Dark Matter from Inflation

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December 22, 2021

Outline

1. INFLATION
2. DARK MATTER FROM INFLATION
3. ENTROPIC DARK MATTER
4. VECTOR DARK MATTER
5. SUMMARY

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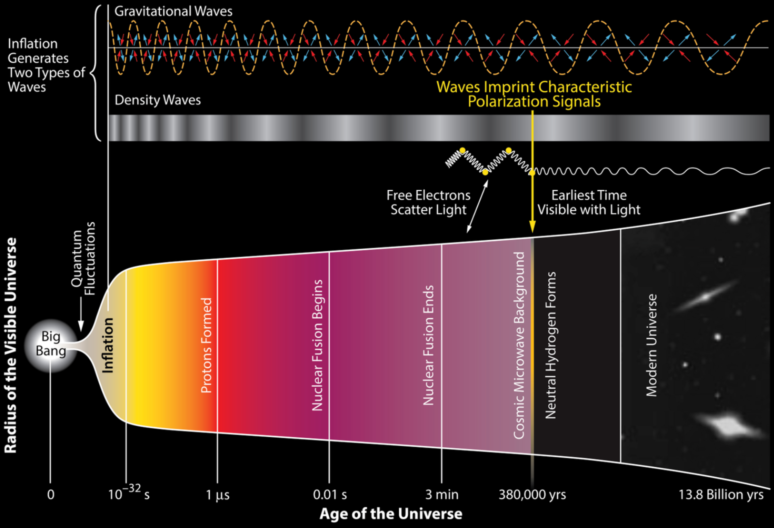
History of the Universe



bicepkeck.org

Flatness problem

From cosmological observations like Type Ia supernova and cosmic microwave background (CMB) radiation:

$$|\Omega_k|_{t=t_0} = |\Omega_{\rm tot} - 1|_{t=t_0} < 1$$

Why our Universe is flat?

Flatness problem

From cosmological observations like Type Ia supernova and cosmic microwave background (CMB) radiation:

$$|\Omega_k|_{t=t_0} = |\Omega_{\rm tot} - 1|_{t=t_0} < 1$$

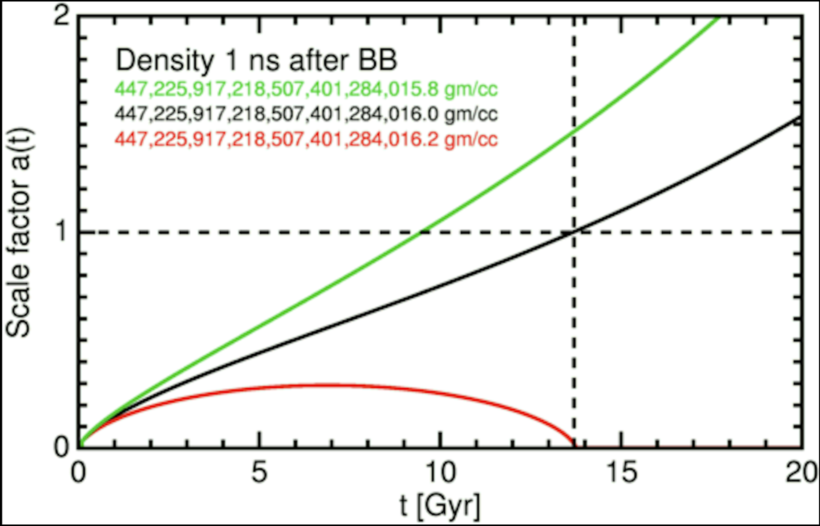
Why our Universe is flat?

- Big bang nucleosynthesis: $|\Omega_{\rm tot} - 1|_{t=1s} < 10^{-18}$
- Electroweak SB scale: $|\Omega_{\rm tot} - 1|_{t=10^{-12}s} < 10^{-30}$

At the Big bang nucleosynthesis we need a fine tuning:

$$0.999999999999999999 \leq |\Omega_{\rm tot}|_{t=1s} \leq 1.000000000000000001$$

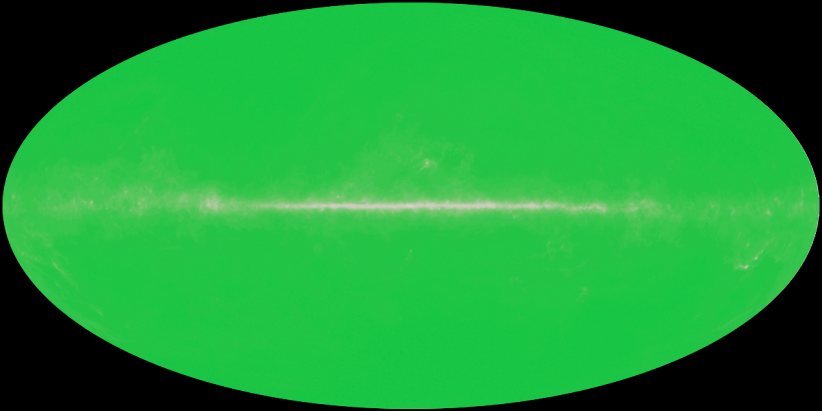
Flatness problem



Ned Wright

Horizon problem

Why the opposite sides of the universe have the same temperature of $T \simeq 2.7$ K?



