What are the two major themes of theoretical high energy physics these days?

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Two major themes

1) The Swampland program

2) Possible connections between QI and geometry

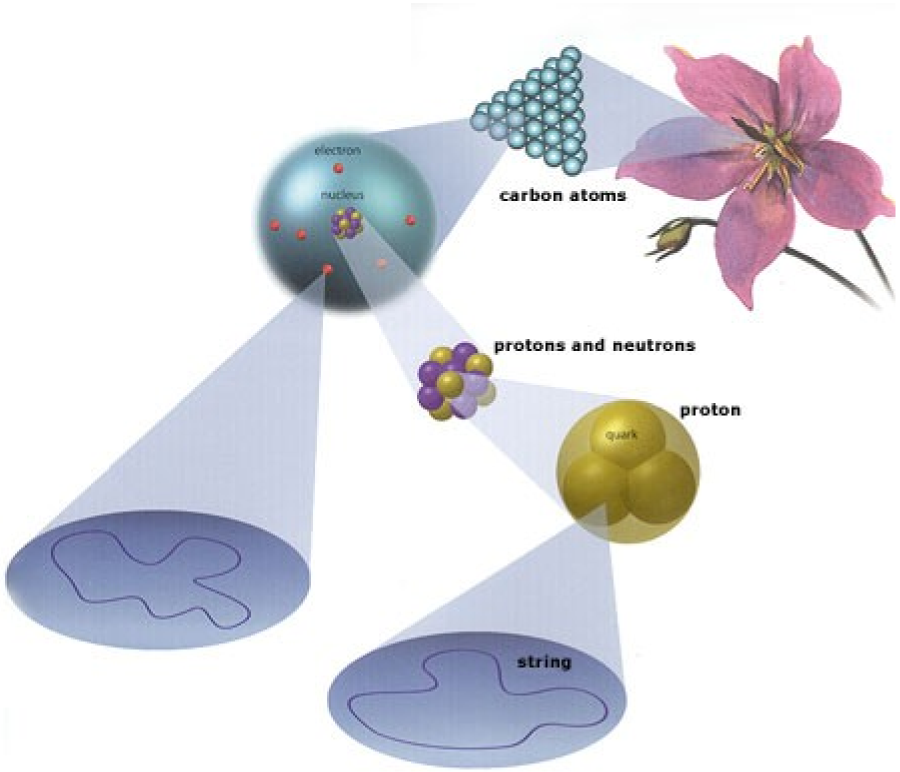
These two concentrate on a same thing, quantum gravity, of course from the different point of view

What is the problem

- Quantum Field Theory perfectly works while gravity is not included.
- But we do need the quantum theories on dynamical backgrounds and it is very non-trivial to get a consistent quantum gravity, mainly because of the non-renormalizability.

