Propagation of Quantum Fields on Quantum Spacetime

Aliasghar Parvizi, UT, IPM

November 10, 2021

Outline

- **Why Quantum Gravity:**
  The main motivations for quantum gravity.
- **Loop Quantum Gravity and Loop Quantum Cosmology :**
  I give here a brief general overview of ideas, techniques and results
- **Propagation of Fields and Emergence of Spacetime :**
  Modes of a massless field on the quantum background probes an effective dressed geometry

Why Quantum Gravity?

Quantum theory seems to be a universal framework for physical theories.

- **Singularities in Nature:** Singularities in Cosmology and black holes physics, we need an encompassing theories, much like quantum mechanics and stationary states of atoms.
- **Unification:** Standard model of particle physics has united all non-gravitational interactions $SU(3) \times SU(2) \times U(1)$. Gravity couples to all form of energy, means gravity has to be implemented in the quantum framework.
- **Invariant Observables:** The role of diffeomorphism group and the notion of 'observables'.

Why Quantum Gravity?

Below the level of full quantum gravity, at the semi-classical and classical limits

- **No back reaction:** equivalence principle holds at the given level of precision.
- **Quantum field theory on curved background:** Notion of particles, Hawking radiation, Davies-Unruh effect,

$$T_H = \frac{\hbar c^3}{8\pi k_B G M} \approx 6.17 \times 10^{-8} \left(\frac{M_\odot}{M}\right) K. \tag{1}$$

Quantum Gravity Corrections

Observational search for quantum gravity in cosmological and black hole settings

- **UV Corrections to Lorentz Symmetry:** The most active quantum-spacetime-phenomenology research area is indeed the one considering possible Planck-scale departures from Poincare/Lorentz symmetries.
- **Improved dynamics of collapse:** The effective quantum dynamics of collapse, removing singularity.
- **Quantum Cosmology:** How a large universe described effectively by classical physics emerges from a small and highly quantum one ?. Pre-inflationary dynamics and non-Gaussianities,

Quantum Gravity Corrections

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33} cm, \tag{2}$$

$$t_p = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44} s, \tag{3}$$

$$m_p = \sqrt{\frac{\hbar c}{G}} \approx 1.22 \times 10^{19} GeV. \tag{4}$$

Effects of quantum gravity can in principle occur at lower energy scales through amplification parameters which strengthen Planckian effects.

- **Emergent Schrödinger equation:** Describes evolution of quantum fields when propagating on a quantum spacetime

Loop Quantum Gravity

Main Ideas:

Take GR
⇒ Find appropriate gauge invariant variables
⇒ Non-Perturbative quantization
⇒ Find corresponding representation
⇒ Construct physical Hilbert space

Background - Independent Non-Perturbative Quantization

Diffeomorphism covariance together with non-perturbative methods naturally lead to a fundamental, in-built discreteness in geometry

Loop Quantum Gravity: Brief Review

Phase space variables: Holonomy and Flux:

Quantum theory would be easier if we use the $SU(2)$ gauge theory version of GR in ADM formalism. Fix a three-dimensional manifold M, with gravitational variables,

$$\begin{aligned} g\ g^{ab} &= E_i^a E_i^b , \\ A_a^i(x) &= \Gamma_a^i(x) + \gamma\ k_a^i(x); \end{aligned}$$

well-defined guage invariant background-independent variables,

$$U_\ell(A) := \mathcal{P}\ \exp \int_\ell A_a^i \tau^i \mathrm{d}\ell^a \qquad \text{and} \qquad E_{f,S} = \int_S f^i E_i^a \mathrm{d}^2 S_a \,, \tag{5}$$

the phase space Γ is the same as in the theory of weak interactions

Loop Quantum Gravity: Brief Review

quantum states

Consider a quantum states that are functionals $\Psi(A)$ of the connection. On these states, the two quantities $\mathcal{T}[\alpha]$ and $E[S, f]$ act naturally: given a loop α in M, we define:

$$\mathcal{T}[\alpha] = -\mathrm{Tr}[U_\alpha], \tag{6}$$

the first as a multiplicative operator, the second as the functional derivative operator

$$E_{f,S} = \int_S \mathrm{d}^2 S_a f^i \frac{\delta}{\delta A^i_a}\,, \tag{7}$$

Loop Quantum Gravity: Brief Review

Loop states

First, given a loop α in M, there is a normalized state $\psi_\alpha(A)$ in $\mathcal{H}$, Namely

$$\psi_\alpha(A) = -\operatorname{Tr} U_\alpha(A). \tag{8}$$

we denote with α also a multiloop, namely a collection of (possibly overlapping) loops $\{\alpha_1, \ldots, \alpha_n,\}$, and we call

$$\psi_\alpha(A) = \psi_{\alpha_1}(A) \times \ldots \times \psi_{\alpha_n}(A) \tag{9}$$

a multiloop state.

