Chiral symmetry restoration in a magnetized rotating system

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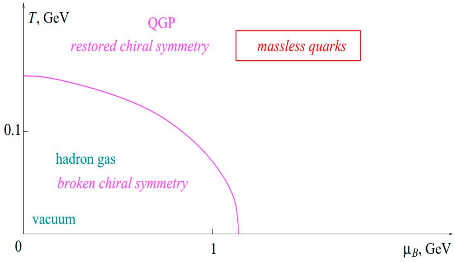
Part I: Concepts and Motivations

Understanding the properties of strongly interacting matter



arXiv: hep-lat/0701002

Fundamental Interest

To undersatnd the universe

Early Universe

Compact Stars

- QCD is described by

$$\mathcal{L} \;=\; -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{\psi}\left(i\not{D} - m\right)\psi,$$

With

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu,$$

and

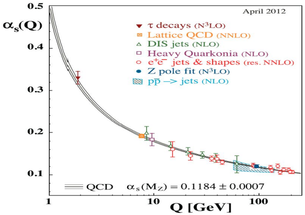
$$D_\mu = \partial_\mu - i g T^a A^a_\mu.$$

- Global symmetries ($m = 0$):

$$SU_L(2) \times SU_R(2) \times U_V(1) \to SU(2)_V \times U_V(1)$$

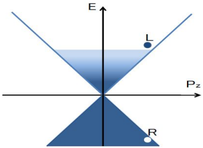
- $U_A(1)$ is anomalously broken.

Asymptotic freedom



Confinement of color

$$< \bar{\psi}_{L/R}(x)\psi_{R/L}(x) > \neq 0$$



Shuryak,Phys. Rept. **61**, 71 (1980)

- Absence of confinement in the $T \to \infty$ limit.

- The main idea : In this limit the quantum fluctuatons of gauge fields can be neglected $\to$ Treated as classical fields
- This can be justified :

In Matsubara frequencies $\omega_n = 2\pi n T$ in limit of $T \to \infty$ only $n = 0$ is relevant $\to$ zero frequency means no dependence on τ $\to$ Integrating over $1/T$ leads to the classical expression for partition function

$$Z = \int dV \exp\left(-\beta \mathcal{H}\right)$$

Polyakov, Phys. Lett. **72B**, 477 (1978)

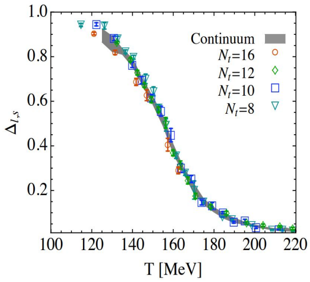
- At hight T the free energy will tends to the

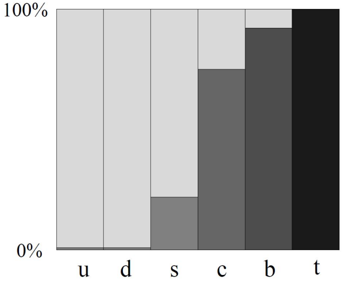
$$F(R_{12}) \underbrace{\Longrightarrow}_{R_{12}\to\infty} \text{constant}$$

Where $R_1$and R_2 are the coordiante of two heavy charges.

- If confinement was present $(F \propto R_{12})$ for small T, then heating the confining fields will lead to a phase transition.

Chiral symmetry breaking



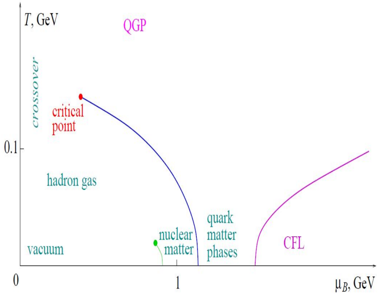


arXiv:1005.3508

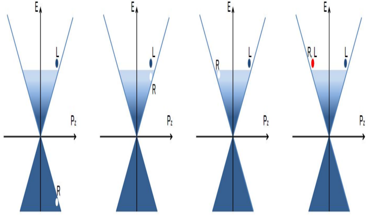
arXiv:0801.4256

$$\Delta_{l,s} = \frac{\langle \bar{\psi}\psi \rangle_{l,T} - \frac{m_l}{m_s}\langle \bar{\psi}\psi \rangle_{s,T}}{\langle \bar{\psi}\psi \rangle_{l,0} - \frac{m_l}{m_s}\langle \bar{\psi}\psi \rangle_{s,0}}, \qquad l = u, d,$$

What is the phase diagram of Quantum Chromodynamics?