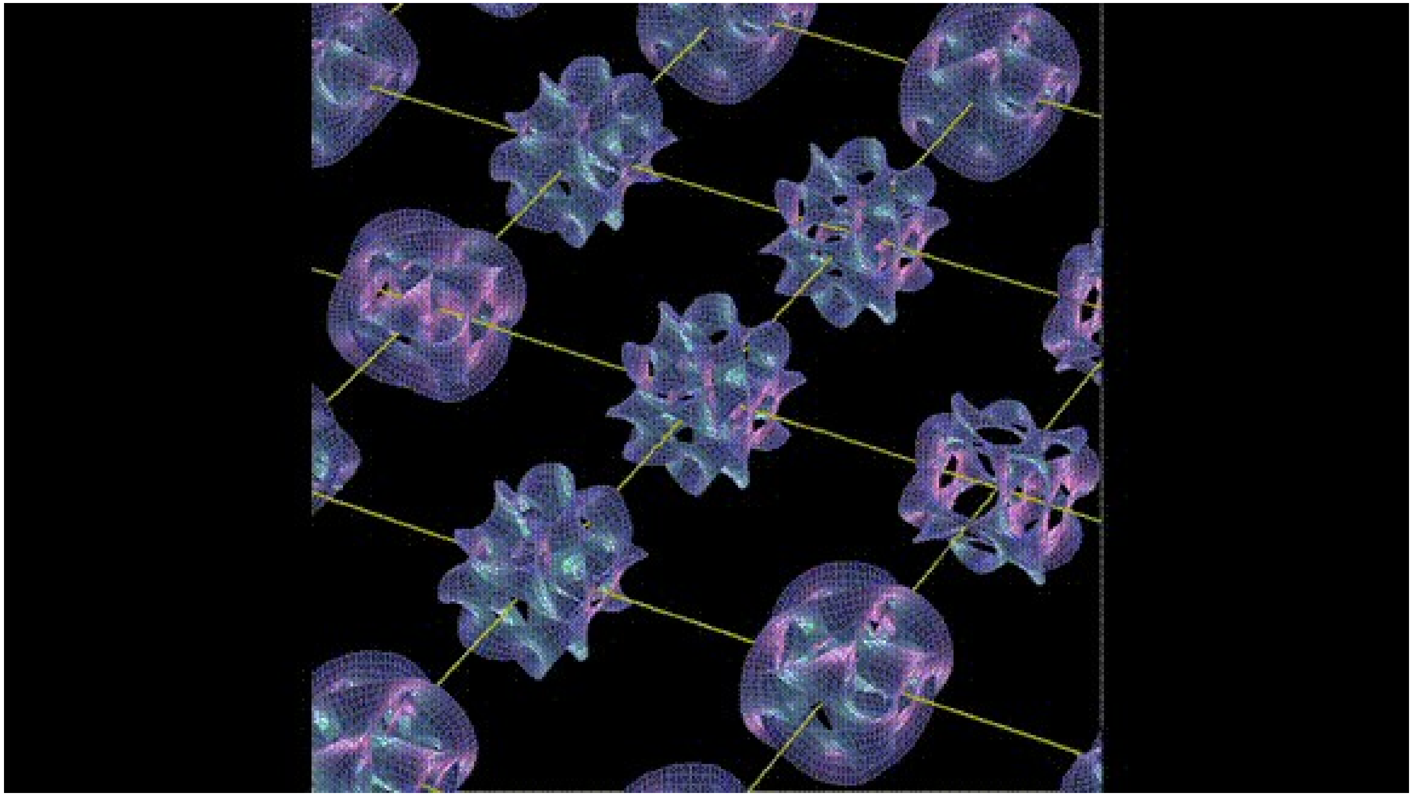
String theory

A consistent framework of the quantum theory of gravity



String Landscape

- (Tiny) extra dimensions and many ways of compactifications



Staring Landscape

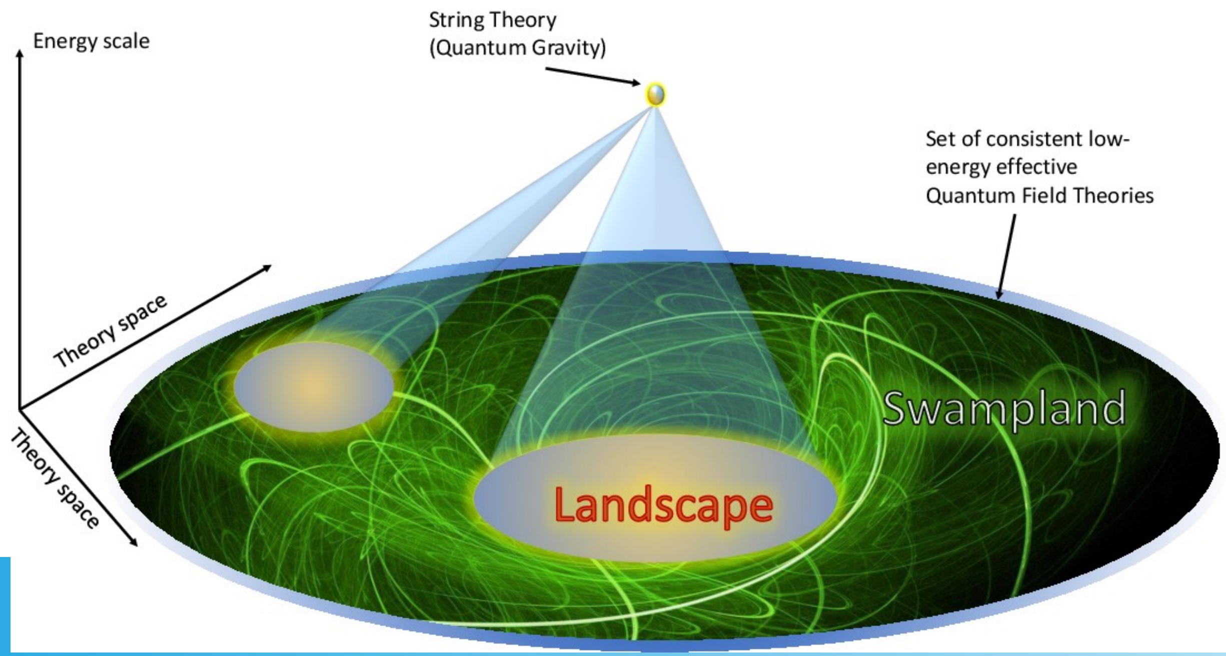
There is a huge number of low energy effective theory so the string theory admits a rich vacuum structure, this is known as the String Landscape.

This feature of the theory has been raised many objections against the string theory.

Swampland (Vafa and collabrators)

But Landscape is just a small island in a big Swampland.

Swampland is actually defined as the set of consistent QFTs that can not be UV completed into QG.



- The important part of the program is to distinguish the Landscape from the Swampland.
- Of course there is not a systematic algorithm to do so, but one may list some criteria for this purpose.

Swampland Program

- No global symmetry, only gauge symmetries are allowed
- All possible charges are present in the spectrum
- Finite range for fields
- The theory must include extended objects
- Gravity is the weakest force
- ...?

Second theme

Relation between QI and Geometry

Quantun Entanglement

Erwin Schrödinger: “The best possible knowledge of a whole, does not necessarily include the best possible knowledge of all its parts.”

• Entanglement is an immediate consequence of Quantum mechanical postulates.

$$|\uparrow\rangle_A |\downarrow\rangle_B \quad vs. \quad \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B + |\downarrow\rangle_A |\uparrow\rangle_B\right) ,$$

EE in QM

- Consider a quantum mechanical system in a pure ground state which is described by $|\psi\rangle$ ($\rho = |\psi\rangle\langle\psi|$).

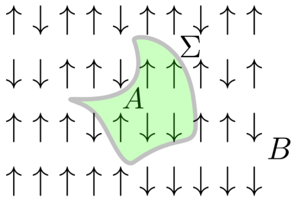


Figure: Note: Σ is imaginary!

