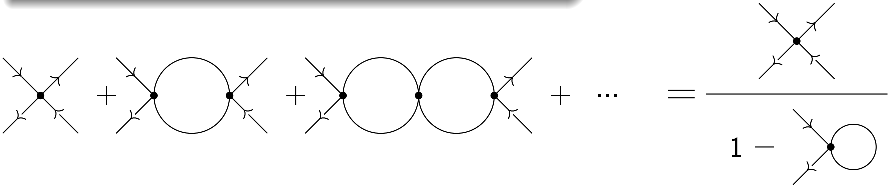
From quarks to hadrons : mesons

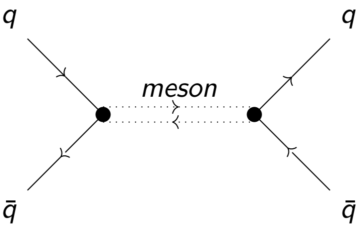
Quark-antiquark bound states

- NJL : **Degrees of Freedom** = quarks.
- Mesons : quark-antiquarks **bound states**.
  $\rightarrow$ Contact interaction **resummed**.

Amplitude

$$iU(k) = \Gamma \frac{2ig_m}{1-2g_m\Pi(k)}\Gamma$$



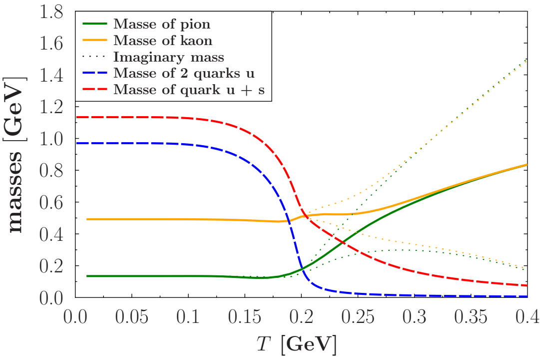


Amplitude of a meson exchanged

$$iU'(k) = \Gamma \frac{-ig_m^2}{k^2-m^2}\Gamma$$

Identify $iU(k)$ and $iU'(k)$

$$1 - 2g_m\Pi(k_0 = m, \vec{k} = 0) = 0$$



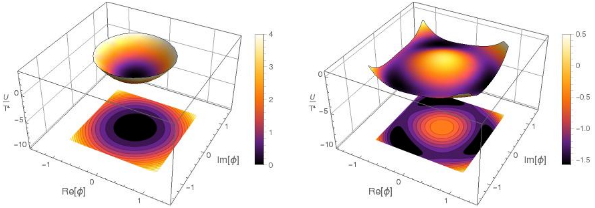
- Low T : meson mass < constituent quarks mass
- Large T : meson mass > constituent quarks mass
  $\rightarrow$ Decay Width - Imaginary mass.
- **Mott** Temperature : meson mass = constituent quarks mass
  $\rightarrow$ Nature realises the state of **least energy**.

Polyakov loop

- Confinement = effective potential $U(\phi, \bar{\phi}, T)$.
- Time component of gauge field A_μ (Polyakov gauge).
  PNJL = Frozen gluons + Thermal gluons.

Polyakov extended NJL Lagrangian

$$\mathscr{L}_{PNJL} = \overline{\psi}_k (i\partial\!\!\!/ - iA_0 - m)\psi_k + G(\overline{\psi}_k \lambda_i \psi_k)^2 + \text{'t Hooft} - U(\phi, \bar{\phi}, T)$$



$U(\phi, \bar{\phi}, T)$ for $T < T_0$ for $T > T_0$

$U(\phi, \bar{\phi}, T)$:

- Gluon mean field surrounding quarks → pressure of the medium
- Parameters fitted with lQCD **pure Yang Mills** pressure P_{YM}.
- T_0 : **critical temperature** for **confinement/deconfinement phase transition** in gluonic matter.

$$\frac{U(\phi,\bar{\phi},T)}{T^4} = -\frac{b_2(T)}{2}\bar{\phi}\phi - \frac{b_3}{6}(\bar{\phi}^3 + \phi^3) + \frac{b_4}{4}(\bar{\phi}\phi)^2$$

with the parameters : $b_2(T) = a_0 + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2 + a_3(\frac{T_0}{T})^3$

a_0	a_1	a_2	a_3	b_3	b_4	T_0
6.75	-1.95	2.625	-7.44	0.75	7.5	270 MeV

- iA_0 modifies quarks thermal distributions :

$$f_{\phi}^{+}(E_i-\mu_i)=\frac{\left[\phi+2\bar{\phi}\exp(\frac{-E_i-\mu_i}{T})\right]\exp(-\frac{E_i-\mu_i}{T})+\exp(-3\frac{E_i-\mu_i}{T})}{1+3\left[\phi+\bar{\phi}\exp(-\frac{E_i-\mu_i}{T})\right]\exp(-\frac{E_i-\mu_i}{T})+\exp(-3\frac{E_i-\mu_i}{T})}$$

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- For $\phi=\bar{\phi}=0$, "poor man's nucleon" : $E_N=3E,\mu_N=3\mu$

  $\rightarrow$ Leads to **statistical quarks suppression**.