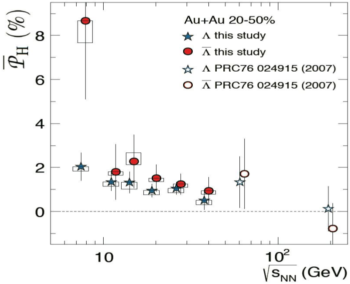


arXiv:hep-lat/0701002



arXiv:1406.1367

- At low T and μ: Dilute gas of hadrons.
- At small μ and finite $T \geq 130 MeV$: Lattice.
- Small T close to the nuclear matter saturation density.

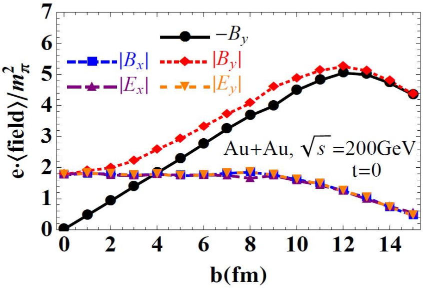


Nature 548, 62-65 (2017)

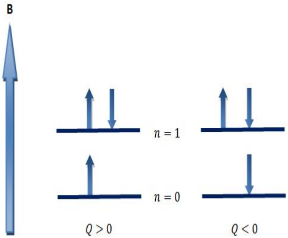
$$|B|_{\sqrt{s}=200\,GeV} = \frac{T_s|\Delta\mathcal{P}|}{2|\mu_\Lambda|} < 8.9\times 10^{11}$$
$$\approx 2.7\times 10^{-3}\, m_\pi^2$$ B. Müller and A. Schäfer, PRD (2018)

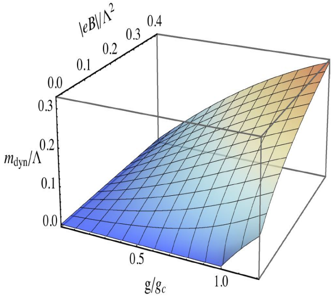
$$\tau_B = \frac{A}{\sqrt{s}},$$ Y. Guo, S. Shi, S. Feng and J. Liao, PLB (2019)

$$B_y(t,\vec{x}) = \begin{cases} \dfrac{B_0(\vec{x})}{1+(t-t_0)^2/\tau_B^2}, & A = 92\,GeV.fm/c \\ \dfrac{B_0(\vec{x})}{[1+(t-t_0)^2/\tau_B^2]^{3/2}}, & A = 125\,GeV.fm/c \\ B_0(\vec{x})\exp(-|t-t_0|/\tau_B) & A = 128\,GeV.fm/c \end{cases}$$



W. T. Deng and X. G. Huang, PRC (2012)





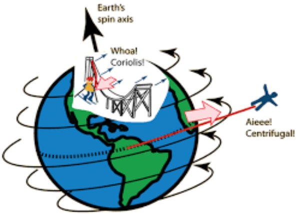
Miransky and Shovkovy, Phys. Rept(2015)

$$\Psi_\alpha(x) = \frac{1}{\sqrt{V}}\sum_{n,s}\int \frac{dp_y dp_z}{(2\pi)^2}\frac{1}{\sqrt{2E_n}}\Big(e^{-i(E_n t - p_y y - p_z z)}a^{(n)}_{p_y p_z, s}\, \Psi^n_{\alpha\rho}(\xi)u_{s,\rho}(\tilde{p}) + e^{i(E_n t - p_y y - p_z z)}b^{(n)\dagger}_{p_y p_z, s}\Phi^n_{\alpha\rho}(\bar{\xi})v_{s,\rho}(\tilde{p})\Big)$$

Part II: Chiral symmetry restoration in a rotating System

In classical physics:

$$m\vec{a} = \vec{F}_I - m\vec{A}_0 - 2m\vec{\Omega} \times \vec{v} - m\dot{\Omega} \times r - m\Omega \times (\Omega \times r)$$



- Many constituents of the matter are simultaneously moving.
- A good way to describe a rotating system is to consider the coordinate transformation to the rotating frame.
- The rotating matter and its constituents are at rest in the rotating frame.