- *Reduced* density operator:

$$\rho_A = \mathrm{Tr}_B\, \rho = \mathrm{Tr}_B |\psi\rangle\langle\psi|\,.$$

Then the EE is

$$S_{EE}(A) = -\mathrm{Tr}\rho_A \log \rho_A\,.$$

A simple Example

Consider a two qubit state as follows

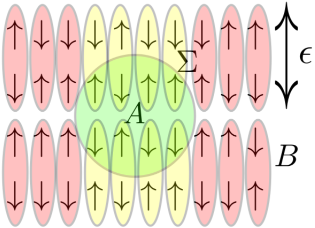
$$|\psi\rangle = \sin\theta|\uparrow\rangle|\downarrow\rangle + \cos\theta|\downarrow\rangle|\uparrow\rangle \quad , \quad 0 \leq \theta \leq \frac{\pi}{2} \, .$$

For this state one finds

$$S_{EE}^{(A)} = -\sin^2\theta \log\sin^2\theta - \cos^2\theta \log\cos^2\theta \, ,$$

- For $\theta = 0, \frac{\pi}{2}$, i.e. for $|\downarrow\rangle|\uparrow\rangle$ and $|\uparrow\rangle|\downarrow\rangle$, $S_{EE}^{(A)} = 0$, two subsystems are disentangled.
- For $\theta = \frac{\pi}{4}$, i.e. for $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)$, $S_{EE}^{(A)} = \log 2$ and two subsystems are maximally entangled.

EE in QFT



- Area law is an interesting feature [Srednicki (03)]

$$S_{EE} \sim \frac{\mathcal{A}(\Sigma)}{\epsilon^{d-2}} \, .$$

Rényi entropy

In a QFT, we first construct the Rényi entropy as

$$S_{RE}(A) = \frac{1}{1-n} \log \mathrm{Tr}\rho_A{}^n \,,$$

The EE reads then

$$S_{EE}(A) = \lim_{n\to 1} S_{RE}(A).$$

Reduced density operator in QFT

The first step is to define the wave functional of the fields

$$\Psi[\phi_0(x,y^i)] = \int_{\phi(x^\mu)|_{\tau=0}=\phi_0(x,y^i)} \mathcal{D}\phi \, e^{-\int d\tau \mathcal{L}[\phi]} \, .$$

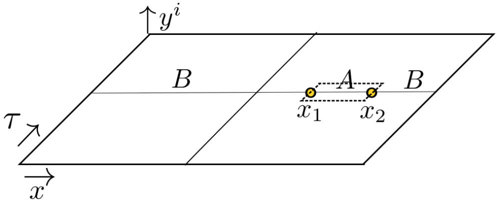


Figure: manifold $\mathcal{M}$

Correspondingly, the reduced density matrix will be

$$\rho_A^{+-} = \int \mathcal{D}\phi_B \; \Psi[\phi_A^+, \phi_B]\bar{\Psi}[\phi_A^-, \phi_B] \, ,$$

Replica Trick

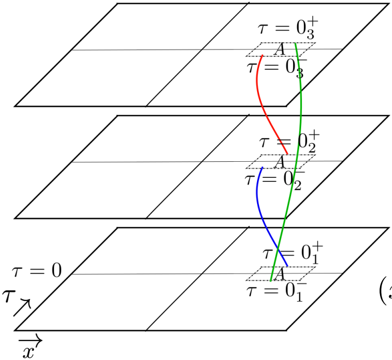


Figure: manifold $\mathcal{R}_n$

1. Making n copies

$$\rho_{A,1}^{+-}\ \rho_{A,2}^{+-} \cdots \rho_{A,n}^{+-}\ .$$

2. Identification

$$(x \in A, \tau = 0_i^+) \sim (x \in A, \tau = 0_{i+1}^-)$$

Partition function on $\mathcal{R}_n$

Finding the RE reduces to computing the partition function on n-sheeted Riemann surface

$$\mathrm{Tr}\hat{\rho}_A^n = Z_1^{-n} \int_{\mathcal{R}_n} \mathcal{D}\phi \, e^{-\int_{\mathcal{R}_n} d\tau \mathcal{L}[\phi]} \equiv \frac{Z_n}{Z_1^n} \, ,$$

then after an analytical continuation in n we will have

$$S_{EE}(A) = -\,\mathrm{Tr}\hat{\rho}_A \log \hat{\rho}_A = -\partial_n \log \, \mathrm{Tr}\rho_A^n \big|_{n=1} = -(n\partial_n - 1) \log Z_n \big|_{n=1} \, .$$

But the deficit angle $\alpha = 2\pi(1-n)$ introduces a conical singularity such that $\mathcal{R}_n \sim \mathcal{C}_n \times \Sigma$.

• The main challenge: **calculation on a manifold with conical singularity.**

EE in general d

In 2d one finds

$$S_{EE}(A) = \frac{c}{3} \log \frac{\ell}{\epsilon} \,.$$

and for higher d

$$S_{EE}(\Sigma) = \frac{s_{d-2}}{\epsilon^{d-2}} + \frac{s_{d-4}}{\epsilon^{d-4}} + \cdots + s_0 \log \epsilon + f \,,$$

where

$$s_{d-2} \propto \mathcal{A}(\Sigma).$$

EE and conformal anomaly

Lets remind ourselves the universal logarithmic term in EE, $s_0 \log \epsilon$. Interestingly it is related to the trace anomaly in even dimensions. To see this consider a Weyl scaling as

$$\ell \to e^{-\omega}\ell \qquad \leftrightarrow \qquad g_{\mu\nu} \to e^{-2\omega} g_{\mu\nu} \,,$$

then

$$\ell \frac{\partial}{\partial \ell}(\log \, \mathrm{Tr}\hat{\rho}_A^n) = 2\int d^d x \, g_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}}[\log Z_n - n \log Z_1] \,.$$

but since

$$T_{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta S}{\delta g_{\mu\nu}} \,,$$