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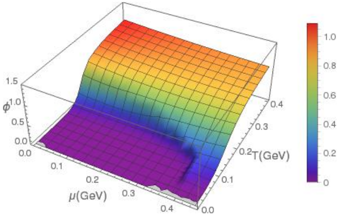
$$f_\phi^+(E_i-\mu_i)=\frac{\left[\phi+2\bar{\phi}\exp(\frac{-E_i-\mu_i}{T})\right]\exp(-\frac{E_i-\mu_i}{T})+\exp(-3\frac{E_i-\mu_i}{T})}{1+3\left[\phi+\bar{\phi}\exp(-\frac{E_i-\mu_i}{T})\right]\exp(-\frac{E_i-\mu_i}{T})+\exp(-3\frac{E_i-\mu_i}{T})}\sim\frac{\exp(-\frac{E_i-\mu_i}{T})}{1+\exp(-\frac{E_i-\mu_i}{T})}$$

- For $\phi=\bar{\phi}=0$, "poor man's nucleon" : $E_N=3E, \mu_N=3\mu$

  $\rightarrow$ Leads to **statistical quarks suppression** below T_c.

- For $\phi=\bar{\phi}=1$, NJL distributions.
- $\phi=0$, center symmetry not broken.
- $\phi\neq 0$ center symmetry broken.



19

Summary of the model

Good things

- ✓ Lagrangian which shares roughly the **symmetries** of the QCD Lagrangian
- ✓ Allows for studying the phase transition at finite μ
- ✓ Degrees of freedom = **quarks** but hadronic matter made from **bound states**

Bad things

- × **No dynamical gluons** in the interaction : **low energy** approximation.
- × 4-point interactions are **non renormalizable** : need of a **cut-off**.

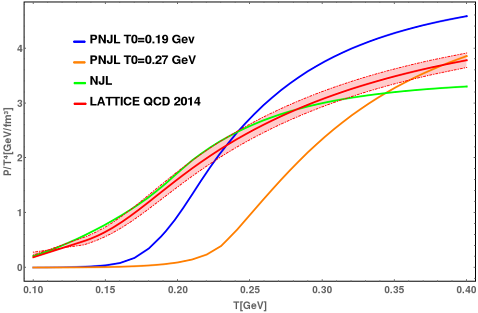
PNJL Equation of State

Equation of state $\rightarrow$ relates macroscopic variables $P \leftrightarrow$ T, μ

- All thermodynamical quantities derived from **Grand potential** Ω_{PNJL}.
- Pressure :$P(T,\mu) = -\Omega(T,\mu)$
- Entropy density : $S(T,\mu) = -\frac{\partial\Omega(T,\mu)}{\partial T} = \frac{\partial P(T,\mu)}{\partial T}$
- Energy density :

$$E(T,\mu) = TS - P + N\mu = -T\frac{\partial\Omega}{\partial T} - P - \mu\frac{\partial\Omega}{\partial\mu} = T\frac{\partial P}{\partial T} - P + \mu\frac{\partial P}{\partial\mu}$$

- Interaction measure : $I(T,\mu) = E - 3P$



- ✓ PNJL "confinement", P = 0 below T_C.
- × Artifact of NJL, unphysical quark pressure at low T.
- → Need hadrons to improve low T.
- × NJL misses gluons at large T.
- × PNJL : gluon contribution with different T_C for confinement. Not in agreement with lQCD.
- → Need a better description at large T.

$$\text{'t Hooft scaling}: g^2 \to g^2 \frac{N_C}{N_C} = (g^2 N_C) \frac{1}{N_C}$$

$$g \to 0,\ N_C \to \infty,\ g^2 N_C \to cst$$

$$g^{2i} N_c^k \equiv (g^2 N_c)^i {N_c}^{k-i}$$

k is the number of fermion lines and i is the number of interaction lines.

$$iS_\Sigma(p) = iS(p)\Big(O(1)O(1) + O((g^2 N_c))O(1) + O((g^2 N_c))O(\frac{1}{N_C}) + O((g^2 N_c)^2)O(\frac{1}{N_C}) + \ldots\Big)$$