Loop Quantum Gravity: Brief Review

spin network states

Next, consider a graph Γ. A “coloring” of Γ is given by the following.

1. Associate an irreducible representation of $SU(2)$ to each link of Γ.
2. Associate an invariant tensor v in the tensor product of the representations $s_1 \dots s_n$, to each node of Γ in which links with spins $s_1 \dots s_n$ meet.

We indicate a colored graph by $\{\Gamma, \vec{s}, \vec{v}\}$, or simply $S = \{\Gamma, \vec{s}, \vec{v}\}$, and denote it a “spin network”.

Loop Quantum Gravity: Brief Review

Constructing Hilbert Space: $\mathcal{H}$

Pick a graph Γ, with n links, denoted $\gamma_1 \dots \gamma_n$, immersed in the manifold M. Let $U_i(A) = U_{\gamma_i},\ i = 1, \dots, n$ be the parallel transport operator of the connection A along γ_i. $U_i(A)$ is an element of $SU(2)$. Pick a function $f(g_1 \dots g_n)$ on $[SU(2)]^n$. The graph Γ and the function f determine a functional of the connection as follows

$$\psi_{\Gamma,f}(A) = f(U_1(A), \dots, U_n(A)). \tag{10}$$

we define the scalar product between any two cylindrical functions by

$$(\psi_{\Gamma,f}, \psi_{\Gamma,h}) = \int_{SU(2)^n} dg_1 \dots dg_n\, \overline{f(g_1 \dots g_n)}\, h(g_1 \dots g_n). \tag{11}$$

where dg is the Haar measure on $SU(2)$.

Dynamics

The definition of the theory is completed by giving the Hamiltonian constraint. I will sketch below the final form of the constraint,

$$\hat{H}|s\rangle = \sum_i \sum_{(IJ)} \sum_{\epsilon=\pm1} \sum_{\epsilon'=\pm1} A_{\epsilon\epsilon'}(p_i ... p_n)\, \hat{D}_{i;(IJ),\epsilon\epsilon'}\, |s\rangle. \tag{12}$$

Here i labels the nodes of the s-knot s; (IJ) labels couples of (distinct) links emerging from i. $p_1 ... p_n$ are the colors of the links emerging from i. $\hat{D}_{i;(IJ)\epsilon\epsilon'}$ is the operator that acts on an s-knot by: (i) creating two additional nodes, one along each of the two links I and J; (ii) creating a novel link, colored 1, joining these two nodes, (iii) assigning the coloring $p_I + \epsilon$ and, respectively, $p_J + \epsilon'$ to the links that join the new formed nodes with the node i. This is illustrated in Figure 2.

Dynamics

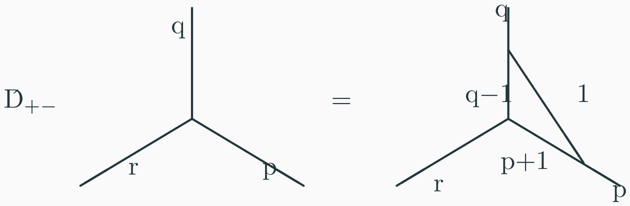


Figure 1: Action of $\hat{D}_{i;(IJ)\epsilon\epsilon'}$.

Loop Quantum Gravity: Brief Review

Area Operator:

The resulting area operator $\hat{A}$ acts as follows on a spin network state $|S\rangle$ (assuming here for simplicity that S is a spin network without nodes on Σ):

$$\hat{A}[\Sigma]\;|S\rangle = \left(\frac{l_0^2}{2} \sum_{i \in \{S \cap \Sigma\}} \sqrt{j_i(j_i+1)} \right)\;|S\rangle \tag{13}$$

where i labels the intersections between the spin network S and the surface Σ, and p_i is the color of the link of S crossing the $i-th$ intersection. This result shows that the spin network states are eigenstates of the area operator

Loop Quantum Gravity

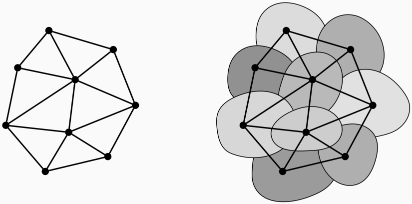


Figure 2: Artist's depiction of quanta of geometry. The left figure is a graph Γ and the right figure shows Γ with its dual cellular decomposition (C. Rovell,2008).