For instance in 2d

$$\ell \frac{\partial}{\partial \ell} S_{EE}(A) = \lim_{n\to 1} n\partial_n \langle T^\mu_\mu \rangle_{\mathcal{M}_n} \sim \lim_{n\to 1} n\partial_n \langle \mathcal{R} \rangle_{\mathcal{M}_n} \,.$$

Holographic EE

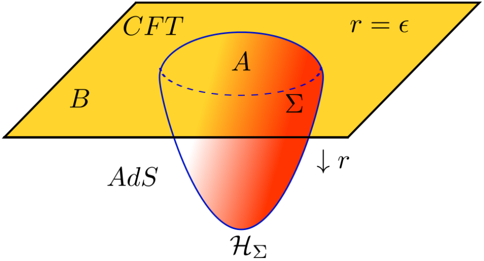
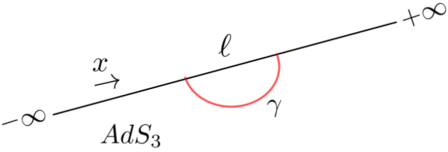


Figure: Ryu-Takayanagi's (RT) proposal (06)

$$S_{HE}(\Sigma) = \text{Min}\, \frac{\mathcal{A}(\mathcal{H}_\Sigma)}{4G_N^{(d+1)}}\,,$$

Holographic EE for a $1+1$ dimensional CFT



In the case of a $1+1$ dimensional CFT we should calculate the geodesic length, γ, in AdS_3 described by

$$ds^2 = \frac{R^2}{r^2}(-dt^2 + dx^2 + dr^2)\,,$$

then using RT proposal the EE reads

$$S_{HE}(\gamma) = \frac{R}{2G_N}\log\frac{\ell}{\epsilon} = \frac{c}{3}\log\frac{\ell}{\epsilon}\,.$$

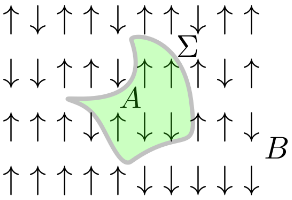
Einstein's equation from QM, Jacobson; 2016

Varying the state and the geometry simultaneously, such that the EE in a small geodesic ball is maximal at fixed volume, $\delta S_{EE}\big|_V = 0$, we say the subsystem is in Entanglement Equilibrium.

Interestingly

$$\delta S_{EE}\big|_V = 0 \ \sim \ G_{\mu\nu} = 8\pi G_N T_{\mu\nu}.$$

What surface maximizes EE?



To give an answer we used the RT prescription in an asymptotic form, this leads to

$$dv_{\mathcal{H}_\Sigma} = r^{-d+1}\left[1 - \frac{1}{2}\left(\frac{d-3}{(d-2)^2}(\,\mathrm{Tr}K)^2 + \,\mathrm{Tr}P\right) r^2 + \cdots\right] dv_\Sigma \, dr \ ,$$

where,

$$P_{\alpha\beta} = \frac{1}{d-2}\left(R_{\alpha\beta} - \frac{R}{2(d-1)} g_{\alpha\beta}\right) \ .$$

EE and the Geometry, Solodukhin; 2008 & A.F.A, Gibbons, Solodukhin; 2014

EE depends on the state and the way through which we isolate a subsystem, i.e. the intrinsic/extrinsic geometry of the manifold/entangling surface.

$$S_{HE}(\Sigma) = \frac{A(\mathcal{H}_\Sigma)}{4G_N} = \frac{1}{4G_N}\frac{A(\Sigma)}{(d-2)\epsilon^{d-2}} + \\ + \frac{1}{4G_N}\frac{1}{2(d-2)(d-4)\epsilon^{d-4}}\int_\Sigma dv_\Sigma \\ \left[R_{aa} - \frac{d}{2(d-1)}R - \frac{d-3}{d-2}(\,\mathrm{Tr}K)^2\right] \; .$$

Entanglement between Math and Physics!

Mathematicians (we also!) want to know what surface minimizes the Willmore functional

$$W(\Sigma) = \frac{1}{4}\int_{\Sigma} (\,\mathrm{Tr} K)^2\,,$$

We have the same question, since

Max of $S_{EE}(\Sigma)$ when $\mathcal{A}(\Sigma)$ is fixed $\sim$ Min of $W(\Sigma)$.

Entropy maximizer in $4D$ is S^2

$$W(\Sigma) = \frac{1}{4}\int_{\Sigma} (\,\mathrm{Tr}K)^2\,.$$

Doing some rewriting we get

$$\frac{1}{2}(\,\mathrm{Tr}K)^2 = R_{\Sigma} + K_{\Sigma}\,,$$

$$R_{\Sigma} = (\,\mathrm{Tr}K)^2 - \,\mathrm{Tr}K^2\,,\; K_{\Sigma} = \,\mathrm{Tr}K^2 - \frac{1}{2}(\,\mathrm{Tr}K)^2\,,$$

but

$$K_{\Sigma} = (K_{ij} - \frac{1}{2}\gamma_{ij}\,\mathrm{Tr}K)^2\,,$$

demanding $K_{\Sigma} = 0 \to K_{ij} = \frac{1}{2}\gamma_{ij}\,\mathrm{Tr}K$. Using the Gauss-Codazzi equations

$$\nabla^j K_{ij} = \nabla_i\,\mathrm{Tr}K \to \,\mathrm{Tr}K = const.$$

$\therefore\; R_{\Sigma} = const. \geq 0 \to \Sigma_0 \sim S^2.$

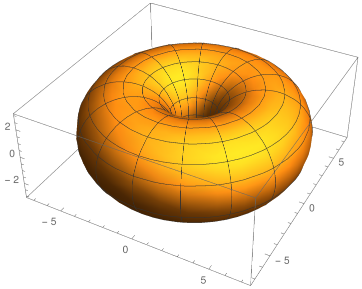
Maximizers of entropy, Willmore conjecture

We Proved (in $4D$) and Conjectured (in higher D) that the Spheres are the global maximizers of the EE.