Beth-Uhlenbeck approach

Grand potential Ω_M : resummation of  +...

$$\Omega_M = \frac{1}{2}\int_0^1 \frac{d\lambda}{\lambda} T \sum_n \int \frac{d^3p}{(2\pi)^3} Tr\left[S^\lambda(i\nu_n,\vec{p})\, \Sigma^\lambda(i\nu_n,\vec{p})\right]$$

$$\Sigma^\lambda(i\nu_n,\vec{p}) = T\sum_m \int \frac{d^3\vec{k}}{(2\pi)^3}\Omega S(i\omega_m,\vec{p}-\vec{k})\bar{\Omega}\frac{2\lambda K}{1-2\lambda K\Pi(i\nu_n-i\omega_m,\vec{k})}$$

$\rightarrow$ Amplitude of exchanged **meson** :

$$\Omega_M = \frac{g_M}{2}\int \frac{d^3p}{(2\pi)^3}\int_0^\infty d\omega\left(1+\frac{1}{\exp(\beta(\omega-\mu_M))-1}+\frac{1}{\exp(\beta(\omega+\mu_M))-1}\right)$$

$$\times \ln\left[\frac{1-2G\Pi(\omega-\mu_M+i\epsilon,\vec{p})}{1-2G\Pi(\omega-\mu_M-i\epsilon,\vec{p})}\right]$$

$\rightarrow$ How to calculate this ? Connexion with S-matrix formalism.

- **S-matrix** formalism from collision theory : describes interaction happening between asymptotic states $t^{-\infty}$- $t^{+\infty}$.
- Asymptotic states : quark, antiquark.
- Interaction : **exchanged meson**.

$$S(E,\vec{p}) = \exp(2i\delta_M(E,\vec{p})) = \frac{1-2G\Pi(\omega-\mu_M-i\epsilon,\vec{p})}{1-2G\Pi(\omega-\mu_M+i\epsilon,\vec{p})}$$

Where δ_M is :

$$\delta_M(E,\vec{p}) = \frac{1}{2i}\ln\left[\frac{1-2G\Pi(\omega-\mu_M-i\epsilon,\vec{p})}{1-2G\Pi(\omega-\mu_M+i\epsilon,\vec{p})}\right]$$
$$\sim\ -Arg[1-2K_M\Pi_M(\omega=\sqrt{s},\vec{p}=0)] = \delta_M(\sqrt{s},T,\mu)$$

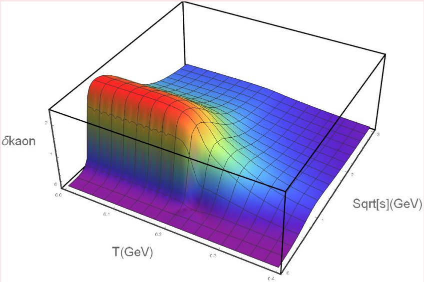
- Corrections all included in the phase shift.
- **Mesons included naturally in the partition function as explicit degrees of freedom.**

Mesonic grand potential

$$\Omega_M = -\frac{g_M}{8\pi^3}\int dpp^2 \int \frac{ds}{\sqrt{s+p^2}} f(s, T, \mu)\delta_M(s, T, mu)$$

Phase shift δ_M

- Mesonic pressure.
- Vanish at large T.
- ✓ Correction to the low T pressure.
- lightest mesons dominate : π, K and scalar partners of π.



a_0	a_1	a_2	a_3	b_3	b_4	T_0
6.75	-1.95	2.625	-7.44	0.75	7.5	270 MeV

Traditional PNJL

Parameter $T_0 = 270 MeV$: critical temperature.
T_c of **pure Yang-Mills** confinement/deconfinement phase transition.

Quarks are here too !

Work of Haas and al : include $q\bar{q}$ excitations.
$T^{eff} = \frac{T - T_c}{T_c} \rightarrow T^{eff}_{YM} \simeq 0.57\, T^{eff}_{rs}$
Rescale the critical temperature to $T_0 = 190 MeV$
No real quark-gluon interaction

https ://arxiv.org/abs/1302.1993, Haas and al.