Dirac equation in a rotating frame:

$$\left[i\gamma^0\left(\partial_t - i\Omega\hat{L}_z - \frac{i\Omega}{2}\sigma^{12}\right) + i\gamma^1\left(\partial_x + \frac{iqeBy}{2}\right) + i\gamma^2\left(\partial_y - \frac{iqeBx}{2}\right) + i\gamma^3\partial_z - m\right]\psi(x) = 0,$$

We restrict ourselves to the case of $\partial m \ll m^2$.

$$i(\gamma\cdot\mathcal{D})\,\mathbb{E}^q_{\lambda,\kappa} = \kappa\mathbb{E}^q_{\lambda,\kappa}(\gamma\cdot\tilde{p}_{\ell,\kappa}),$$

With $\kappa = \pm 1$ and

$$\mathbb{E}^q_{\lambda,\kappa} = e^{-i\kappa(E_\kappa t - p_z z)}\left(\mathcal{P}_+ f^+_{s_q,\lambda} + \mathcal{P}_- f^-_{s_q,\lambda}\right) \equiv e^{-i\kappa(E_\kappa t - p_z z)}\mathbb{P}^q_\lambda(x)$$

$$\gamma\cdot\mathcal{D} = i\gamma^0\left(\partial_t - i\Omega\hat{L}_z - \frac{i\Omega}{2}\sigma^{12}\right) + i\gamma^1\left(\partial_x + \frac{iqeBy}{2}\right) + i\gamma^2\left(\partial_y - \frac{iqeBx}{2}\right) + i\gamma^3\partial_z,$$

$$\mathcal{P}_\pm = \frac{1\pm i\gamma^1\gamma^2}{2}, \qquad \hat{J}_z\mathbb{E}^q_{\lambda,\kappa} \equiv \left(\hat{L}_z + \frac{\Sigma_z}{2}\right)\mathbb{E}^q_{\lambda,\kappa} = \left(\ell + \frac{1}{2}\right)\mathbb{E}^q_{\lambda,\kappa},$$

$$E_{\lambda,\kappa} = \pm\sqrt{p_z^2 + 2\lambda\,|qeB| + m^2} - \kappa\Omega\big(\ell + \frac{1}{2}\big) \equiv \epsilon - \kappa\Omega j,$$

Gap equation

$$\frac{m}{G} = \frac{m}{2\pi}\sum_{\ell,k,n}(\mathcal{A}^{s_q}_{\ell,k})^2\int dk_z \frac{(\Phi^-_{s_q,\lambda_{\ell,k}})^2+(\Phi^+_{s_q,\lambda_{\ell,k}})^2}{\sqrt{k_z^2+2\lambda\,|qeB|+m^2}}\left[1-f(\epsilon+\Omega j)-f(\epsilon-\Omega j)\right]$$

Where

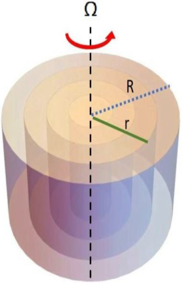
$$\Phi^-_{s_q,\lambda_{\ell,k}} = \frac{1}{|\ell+1|!}\sqrt{\frac{|qeB|\left(\mathcal{M}_{s_q}+|\ell+1|\right)!}{2\pi\mathcal{M}_{s_q}!}}\,e^{-x/2}x^{|\ell+1|/2}{}_1F_1\left(-\mathcal{M}_{s_q};|\ell+1|+1;x\right),$$

$$\Phi^+_{s_q,\lambda_{\ell,k}} = \frac{1}{|\ell|!}\sqrt{\frac{|qeB|\left(\mathcal{N}_{s_q}+|\ell|\right)!}{2\pi\mathcal{N}_{s_q}!}}\,e^{-x/2}x^{|\ell|/2}{}_1F_1\left(-\mathcal{N}_{s_q};|\ell|+1;x\right),$$