Each 3-cell in the decomposition is a topological tetrahedron dual to a node n of Γ; each face is dual to a link ℓ. Each 3-cell can be visualized as an 'atom of geometry'. Its volume 'resides' at the node, and areas of its faces 'reside' at the point at which the face intersects the link of the graph. Thus, classical Riemannian structures arise only on coarse graining. arXiv:2104.04394, arXiv:gr-qc/0404018

Loop Quantum Cosmology

Implementing techniques of loop quantum gravity to cosmological models.

We start with:

$$g_{ab}dx^a dx^b = -N^2_{x_0}(x_0)dx_0^2 + a^2(x_0)d\mathbf{x}^2 \ . \tag{14}$$

and the gravitational Hamiltonian

$$H_{\mathrm{gr}} = \int d^3x N_t \mathcal{H}_{\mathrm{gr}} = \ \frac{3\pi G}{2\alpha} b^2 |v| \ , \tag{15}$$

where $b/\gamma = \dot{a}/a$ is the Hubble parameter with γ being the so called Barbero-Immirzi parameter of LQG. Moreover, $\alpha = 2\pi\gamma\sqrt{\Delta}\ell^2_{\mathrm{Pl}} \approx 1.35\ell^3_{\mathrm{Pl}}$ (where Δ is the so-called ‘area gap’ given by $\Delta = 3\sqrt{3}\pi\gamma\ell^2_{\mathrm{Pl}}$), and $v = \ell^3 a^3/\alpha$ is the *oriented volume* with b being its conjugate momentum satisfying $\{v, b\} = 2$.

Loop Quantum Cosmology

In Hilbert space of LQC, $\hat{v}$ is the volume operator of the quantum background geometry:

$$\hat{v}|v\rangle = v|v\rangle \quad \text{and} \quad \hat{\mathcal{N}}|v\rangle = |v+1\rangle \ , \tag{16}$$

where, $\hat{\mathcal{N}} \equiv \widehat{\exp(ib/2)}$, and $\{|v\rangle\}$ is the basis of eigenstates of the volume operator $\hat{v}$ satisfying $\langle v|v'\rangle = \delta_{v,v'}$.

Gravitational Hamiltonian constraint becomes operator $\hat{H}_{\text{geo}} = \hat{H}_{\text{grav}} + \hat{H}_T$ on $\mathscr{H}^o_{\text{kin}} = \mathscr{H}_{\text{grav}} \otimes \mathscr{H}_T$, arXiv:1108.1145[gr-qc], where,

$$\hat{H}_{\text{gr}} \ = \ -\frac{3\pi G}{8\alpha}\sqrt{|\hat{v}|}\big(\hat{\mathcal{N}} - \hat{\mathcal{N}}^{-1}\big)^2\sqrt{|\hat{v}|} \ , \tag{17}$$

physical states must satisfy

$$i\hbar\partial_T\Psi_o(v, T) \ = \ \hat{H}_{\text{gr}}\Psi_o(v, T) \ . \tag{18}$$

Loop Quantum Cosmology

LQC evolution operator, $\hat{\Theta}$, where an action of the operator Θ equals

$$[\Theta\psi](v) = f_{-}(v)\Psi(v-4) - f_{o}(v)\Psi(v) + f_{+}(v)\Psi(v+4), \qquad (19)$$

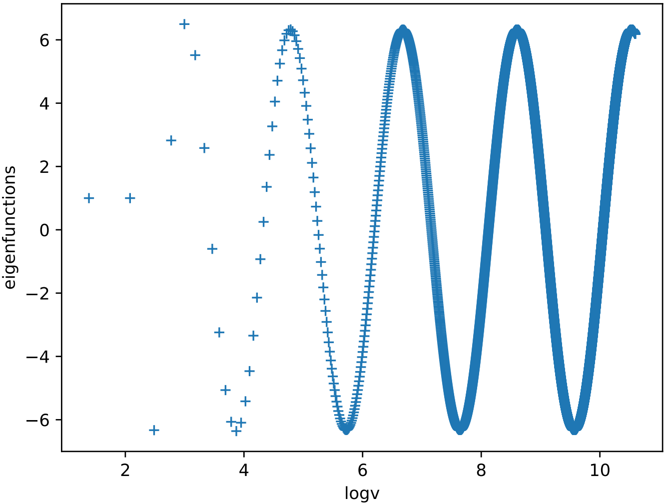
$$\Psi(v,\phi) = \int_{-\infty}^{\infty} dk\,\Psi(k)\, e_{k}^{(s)}(v)\, e^{i\omega\phi} \ . \qquad (20)$$