Phenomenological quark-gluons interaction

Temperature dependance in the rescaling :

$$\tau = 0.57\frac{T - T_0(T)}{T_0(T)}$$

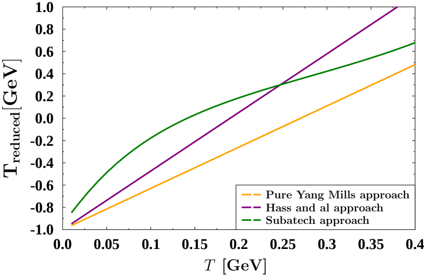
where : $T_0 = a + bT + cT^2 + dT^3 + e\frac{1}{T}$

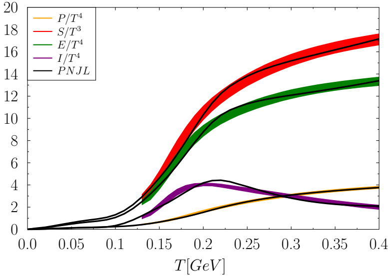
and : $b_2(T) = a_0 + \frac{a_1}{1+\tau} + \frac{a_2}{(1+\tau)^2} + \frac{a_3}{(1+\tau)^3}$

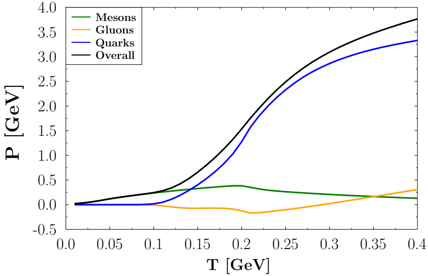
a	b	c	d	e
0.082	0.36	0.72	-1.6	-0.0002

Parameters fitted to reproduce lattice pressure.

Same asymptotic pressure.







https ://arxiv.org/abs/1407.6387v2, HotQCD Collaboration

We reproduce lattice results at $\mu = 0$

Effective model based on a **Lagrangian** that shares **QCD symmetries** and match **lattice** results.

Allows **finite** μ study.

Lattice at finite μ

Taylor expansion at $\mu = 0$: $\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + \dots$

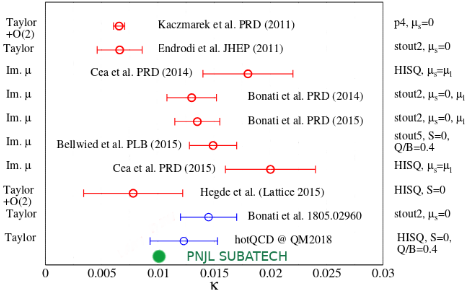
κ : curvature of critical line

"On the critical line of 2+1 flavor QCD" Cea, Cosmai, Papa

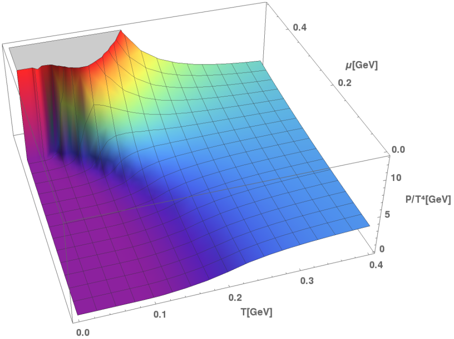
$$\kappa = -T_c(0) \frac{\partial T_c(\mu_B)}{\partial \mu_B^2}\bigg|_{\mu_B=0}$$

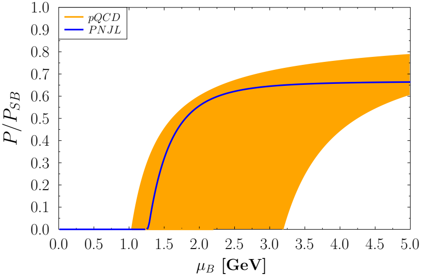
Our κ agrees with lattice results

At $\mu_B = 0$, we get the critical temperature : $T_c = 204 MeV$ and $\kappa = 0.00989$



Aleksi Kurkela and Aleksi Vuorinen, Cool quark matter, Phys. Rev. Lett. 117, 042501 (2016)

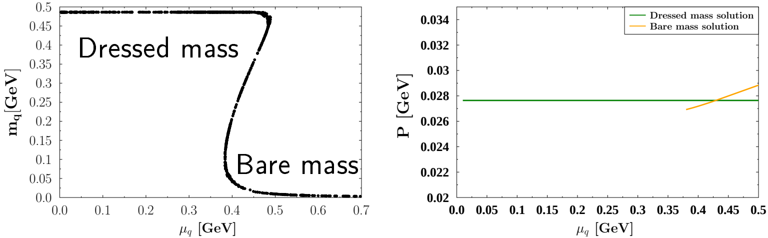




Finite μ

- Match pQCD predictions at large μ
- Phase diagram of QCD at finite T and μ
- CEP ? 1^{st} order phase transition ?

- Solutions for bare and dressed quarks mass.
- Region with 3 solutions $\rightarrow 1^{st}$ order transition
- μ_C at $T = 0$? $\rightarrow$ Maxwell construction.