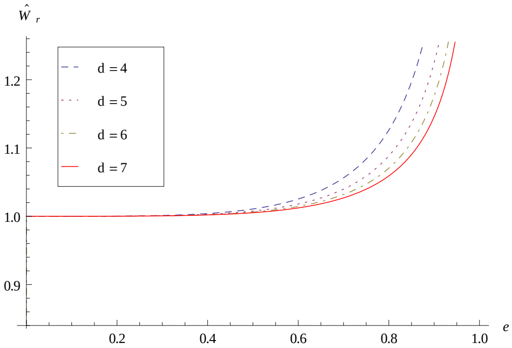
$$g = 0 \to W(\Sigma) \geq W(S^2) = 4\pi$$
$$g = 1 \to W(\Sigma) \geq W(\mathsf{T}^2_{cliff}) = 2\pi^2 \ .$$

In $g = 1$ class Clifford torus is the entropy maximizer.

$$S_{EE}(\Sigma_{g=1}) \leq S_{EE}(\mathsf{T}^2_{Cliff})$$



Maximizers of entropy, Ellipsoid in Higher D



We have ploted $\widehat{W}(E^{d-2})/\widehat{W}(S^{d-2})$

$$\frac{x_1^2}{a_1^2} + \ldots + \frac{x_{d-1}^2}{a_{d-1}^2} = 1 \ , \ (a_1 = a_2 = \cdots = a_{d-2} = a) \neq (a_{d-1} = b) \, .$$

with $e = \sqrt{1 - \frac{a^2}{b^2}}$

$$e = \sqrt{1 - \frac{a^2}{b^2}} \, .$$

Maximizers of entropy, $S^m \times S^n$ geometry in Higher D

As a generalization of a torus, for $S^m \times S^n$ geometries we have

- $d = m + n + 2 = 4$

d=4	$S^1 \times S^1$
x_{min}	0.707
$\widehat{W}_{r,min}$	1.571

- $d = m + n + 2 = 5$

d=5	$S^2 \times S^1$	$S^1 \times S^2$
x_{min}	0.886	0.816
$\widehat{W}_{r,min}$	1.391	1.333

- $d = m + n + 2 = 6$

d=6	$S^3 \times S^1$	$S^2 \times S^2$	$S^1 \times S^3$
x_{min}	0.968	1	1
$\widehat{W}_{r,min}$	1.324	1.237	1.116

- $d = m + n + 2 = 7$

d=7	$S^4 \times S^1$	$S^3 \times S^2$	$S^2 \times S^3$	$S^1 \times S^4$
x_{min}	0.9987	1	1	1
$\widehat{W}_{r,min}$	1.289	1.226	1.152	1.076

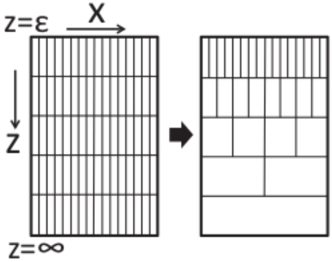
Quantum Complexity

- Complexity: How much a job is hard to do!
- How much a circuit which processes an algorithm must be complicated.
- in QM: How much the preparation of a quantum state is complicated.

$$|\uparrow\rangle|\uparrow\rangle \quad \to \quad |\uparrow\rangle|\downarrow\rangle \quad vs. \quad \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)$$

State in QFT, Takayanagi, 2017

$$\Psi[\chi_0] = \int \prod_x \prod_{\epsilon \le z < \infty} \mathcal{D}\chi e^{-S[\chi]} \times \delta\left(\chi(z=\epsilon, x) - \chi_0(x)\right) .$$



$$ds^2 = dz^2 + dx^2 \xrightarrow{\text{optimization}} g_{ij}(z,x) .$$

What is the form of g_{ij}?

complexity in 2 dimensions

In 2d all of the metrics are conformally flat

$$g_{ij}(z,x) = e^{2\phi}(dz^2 + dx^2)\,.$$

The question is now which ϕ optimizes the path integral?

Note:

$$\mathcal{D}\chi|_{e^{2\phi}\eta_{ij}} = e^{S_L}\mathcal{D}\chi|_{\eta_{ij}}\ ,$$

where S_L is the Liouville action

$$S_L = \frac{c}{24\pi}\int d^2x\left((\partial\phi)^2 + \mu e^{2\phi}\right)\ .$$

complexity in 2 dimensions, Takayanagi, Susskind, Myers, ... 2016-...