Results: The effective dynamics is

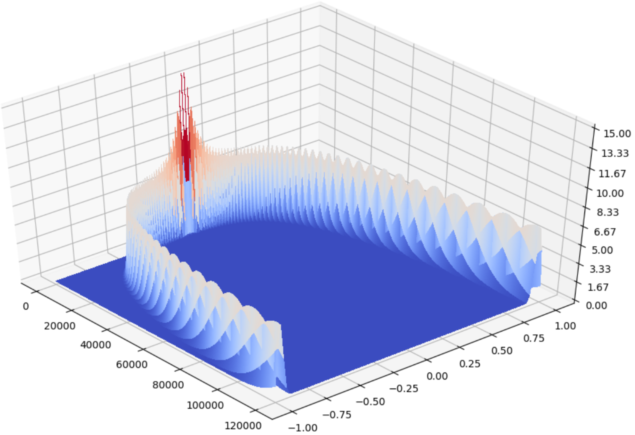
$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{cr}}\right), \quad \text{where} \quad \rho_{cr} = \frac{\sqrt{3}}{16\pi^2\gamma^3 G^2\hbar} \ . \qquad (21)$$

arXiv:1108.0893,arXiv:gr-qc/0601085[gr-qc]

Loop Eigenfunctions



Loop Wave function



Quantum Trajectory

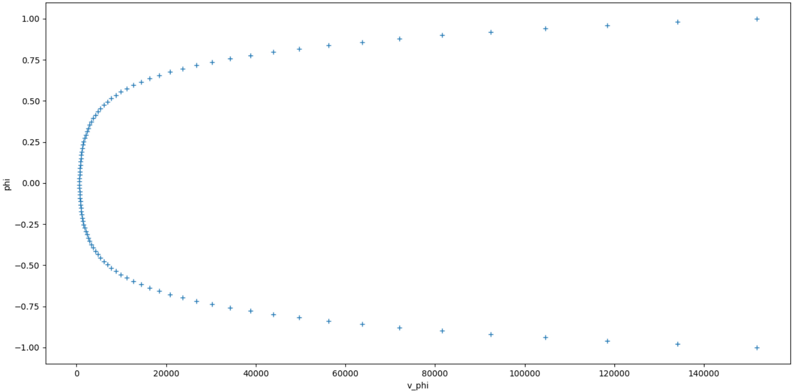
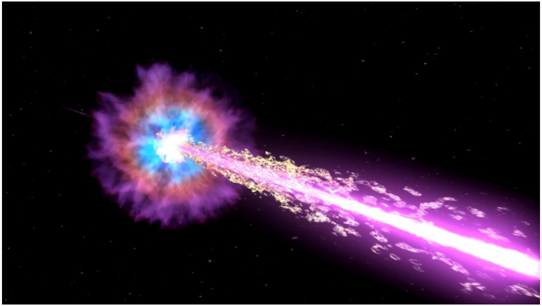


Figure 3: The expectation values of $\widehat{|v|_\phi}$ are plotted for the corresponding wave function.

Our Main motivation

Collapse Scenario and GRBs

The highest observable energies in the universe is provided by cosmic gamma rays. Considering the fact that gamma ray bursts (GRBs) are physically connected with the core-collapse supernovae (SNe), it is not unreasonable to expect that when observing GRBs we directly observe the gravitational collapse of a massive and compact star core.



Full Quantum System

Total Hamiltonian constraint, on the full phase space:

$$\begin{aligned} H &= \int d^3x \big[p_T + \mathcal{H}_{\rm gr} \big] + H_{\rm EM} \\ &= \ell^3 \big[p_T + \mathcal{H}_{\rm gr} \big] + H_{\rm EM} \;\approx 0 \; . \end{aligned} \tag{22}$$

the Schrodinger equation for each mode and polarisation of the field, is written as

$$\begin{aligned} i\hbar \partial_T \Psi(v, Q, T) &= \Big[\hat{\mathcal{H}}_{\rm gr} + \hat{H}^{(r)}_{T,\mathbf{k}} \Big] \Psi(v, Q, T) \\ &=: \hat{\hat{\tilde{\mathcal{H}}}}_{\mathbf{k}} \Psi(v, Q, T) \; , \end{aligned} \tag{23}$$

where

$$\hat{H}^{(r)}_{T,\mathbf{k}} := \frac{1}{2\ell^3} \Big[\widehat{a^{-3}} \otimes \big(\hat{P}^r_{\mathbf{k}} \big)^2 + k^2 \hat{a} \otimes \big(\hat{Q}^r_{\mathbf{k}} \big)^2 \Big]. \tag{24}$$

Emerging geometries

Zeroth order effect

When a test-field approximation is considered, for any time T one can decompose the total wave function as

$$\Psi_{\text{int}}(v, Q, T) \;=\; \Psi_o(v, T_0) \otimes \psi(Q, T) \;, \tag{25}$$