More accurate from pressure : $\mu_C = 0.425$ GeV at T=0.

Critical End Point

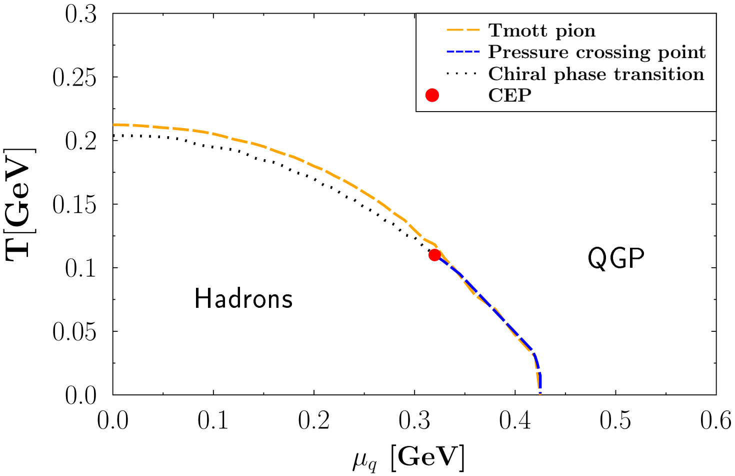
6 equations - 6 unknowns

- Masses from the 2 gap equations $\rightarrow m_q,\ m_s$
- Polyakov loops from minimisation of Ω : $\frac{\partial \Omega_{PNJL}}{\partial \phi, \bar{\phi}} = 0 \rightarrow \phi,\ \bar{\phi}$
- CEP (2nd order phase transition) from $\frac{\partial \mu}{\partial mq} = 0 \quad \frac{\partial^2 \mu}{\partial mq^2} = 0$

$\rightarrow m_q^{CEP},\ m_s^{CEP},\ \phi_{CEP},\ \bar{\phi}_{CEP},\ T_{CEP},\ \mu_{CEP}$

$$(T_{CEP} = 0.11\ GeV, \mu_{CEP} = 0.32\ GeV)$$

Alexandre Biguet, PhD thesis, https ://tel.archives-ouvertes.fr/tel-01453184/document

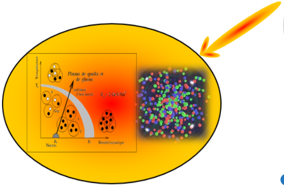


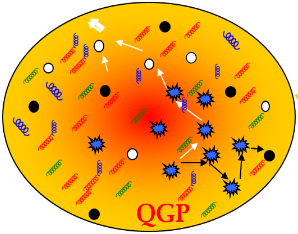
Chiral phase transition : m_q inflexion point.

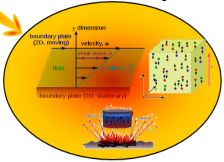
Transport coefficients

Pictures Hamza Berrehrah

In collaboration with Olga Soloveva and Elena Bratkovskaya, Frankfort university







- Thermodynamic properties (T, μ)
  - $\mathbf{P}, \epsilon, \mathbf{s}, \mathbf{I}$ (T, μ)
  - c_s^2 (T, μ)

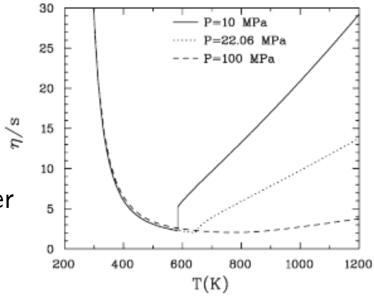
- Microscopic models hard to compare

  ← Must all give same →

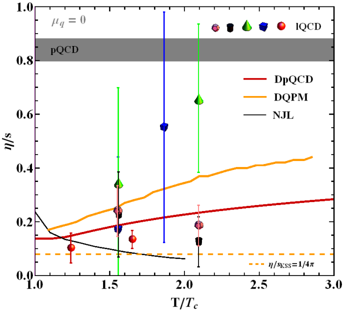
- Transport properties (T, μ)
  - Shear viscosity
  - Electric conductivity

- Shear viscosity : phase transitions (1st order, cross over), degrees of freedom (quarks, hadrons, quasiparticles), coupling of the QGP (strong, weak).