Optimizing the path integral, i.e. solving the e.q.m for the Liouville action we will get

$$e^{2\phi} = \frac{1}{z^2} \to g_{ij} = \frac{1}{z^2}(dz^2 + dx^2)\,,$$

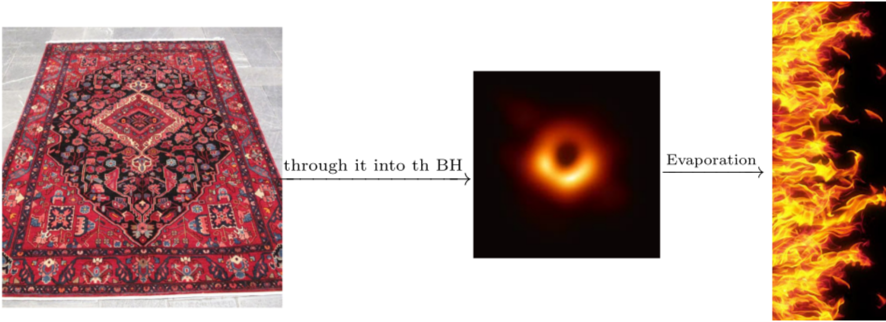
This is the time constant slice of AdS_3 !

Optimization of the complexity $\sim$ Solving the Einstein equation

To sum up

The quantum mechanical patterns, admit some geometrical dual pictures which are natural outcome of the dynamical equations of gravity.
There is still a long way to go!

Information Paradox (After the collapse)



Question 1

Why pure states are more informative than the mixed states?

$$\rho_{pure} = |\psi\rangle\langle\psi| \quad vs. \quad \rho_{mixed} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

for pure states we can find operators whose eigenstates are proportional to $|\psi\rangle_{pure}$, then measuring the observable we get an eigenvalue with perfect accuracy. This is of course not the case for the mixed states.

Question 2

Does this loss of information mean that we can not reconstruct the initial state from the final mixed state?
NO! if we have a large ensemble of the system with mixed state we may compensate the statistical inaccuracy with repeating the measurements of a sufficiently large set of the observables.

Example

For a spin 1/2 system, a typical mixed state is

$$\rho = \rho_{11}|\uparrow\rangle\langle\uparrow| + \rho_{12}|\uparrow\rangle\langle\downarrow| + \rho_{21}|\downarrow\rangle\langle\uparrow| + \rho_{22}|\downarrow\rangle\langle\downarrow|$$

defining the polarization vector $\vec{P}$ such that $\rho = \frac{1}{2}(I + \vec{P}\cdot\sigma)$ this state can be characterized as

$$\vec{P} = (\rho_{12} + \rho_{21}, i\rho_{12} - i\rho_{21}, \rho_{11} - \rho_{22})$$

Taking this mixed state as a final state we search for the initial pure state satisfying

$$\rho_{ab}^{final} = S_{ab}^{cd}\rho_{cd}^{initial}$$

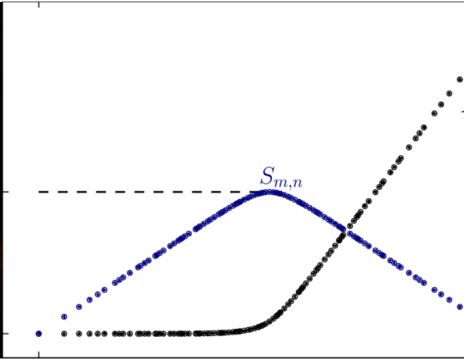
An example for superscattering matrix for the above model is

$$S_{ab}^{cd} = \lambda\delta_a^c\delta_b^d + \frac{1-\lambda}{2}\delta_{ab}\delta^{cd}\ ,\ 0 \leq \lambda \leq 1$$

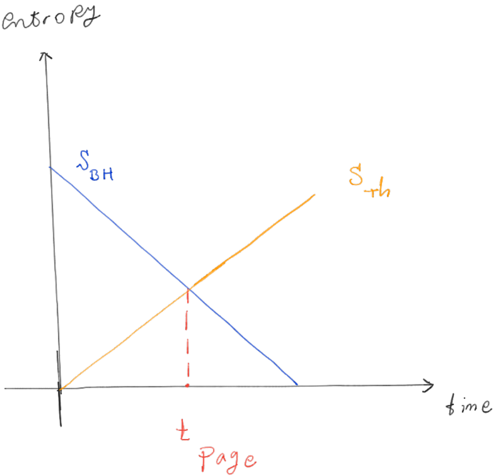
which multiplies the polarization vector with λ.

Page resolution, Page, 1993

Just wait for the burning of the half of the coal. Now you can recover the information!



Information paradox in BHs



What Hawking did not consider?

- ✔ Coarse grained entropy→Statistical ignorance→Having the density matrix we measure not all observables, but a subset of coarse grained observables→Thermodynamical entropy, S_{th}

Information: $I = S_{max} - S_{fg} \approx S_{th} - S_{vN} = \log d_H + \mathrm{Tr}\rho \log \rho.$

What Hawking did not consider?

- ✔ Coarse grained entropy→Statistical ignorance→Having the density matrix we measure not all observables, but a subset of coarse grained observables→Thermodynamical entropy, S_{th}
- ✔ Fine grained entropy→Ignorance about the precise quantum state of the system→ von Neumann entropy, $S_{vN} = -Tr\rho\log\rho$

Information: $I = S_{max} - S_{fg} \approx S_{th} - S_{vN} = \log d_H + \mathrm{Tr}\rho\log\rho.$

A very useful framework would be 2 dimensional gravity. Gravity is trivial (topological) in 2d, but adding a dilaton it becomes non-trivial→Jackiw-Teitelboim gravity (85 & 83)

$$S_{JT} = \frac{1}{16\pi G_N}\int \phi(R + \frac{2}{L^2}) + \frac{1}{8\pi G_N}\int_{bdy}\phi_b(K-1)\,.$$

The solution is the AdS_2+running dilaton, what remains for us is a time re-parametrization and thus a black-hole.

$$S_{on-shell} = \frac{1}{8\pi G_n} \int du \; \phi_b \{\tau(u), u\} \,,$$

where

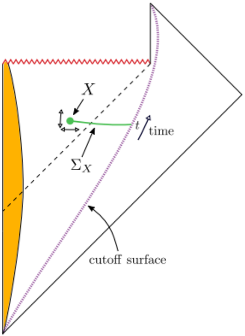
$$\{\tau(u), u\} = \frac{\tau'''}{\tau'} - \frac{3}{2} (\frac{\tau''}{\tau'})^2 \,,$$

and this corresponds to the spectrum of the SYK model.