In this case, the Schrodinger equation reduces to an evolution equation for $\psi(Q, T)$ only:

$$i\hbar \partial_T \psi = \frac{1}{2\ell^3} \Big[\big\langle \widehat{a^{-3}}(T) \big\rangle_o \big(\hat{P}^r_{\mathbf{k}} \big)^2 + k^2 \big\langle \hat{a}(T) \big\rangle_o \big(\hat{Q}^r_{\mathbf{k}} \big)^2 \Big] \psi. \tag{26}$$

Emerging geometries

Zeroth order effect

$$i\hbar\partial_{x_0}\psi(x_0,Q^r_{\mathbf{k}}) = \frac{N_{x_0}}{2a^3}\left[\left(\hat{P}^r_{\mathbf{k}}\right)^2 + k^2 a^4 \left(\hat{Q}^r_{\mathbf{k}}\right)^2\right]\psi(x_0,Q^r_{\mathbf{k}}),$$

by comparison with

$$i\hbar\partial_T\psi = \frac{1}{2\ell^3}\left[\left\langle\widehat{a^{-3}}(T)\right\rangle_o\left(\hat{P}^r_{\mathbf{k}}\right)^2 + k^2\left\langle\hat{a}(T)\right\rangle_o\left(\hat{Q}^r_{\mathbf{k}}\right)^2\right]\psi, \tag{27}$$

can be seen as an (Schrödinger) evolution equation for the **k**-th mode of the EM field on a dressed space-time with $(M,\bar{g}_{ab})$:

$$\bar{g}_{ab}\,dx^a\,dx^b \;=\; -\bar{N}^2_T(T)\,dT^2 + \bar{a}^2(T)\,d\mathbf{x}^2\;, \tag{28}$$

$$\bar{N}_T(T) \;=\; \ell^{-3}\left[\left\langle\widehat{a^{-3}}(T)\right\rangle_o\left\langle\hat{a}(T)\right\rangle_o^3\right]^{\frac{1}{4}},$$

$$\bar{a}(T) \;=\; \left[\frac{\left\langle\hat{a}(T)\right\rangle_o}{\left\langle\widehat{a^{-3}}(T)\right\rangle_o}\right]^{\frac{1}{4}}. \tag{29}$$

Emerging geometries

Next order effect

For back-reaction of quantum field, our task will be then to obtain solutions to the full eigenvalue problem:

$$\begin{aligned}\Big\{\hat{\mathcal{H}}_{\rm gr} &+ \frac{1}{2\ell^3}\Big[\widehat{a^{-3}} \otimes \big(\hat{P}^r_{\mathbf{k}}\big)^2 \\ &+ k^2\,\hat{a} \otimes \big(\hat{Q}^r_{\mathbf{k}}\big)^2\Big]\Big\}\tilde{\Psi}(v, Q) = E_{\mathbf{k}}\tilde{\Psi}(v, Q),\end{aligned} \tag{30}$$

where $E_{\mathbf{k}}$ is the total energy eigenvalue of each mode.
with solution

$$\tilde{\Psi}(v, Q) \;=\; \sum_{\mu, l} c_{\mu l}\xi^{\mu}_{l}(v) \otimes \chi^{l}(v, Q), \tag{31}$$

Emerging geometries

Next order effect

For each mode of the field

$$\hat{H}^{(r)}_{T,\mathbf{k}}\ \chi^l(Q;v) = \epsilon^{(l)}_{\mathbf{k}}(\hat{v})\ \chi^l(Q;v)\ , \tag{32}$$

then the eigenvalue equation of the background is:

$$\left[\hat{\mathcal{H}}_{\rm gr} + \lambda\epsilon^{(l)}_{\mathbf{k}}(\hat{v})\right]\xi^\mu_l(v) = E^\mu_l\xi^\mu_l(v)\ , \tag{33}$$

with

$$\begin{aligned}\epsilon^{(l)}_{\mathbf{k}}(\hat{v}) \quad = \quad & \frac{1}{2\ell^3}\Big[\widehat{a^{-3}}\big\langle\chi^l\big|\big(\hat{P}^r_{\mathbf{k}}\big)^2\big|\chi^l\big\rangle \\ & \qquad\qquad + k^2\,\hat{a}\big\langle\chi^l\big|\big(\hat{Q}^r_{\mathbf{k}}\big)^2\big|\chi^l\big\rangle\Big].\end{aligned} \tag{34}$$