Water



Laszlo P. Csernai and al Phys. Rev. Lett. 97, 1523034 2006

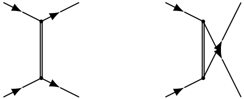


H.Berrehrah Hot Quarks 14

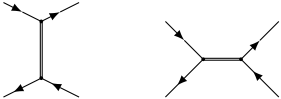
- Electric conductivity : aptitude of a medium to allow electric charges to move freely.
  $\rightarrow$ transport of electric charge in the QGP, response of the medium to electric field (polarisation)

Elastic cross section with PNJL

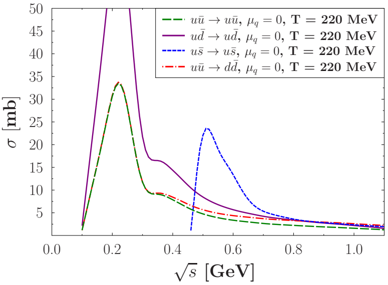
- Transport coefficients calculated from **microscopic cross section**.
- Elastic q-q, q-$\bar{q}$ cross sections with **meson exchange**.
- Quark-quark process :
  Meson is **space-like** $\rightarrow$ no pole.

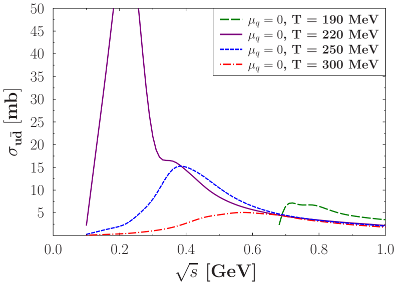


- Quark-antiquark process :
  Meson is **time-like** in s channel $\rightarrow$ **Pole**.



- **Resonance at sqrt(s) = meson mass in s channel of quark-antiquark cross sections**.





- No cross section before threshold $\sqrt{s} = Max(m_1 + m_2, m_3 + m_4)$.
- q-$\bar{q}$ $\to$ **resonant** at $\sqrt{s} = m_{meson}$
- Cross section maximum around phase transition.
- Peak shifted and lowered with increasing T as meson mass increases
- Cross sections vanishing at large T.

$$\eta(T,\mu_q) = \frac{1}{15T}\sum_{i=q,\bar{q}}\int \frac{d^3p}{(2\pi)^3}\frac{\mathrm{p}^4}{E_i^2}\tau_i(T,\mu_q)\cdot d_q f_i^{\phi}$$

$$\sigma_0(T,\mu_q) = \frac{e^2}{3T}\sum_{i=q,\bar{q}} q_i^2 \int \frac{d^3p}{(2\pi)^3}\frac{\mathrm{p}^2}{E_i^2}\cdot\tau_i(T,\mu_q)\; d_q\; f_i^{\phi}$$

- Shear viscosity and electric conductivity calculated from Boltzmann equation in RTA approximation, proportional to relaxation time

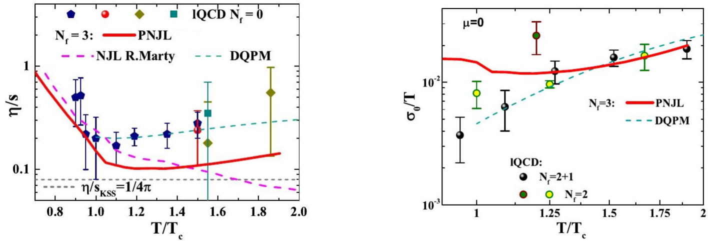
$$\tau_i^{-1}(p_i,T,\mu_q) = \sum_{j=q,\bar{q}}\int \frac{\mathrm{d}^3p_j}{(2\pi)^3}\; d_q f_j^{(0)}(E_j,T,\mu_q)v_{\mathrm{rel}}\sigma_{ij\to cd}(s,T,\mu_q)$$

- Relaxation time inversely proportional to cross sections.

  $\rightarrow$ Interactions help the system to reach equilibrium

  $\rightarrow$ Maximum of the cross section, minimum of the the shear viscosity, electric conductivity.

Comparison with lattice results at $\mu = 0$



- Maximum of cross section → minimum of η/S.

✓ Good agreement above T_C. 2 improvements compare to R. Marty :

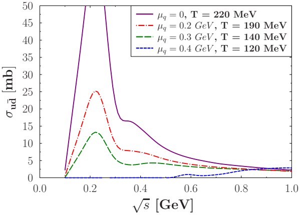
  - **Entropy** agrees with lattice results → better than R. Marty.
  - Use of A_{corr} → Cross section lower → η/S larger at large T.

× lQCD shear viscosity : only gluons.

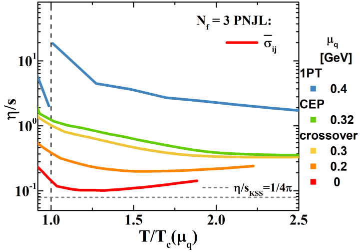
✓ Electric conductivity : **only quarks**, gluons electrically neutral.