Step 1, fg entropy in gravitational systems, Engelhardt, Wall, fig from 2006.06872

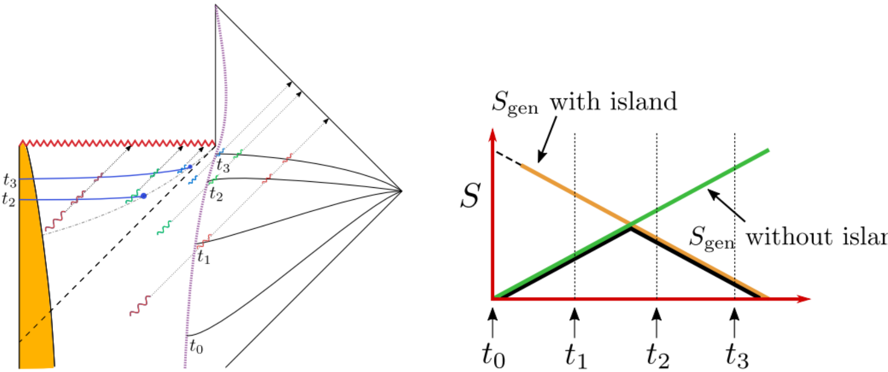
We rely on semi classical computation, i.e. quantum field theory on classical geometry.

$$S_{gen} = \mathrm{Min}_X \left\{ \mathrm{Ext}_X \left[\frac{A(X)}{4G_N} + S_{semi-classical}(\Sigma_X) \right] \right\}$$



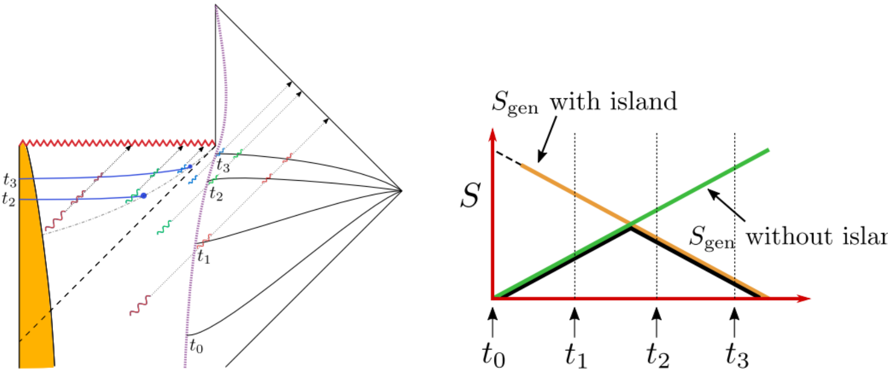
Step 1, Island formula, Almheiri, Maldacena, Engelhardt, Wall, ... fig from 2006.06872

$$S_{\text{gen}} = \text{Min} \left\{ \text{Ext} \left[\frac{A(\partial I)}{4G_N} + S_{\text{vN}}(R \cup I) \right] \right\},$$



Scrambling time, fig from 2006.06872

Heyden-Preskill hypothesis: Throughing a qubit into the BH, one should wait till the scrambling time, $t_s \sim M \log M$, to recover the information.



Higher derivative terms M. Alishahiha, A.F.A, A. Naseh (2020)

$$I = I_{\text{gravity}} + I_{\text{matter}}$$

with

$$I_{\text{gravity}} = \frac{1}{16\pi G_N} \int d^4x\sqrt{g}\Big(R[g] + \lambda_1 R^2[g] + \lambda_2 R_{\mu\nu}[g]R^{\mu\nu}[g] + \lambda_{\text{GB}}\,\mathcal{L}_{GB}[g]\Big),$$

where

$$\mathcal{L}_{\text{GB}}[g] = R_{\mu\nu\rho\sigma}[g]R^{\mu\nu\rho\sigma}[g] - 4R_{\mu\nu}[g]R^{\mu\nu}[g] + R^2[g],$$

Solution

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \qquad f(r) = 1 - \frac{r_h}{r}$$

What would be different?