Emerging geometries

Next order effect

we can expand the perturbed geometric eigenfunctions ξ^μ_l:

$$\xi^\mu_l \;=\; N(n)\Big[\xi^\mu_o + \sum_{\nu\neq\mu}\sum_n \lambda^n \beta^{n(l)}_{\mu\nu}\xi^\nu_o\Big], \tag{35}$$

to the first order of perturbation (i.e., for $n = 1$), we get

$$\beta^{1(l)}_{\mu\nu} \;:=\; \frac{\epsilon^{(l)}_{\mu\nu}(k)}{E^\mu_o - E^\nu_o}\ , \tag{36}$$

where

$$\begin{aligned}
\epsilon^{(l)}_{\mu\nu} \;&:=\; \big\langle\xi^\nu_o\big|\epsilon^{(l)}_{\mathbf{k}}(\hat{v})\big|\xi^\mu_o\big\rangle \\
&=\; \frac{1}{2\ell^3}\Big[\big\langle\xi^\nu_o\big|\widehat{a^{-3}}\big|\xi^\mu_o\big\rangle\big\langle\chi^l\big|\big(\hat{P}^r_{\mathbf{k}}\big)^2\big|\chi^l\big\rangle \\
&\qquad\qquad +k^2\big\langle\xi^\nu_o\big|\hat{a}\big|\xi^\mu_o\big\rangle\big\langle\chi^l\big|\big(\hat{Q}^r_{\mathbf{k}}\big)^2\big|\chi^l\big\rangle\Big] \\
&=:\; A^{(l)}_{\mu\nu} + k^2 B^{(l)}_{\mu\nu}\ .
\end{aligned} \tag{37}$$

Next order effect

Then total wave function (31) will be,

$$\begin{aligned}\tilde{\Psi}(v,Q) &= \sum_{\mu,l}\left[c^{o}_{\mu}\xi^{\mu}_{o}\otimes b_{l}\chi^{l}+\lambda\sum_{\nu\neq\mu}c_{\mu l}\beta^{1(l)}_{\mu\nu}\xi^{\nu}_{o}\otimes\chi^{l}\right]\\ &=: \Psi_{o}(v)\otimes\psi(Q)+\delta\Psi(v,Q),\end{aligned} \tag{38}$$

we can decompose the perturbation wave function as

$$\delta\Psi(v,Q) \;\approx\; \delta\Psi_{o}(v)\otimes\psi(Q)\ , \tag{39}$$

where

$$\delta\Psi_{o}(v) \;=\; \lambda\sum_{\mu}\sum_{\nu\neq\mu}c^{o}_{\mu}\beta^{1}_{\mu\nu}\xi^{\nu}_{o}\ . \tag{40}$$

Emerging geometries

Next order effect

For time-dependent Schrödinger equation we obtain:

$$\begin{aligned} i\hbar\partial_T\psi(Q,T) &= \frac{1}{2\ell^3}\Big[\Big(\langle\widehat{a^{-3}}\rangle_o+\lambda\langle\widehat{a^{-3}}\rangle_\delta\Big)\big(\hat{P}^r_{\mathbf{k}}\big)^2 \\ &\quad +k^2\Big(\langle\hat{a}\rangle_o+\lambda\langle\hat{a}\rangle_\delta\Big)\big(\hat{Q}^r_{\mathbf{k}}\big)^2\Big]\psi, \end{aligned} \tag{41}$$

where we defined $\langle\widehat{a^{-3}}\rangle_\delta\equiv\langle\Psi_o|\widehat{a^{-3}}|\delta\Psi_o\rangle$ and $\langle\hat{a}\rangle_\delta\equiv\langle\Psi_o|\hat{a}|\delta\Psi_o\rangle$, and

$$\langle\widehat{a^{-3}}\rangle_\delta = \sum_{\sigma,\mu}\sum_{\nu\neq\mu}(c^o_\sigma)^*c^o_\mu\frac{\langle\xi^\sigma_o|\widehat{a^{-3}}|\xi^\nu_o\rangle}{E^\mu_o-E^\nu_o}\epsilon^{(l)}_{\mu\nu}(k), \tag{42}$$

$$\langle\hat{a}\rangle_\delta = \sum_{\sigma,\mu}\sum_{\nu\neq\mu}(c^o_\sigma)^*c^o_\mu\frac{\langle\xi^\sigma_o|\hat{a}|\xi^\nu_o\rangle}{E^\mu_o-E^\nu_o}\epsilon^{(l)}_{\mu\nu}(k)\ . \tag{43}$$

Emerging geometries

Next order effect

Perturbed Schrödinger equation can be seen as an evolution equation for **k**-th mode of the EM field on a dressed space-time with $(M, \tilde{g}_{ab})$:

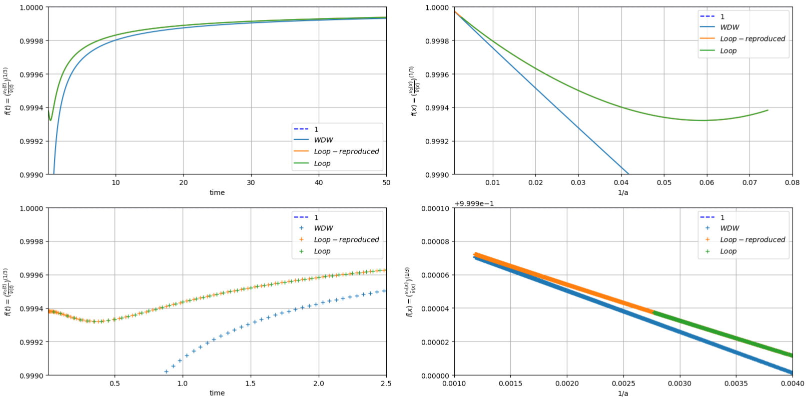
$$\tilde{g}_{ab} dx^a dx^b = -\tilde{N}_T^2(T) dT^2 + \tilde{a}^2(T) d\mathbf{x}^2 \ , \tag{44}$$

we obtain

$$\tilde{N}_T = \ell^{-3}\Big[\Big(\big\langle\widehat{a^{-3}}\big\rangle_o + \lambda\langle\widehat{a^{-3}}\rangle_\delta\Big) \times\Big(\big\langle\hat{a}\big\rangle_o + \lambda\langle\hat{a}\rangle_\delta\Big)^3\Big]^{\frac{1}{4}}, \tag{45}$$

$$\tilde{a}(T) = \left[\frac{\big\langle\hat{a}\big\rangle_o + \lambda\langle\hat{a}\rangle_\delta}{\big\langle\widehat{a^{-3}}\big\rangle_o + \lambda\langle\widehat{a^{-3}}\rangle_\delta}\right]^{\frac{1}{4}} . \tag{46}$$

Backreaction Deviation



Evolution parameters: pt= 1.0e+03; Dust field momentum; V_0 = 6.0e+08; Initial volume
k = 50, ; Beta = 0.5; Perturbation parameters; Integration time: t = [178.0, 0.0]

Figure 4: Deviation of perturbed quantum background with respect to unperturbed one.

Emerging geometries

Results:

Speed of light measured by low energy cosmological observer reads [Lewandowski, Nouri-Zonoz, P. and Tavakoli,2017]

$$|v| \approx 1 + \lambda(L_1 + 3L_2 p^2) + \mathcal{O}(p^4, \lambda^2). \tag{47}$$

Here, L_1 and L_2 are functions of quantum fluctuation of the background geometry and the field mode:

$$L_1 = \frac{1}{2} \sum_{\sigma,\mu} \sum_{\nu \neq \mu} \frac{(c_\sigma^o)^* c_\mu^o Z_{\sigma\nu}}{E_o^\mu - E_o^\nu} A_{\mu\nu}^{(l)} , \tag{48}$$

$$L_2 = \frac{\bar{a}^2}{2} \sum_{\sigma,\mu} \sum_{\nu \neq \mu} \frac{(c_\sigma^o)^* c_\mu^o Z_{\sigma\nu}}{E_o^\mu - E_o^\nu} B_{\mu\nu}^{(l)} , \tag{49}$$

Emerging geometries: Collapse Scenario

Black hole implications

We construct the whole spacetime structure of our model of quantum gravitational collapse by using matching conditions,

$$ds_+^2 = -F_k(t,X)\,dt^2 + F_k^{-1}(t,X)\,dX^2 + X^2\,d\Omega^2, \tag{50}$$

$$\begin{aligned} F(u,X) &= 1 - \frac{2G\tilde{M}(X,u)}{X} \\ &= 1 - \frac{2G}{X}\langle \hat{M}_T\rangle - \frac{2G}{X^{1-\beta}}\tilde{R}^{-\beta}\left[\langle \hat{M}\rangle_0\left(1 - \frac{3}{4\pi\rho_{\mathrm{cr}}}\frac{\langle \hat{M}\rangle_0}{\tilde{R}^3}\right) - \langle \hat{M}_T\rangle\right]. \end{aligned} \tag{51}$$

where,

$$\langle \hat{M}\rangle_0 = \langle \hat{M}_T\rangle + \frac{r_{\mathrm{b}} N_k \hbar k}{\tilde{R}}\,. \tag{52}$$