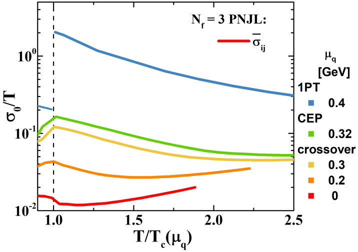
× Bad around and below T_C : quark-hadron cross sections missing.

Finite μ results



- Resonance decreasing at finite μ.
- **Increase of η/S and σ/T.**





- **No sign of CEP** from the shear viscosity or the electric conductivity.
- **Sign of 1^{st} order** phase transition, **vertical discontinuity** at T_C.

Conclusion (1)

- PNJL : effective model to study the phase diagram at finite μ.

PNJL + T0(T) + Pressure beyond mean field (mesons)

=

- ✓ Phase diagram of QCD matter
- ✓ Lattice equation of state at $\mu = 0$.
- ✓ Lattice equation of state at $\mu \simeq 0$.
- ✓ pQCD results for pressure at large μ
- ✓ Cross over transition at low μ
- ✓ First order phase transition at low T
- ✓ CEP coordinates : ($T_{CEP} = 0.11\ GeV$,$\mu_{CEP} = 0.32\ GeV$)

Conclusion (2)

✓ Shear viscosity and electric conductivity in agreement with the lattice results at vanishing μ.

- No sign of CEP from the shear viscosity and the electric conductivity, but sign of first order phase transition.

Perspectives

- Pressure beyond mean field, but chiral condensate calculated in mean field. *E. Quack and S. P. Klevansky, Phys rev C, V49, nb6 (1994)*

  O(10%) and 16% for the masses

- Baryons description need to be improved and added as a beyond mean field correction.

- Add quark-hadron cross sections to better describe the transport coefficient around T_C.

Thank you for your attention ! !

- Boltzman equation :

$$\frac{df(x,k)}{dt} = C[f(x,k)] \quad (4)$$

- Non equilibrium :

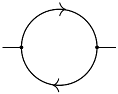
$$f(x,k) = f_0(x,k) + \delta f(x,k) \rightarrow \frac{df}{dt} \sim \frac{df_0}{dt} \quad (5)$$

- Collision term $C[f(x,k)] \rightarrow$ **Relaxation Time Approximation RTA** :

$$C[f(x,k)] = -E\frac{\delta f}{\tau}. \quad (6)$$

- Non equilibrium described by δf, equilibrium disappear in collision term.
- System goes back to equilibrium with a **characteristic time** τ.
- Shear viscosity and electric conductivity calculated from the RTA.

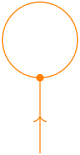
Polarisation function Π

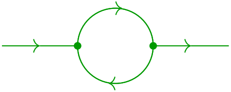


$$\Pi^{ps}(k) = -\frac{TN_c}{2}\sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} Tr(i\gamma_5 S(i\omega_n, \vec{p}) i\gamma_5 S(i\omega_n - \nu_n, \vec{k} - \vec{p}))$$

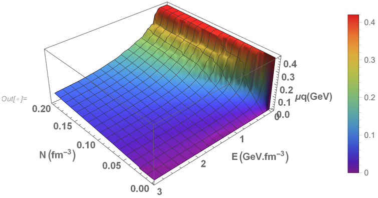
Written in terms of the one-fermion loop A_{corr} and the two-fermion loop B_0

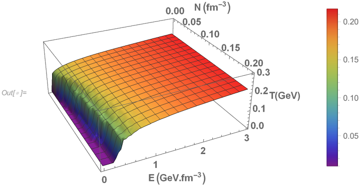
$$\Pi^{ps}_{ff'}(k_0, \vec{k}) = -\frac{N_c}{4\pi^2}[A_{corr}(\Lambda, \infty) + A_{corr}(\Lambda, \infty) + [(m_f - m_{f'})^2 - (k_0 + \mu_f - \mu_{f'})^2 + \vec{k}^2] B_0(\vec{k}, k_0, \Lambda)]$$





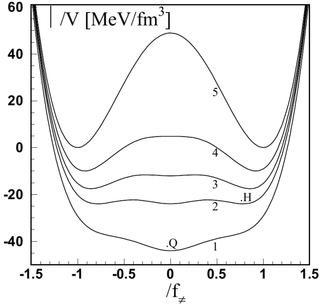
Theory : P(T,μ), E(T,μ), Nb(T,μ)
Experiment : P(E,Nb), T(E,Nb), μ(E,Nb)
Use of the monotonic property of E and Nb to inverse the equation of state.