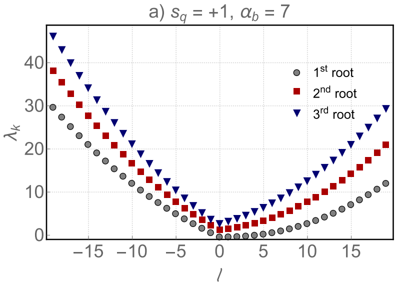
And $x=|qeB|R^2/2$,

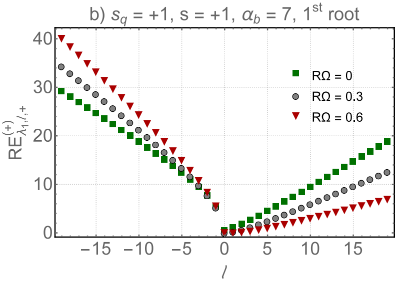
$$\mathcal{N}_{s_q} = \lambda+\frac{s_q(\ell+1)-|\ell|-1}{2},$$

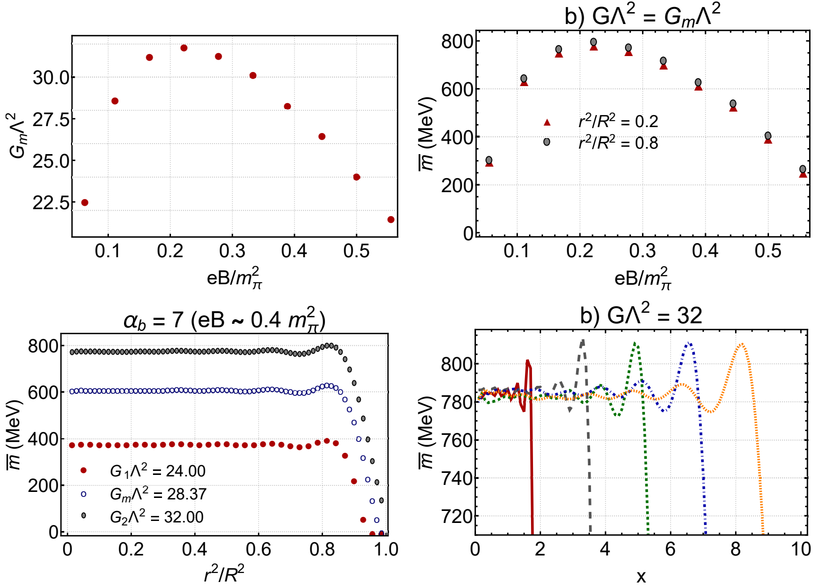
$$\mathcal{M}_{s_q} = \lambda+\frac{s_q\ell-|\ell+1|-1}{2}.$$



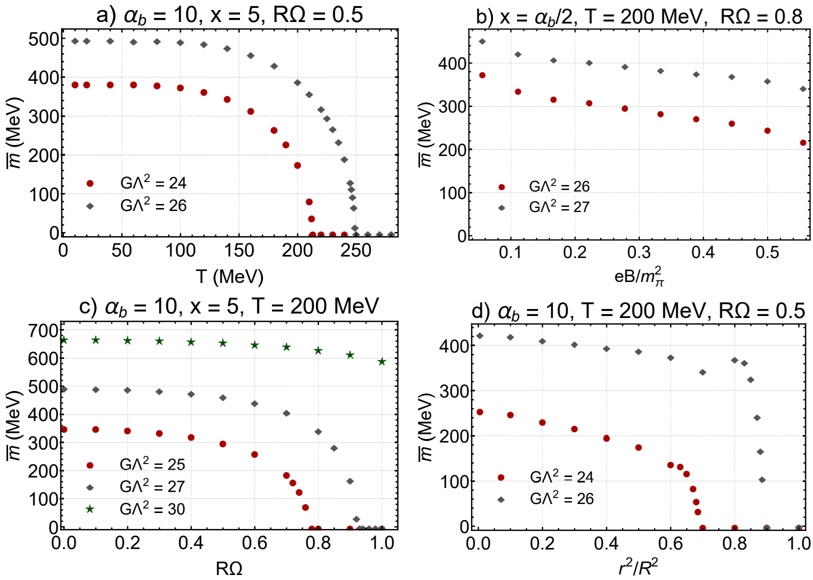
N. Sadooghi, S.M.A.T.M and F. Taghinavaz, arXiv: 2108.12760