Black hole implications: Lensing

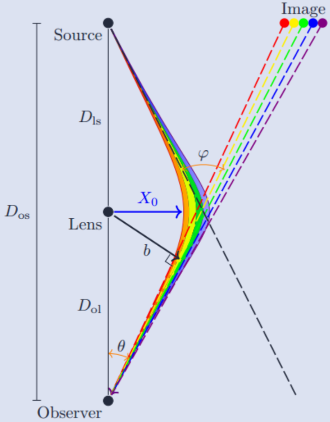
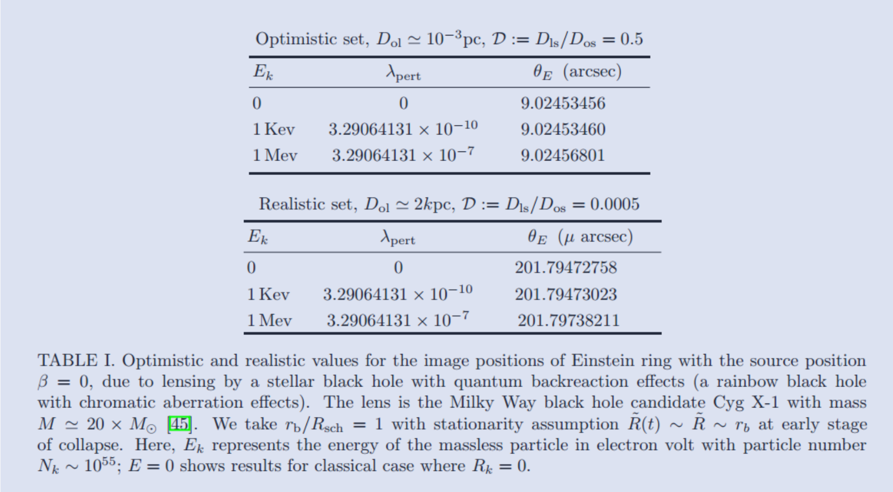


FIG. 2. Lensing configuration: deflection of different modes by a point mass, where lens, observer and source are highly aligned. Each mode probes different curvature and creates images at different angular positions, this chromatic gravitational aberration can be called " Quantum Gravitational Prism".

Black hole implications: Lensing



Optimistic set, $D_{\rm ol} \simeq 10^{-3}$pc, $\mathcal{D} := D_{\rm ls}/D_{\rm os} = 0.5$

E_k	$\lambda_{\rm pert}$	θ_E (arcsec)
0	0	9.02453456
1 Kev	$3.29064131 \times 10^{-10}$	9.02453460
1 Mev	$3.29064131 \times 10^{-7}$	9.02456801

Realistic set, $D_{\rm ol} \simeq 2k$pc, $\mathcal{D} := D_{\rm ls}/D_{\rm os} = 0.0005$

E_k	$\lambda_{\rm pert}$	θ_E (μ arcsec)
0	0	201.79472758
1 Kev	$3.29064131 \times 10^{-10}$	201.79473023
1 Mev	$3.29064131 \times 10^{-7}$	201.79738211

TABLE I. Optimistic and realistic values for the image positions of Einstein ring with the source position $\beta = 0$, due to lensing by a stellar black hole with quantum backreaction effects (a rainbow black hole with chromatic aberration effects). The lens is the Milky Way black hole candidate Cyg X-1 with mass $M \simeq 20 \times M_\odot$ [45]. We take $r_{\rm b}/R_{\rm sch} = 1$ with stationarity assumption $\tilde{R}(t) \sim \tilde{R} \sim r_b$ at early stage of collapse. Here, E_k represents the energy of the massless particle in electron volt with particle number $N_k \sim 10^{55}$; $E = 0$ shows results for classical case where $R_k = 0$.

What we have done so far

Projects:

- **Jerzy Lewandowski, Mohammad Nouri-Zonoz, Ali Parvizi, Yaser Tavakoli, arXiv:1709.04730 [gr-qc]** Quantum theory of electromagnetic fields in a cosmological quantum spacetime.
- **A Parvizi, T Pawłowski, Y Tavakoli, J Lewandowski, arXiv:2110.03069** Rainbow Black Hole From Quantum Gravitational Collapse

Colleagues:

- **Mohammad Nouri-Zonoz** University of Tehran, Iran.
- **Yaser Tavakoli** University of Guilan, Iran.
- **Jerzy Lewandowski** University of Warsaw, Poland.
- **Tomasz Pawłowski** University of Wroclaw, Poland.

Next Steps

- **Time delays:** Time delays in cosmological and black hole settings.
- **Solving geodesic equations:** Solving geodesic equations using numerical methods without simplifying assumptions.
- **Stochastic Spacetime:** emergence of stochastic spacetime.

Questions and Comments?

Untitled, 1968, Mark Rothko