Nucleation

- Non equilibrium phase transition
- In 1^{st} order phase transition : metastable phases.
- Fluctuation of energy density :
  $\rightarrow$ bubbles of high baryonic density in Plasma phase.



Coupling Constant integration

- Technic to relate Ω to green functions.
- Rescale the four point interaction coupling constant : $K \rightarrow \lambda K$, with $0 < \lambda < 1$
- Differentiate Ω with respect to λ.
- Real system $\lambda = 1$, non interacting system $\lambda = 0$.
- $Omega(\lambda = 1) - Omega(\lambda = 0) = \int_0^1 \frac{d\lambda}{\lambda} ...$

Boltzman equation

- $\frac{df(x,p^*,t}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial p^*}\frac{dp^*}{dt} = C_a$
- p^* dispersion relation : quasiparticle properties.
- Mass dynamically shifted.

Shear viscosity

- Difference in velocity between adjacent layers of the fluid : velocity gradient.
- To keep one layer of fluid moving at a greater velocity than the adjacent layer : shearing stress.
- Shear viscosity : shearing stress/velocity gradient
- Specific shear viscosity, comparison of viscosity at different T scale.
- Fluctuation-dissipation theorem - Green-Kubo formalism
- Viscosity, conductivity inversely proportional to interaction rate, like with RTA approximation.

lQCD

- space-time discretised over lattice.
- Any closed loop is gauge invariant : simplest gauge action is the smallest plaquette.
- Gauge invariant, no need of gauge fixing.
- Infinite lattice + zero lattice spacing $\rightarrow$ QCD continuum
- Heavy computational cost : extrapolation from simulations with several lattice spacing.
- $Z = \int \mathscr{D}_U \exp(-S_{YM})\mathrm{Det}(M(U, \mu, m_q))$
- Quenched approximation : Det(M) = cst. No quark-antiquark excitation in gluon propagator.
- Finite chemical potential : Det(M) is complex $\rightarrow$ Sign problem.
- Integral solvec with Monte-Carlo method.

- Regularisation scheme
  - Make a divergent integral finite.
  - Scheme $\rightarrow$ final result independant of the scheme
  - Different possible regularisation schemes
- Resummation :
  - Series with next order not that small :
    Need to resum all the terms to show that it is convergent.
  - Series which is not convergent : change the paremeters of expansion a into a*b and resum on this parameter.
- Renormalisation :
  - Define a system at a given scale
  - Zoom out the system to reduce the degrees of freedom.
  - Use of regularisation and resummation to obtain finite values.
  - The result for an observable must be independant of the the scheme and scale choice.

DQPM

- Propagators with imaginary part.
- Imaginary part defines spectral functions and width of degrees of freedom
- Off-shell particles and dynamics of the spectral information

How to obtain cross section from Lagrangian ?

- Calculate Feynman rules from the Lagrangian
  - Equation of motion of kinetic terms : propagators
  - Interaction term : vertex of interaction.
- Feynman rules give the amplitude associated to the Feynman diagrams averaged initial states and summed over final states.
- The final state phase space of the collisions
- The incident flux of particles

Sign problem

- Partition function : $Z = \int \mathscr{D}_U \mathscr{D}_{\bar{\psi}} \mathscr{D}_{\psi} \exp(-S)$
- With the action :
  $S = \int d^4x \bar{\psi}(\gamma_\nu(\partial_\nu + iA_\nu) + \mu\gamma_4 + m)\psi = \int d^4x \bar{\psi} M \psi$
- μ appears as an A_4 imaginary quadrivector and :
  $M = \gamma_\nu \partial_\nu + i\gamma_\nu A_\nu + \mu\gamma_4 + m$
- We then have :
  $M^\dagger(\mu) = M(-\mu^*)$
- The action is now complex. It can be seen using the hermiticity of the γ_5 matrix. M hermiticity valide at $\mu = 0$ and but not for finite μ.

$U_A(1)$ anomaly

- Classical action invariant $\rightarrow$ symmetry.
- Quantum action not invariant $\rightarrow$ symmetry broken.
- Symmetry broken by quantum fluctuation : Anomalies !

S matrix

$S(p, E) = \exp(2i\delta(\vec{p}, E))) = \frac{F_J(\vec{k}, E^*)}{F_J(\vec{k}, E)}$

The zeroes of the Jost function are the poles of the S-matrix.

- S-matrix has a pole at k $= +$iκ : Bound states have exponentially decaying solutions.
- Poles in the lower half plane can be written as k $=$ -i$\kappa + \gamma$
    - γ= 0, resonances
    - γ = 0, antibound or virtual states.