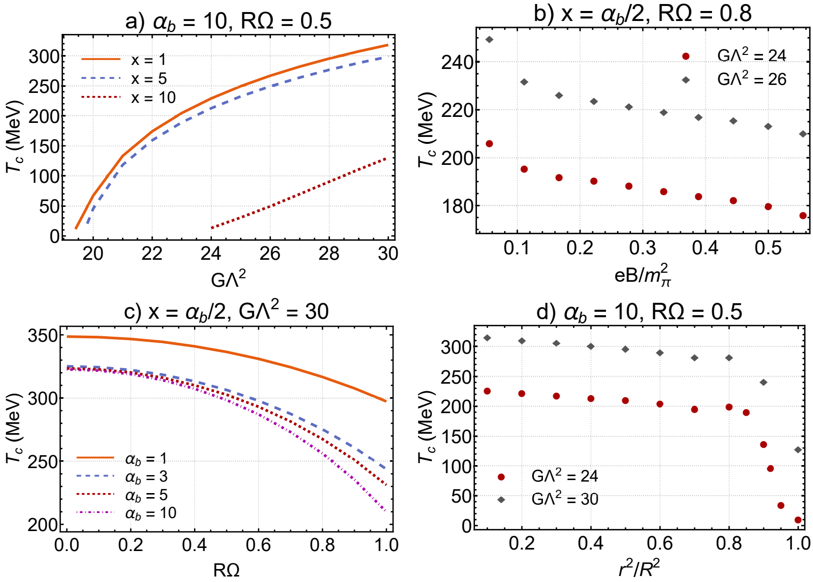






$G_m\Lambda^2$
30.0
27.5
25.0
22.5
0.1 0.2 0.3 0.4 0.5
eB/m_π^2
b) $G\Lambda^2 = G_m\Lambda^2$
$\overline{m}$ (MeV)
800 600 400 200 0
$r^2/R^2 = 0.2$
$r^2/R^2 = 0.8$
0.1 0.2 0.3 0.4 0.5
eB/m_π^2
$\alpha_b = 7$ (eB ~ 0.4 m_π^2)
$\overline{m}$ (MeV)
800 600 400 200 0
$G_1\Lambda^2 = 24.00$
$G_m\Lambda^2 = 28.37$
$G_2\Lambda^2 = 32.00$
0 0.2 0.4 0.6 0.8 1.0
r^2/R^2
b) $G\Lambda^2 = 32$
$\overline{m}$ (MeV)
800 780 760 740 720
0 2 4 6 8 10
x


a) α_b = 10, x = 5, RΩ = 0.5
$\overline{m}$ (MeV)
T (MeV)
GΛ^2 = 24
GΛ^2 = 26
b) x = α_b/2, T = 200 MeV, RΩ = 0.8
$\overline{m}$ (MeV)
eB/m_π^2
GΛ^2 = 26
GΛ^2 = 27
c) α_b = 10, x = 5, T = 200 MeV
$\overline{m}$ (MeV)
RΩ
GΛ^2 = 25
GΛ^2 = 27
GΛ^2 = 30
d) α_b = 10, T = 200 MeV, RΩ = 0.5
$\overline{m}$ (MeV)
r^2/R^2
GΛ^2 = 24
GΛ^2 = 26



a) α_b = 10, RΩ = 0.5
x = 1
x = 5
x = 10
T_c (MeV)
GΛ^2
b) x = α_b/2, RΩ = 0.8
GΛ^2 = 24
GΛ^2 = 26
T_c (MeV)
eB/m_π^2
c) x = α_b/2, GΛ^2 = 30
α_b = 1
α_b = 3
α_b = 5
α_b = 10
T_c (MeV)
RΩ
d) α_b = 10, RΩ = 0.5
GΛ^2 = 24
GΛ^2 = 30
T_c (MeV)
r^2/R^2

Thank You For Your Attention!