Using the cone regularization method:

$$S_{\text{gravity}} = \frac{A[\partial I]}{4G_N} + \frac{1}{4G_N}\int_{\partial I}\left(2\lambda_1 R[g] + \lambda_2\left[R_{\mu\nu}[g]n_i^\mu n_i^\nu - \frac{1}{2}K_iK_i\right] + 2\lambda_{\text{GB}}\, R[\partial$$

and from the conformal anomaly or heat kernel method:

$$S_{\text{vN}} = \frac{A[\partial I]}{\epsilon^2} + \alpha\int_{\partial I}\left(\frac{2C}{3}R[g] - C\left[R_{\mu\nu}[g]n_i^\mu n_i^\nu - \frac{1}{2}K_iK_i\right] + (A-C)R[\partial I]\right)\log\epsilon + S_{\text{v}}$$

then

$$S_{\text{gen}}(R) = \text{Min}\Bigg\{\text{Ext}\Bigg[\frac{A[\partial I]}{4G_{\text{N,ren}}}$$

$$+\frac{1}{4G_{\text{N,ren}}}\int_{\partial I}\left(2\lambda_{1,\text{ren}}R[g] + \lambda_{2,\text{ren}}\sum_{i=1}^{2}\left[R_{\mu\nu}[g]n_i^\mu n_i^\nu - \frac{1}{2}K_iK_i\right] + 2\lambda_{\text{GB,ren}}R[\partial I]\right.$$

$$\left. + S_{\text{vN.fin}}(R\cup I)\Bigg]\Bigg\}.$$

Result for the one sided asymptotically flat BH

$$S_{\mathsf{gen}} = S_{\mathsf{th}} - \frac{A}{12}\left(1 - \log\left[\frac{16e^{\frac{b}{r_h}}(b-r_h)^2 r_h^3}{b}\right]\right) + \mathcal{O}(G_{\mathsf{N,ren}}).$$

$$t_{\text{Page}} \sim \frac{6S_{\text{th}}}{A} r_h.$$

Result for the asymptotically AdS BH

For critical Gravity

$$I_{\text{critical}} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}R^2\right)\right],$$

we have

$$S_{\mathsf{gen}} = 2S_{\mathsf{th}} + \frac{A}{6}\left(\frac{3br_h}{\ell^2} - \frac{\pi}{\sqrt{3}} + \log\left(\frac{16\ell^6}{9\sqrt{3}r_h^2}\right)\right) + \mathcal{O}(G_{\mathsf{N,ren}}).$$

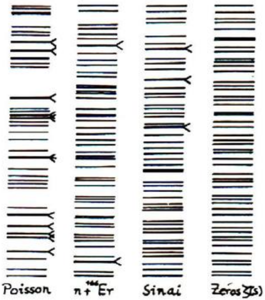
$$t_{\text{Page}} \sim \frac{12S_{\text{th}}}{A}r_h$$

To sum up

Even for the non-unitary theories the black hole evaporation follows the Page curve!

Matrix models/2D gravity duality Witten; Saad, Shenker, Stanford; Johnson, ...

- Black holes are strongly chaotic.
- It has been proved that some random matrix model such as the SYK model are also chaotic at the strongest level



The neighboring energy levels tend to repel each other and as a result one observes a long-range rigidity in the pattern of the energy levels.

Consequences

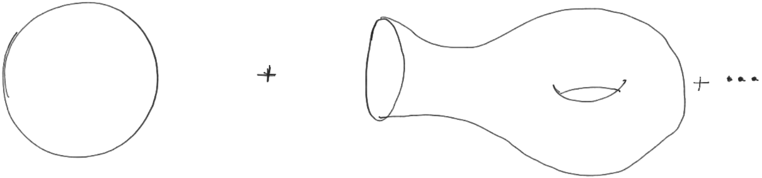
- The investigation should be non-perturbative.
- A topological order can be observed.
- Conjecture: Chaotic systems can be described by random matrices. The Matrix models/2D gravity duality has root in this conjecture.

Performing the non-perturbative analysis we need some sort of the recursion relations.
For the matrix models

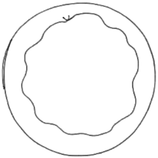
$$\mathcal{Z} = \int \mathcal{D}H e^{-LV(H)},$$

This recursion relation does exist and all loop (double line) integrals can be re-expressed in terms of the lower ones recursively. So partition function $Z(\beta)$ and its correlators can be computed in terms of the density of states $\rho(E)$.

In the other side, we aim to compute the Partition function for JT gravity, *i.e. the Euclidean path integral with (near) AdS_2 and boundary dilaton ϕ_b at the boundary.*
To do so we need to sum over the topologies.

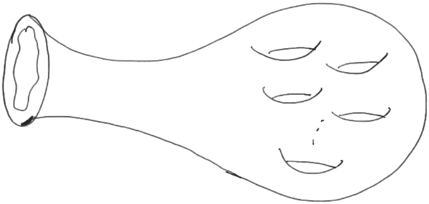


The leading term gives the density of states and the subleading terms are of the following forms

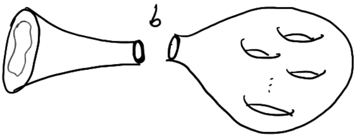


$$\longrightarrow \quad \rho(E) \sim \sinh(\sqrt{E})$$

The subleading terms have the following forms

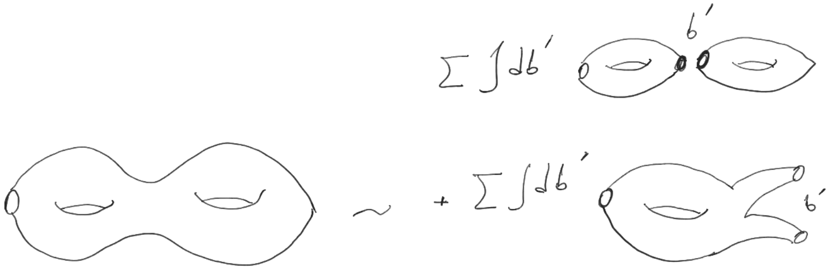


$$Z(\beta) = \int bdb \; Z_{\text{Trump}}(\beta, b) \times V_g(b)$$



Weil-petterson Volume

Mirzakhani's recursion relation



In the memory of Marya Mirzakhani