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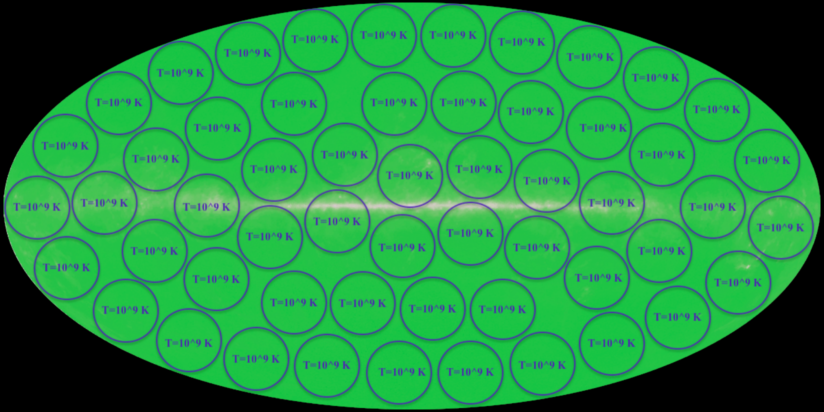
Horizon problem

There are many($\sim 10^8$) casually disconnected regions with the same temperatures $T \simeq 10^9$ K at Big Bang nucleosynthesis!



Solution to the Big Bang problems

Inflation solves the Big Bang problems:

- **Flatness (problem):**
  Inflation implies $0.999... \leq |\Omega_{\rm tot}|_{t=1s} \leq 1.000...1$ or $|\Omega_{\rm tot}| \rightarrow 1.000...$. is the attractor solution.

Solution to the Big Bang problems

Inflation solves the Big Bang problems:

- **Flatness (problem):**
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- **Horizon (problem):**
  Inflation creates our observable universe from one causally disconnected region through the rapid exponential expansion.

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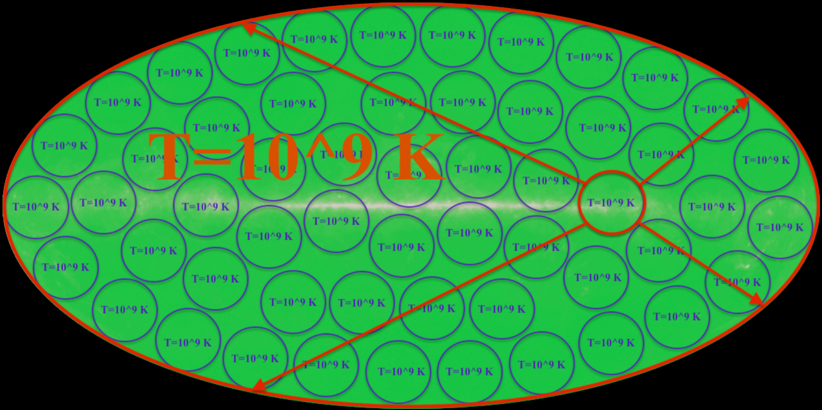


Inflation

- $a(t)$ is the scale factor which characterizes the size of the Universe
- The expansion rate (Hubble parameter) is positive
  $H \equiv \dot{a}/a > 0$ [$\dot{a} > 0$ in expanding universe]

Inflation is a short accelerated expansion $\ddot{a} > 0$ at early times, say before the Big Bang nucleosynthesis

- The comoving Hubble horizon $(aH)^{-1} = \dot{a}^{-1}$ is
  - increasing in decelerating universe ↑
  - decreasing in accelerating universe ↓
- Since $(\dot{a}^{-1})^{\cdot} = -\ddot{a}/\dot{a}^2 = -(1-\epsilon)/a$ where $\epsilon \equiv -\dot{H}/H^2$, we need $\epsilon < 1$

How to get inflation?

Friedmann Equations $\left[M_{\rm Pl} = (8\pi G)^{-1/2}\right]$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{\rho}{3M_{\rm Pl}^2} \qquad\qquad \frac{\ddot a}{a} = -\frac{1}{6M_{\rm Pl}^2}(\rho + 3p)$$

Inflation needs $\ddot a/a > 0$ which implies $\rho + 3p < 0$!

Scalar field:

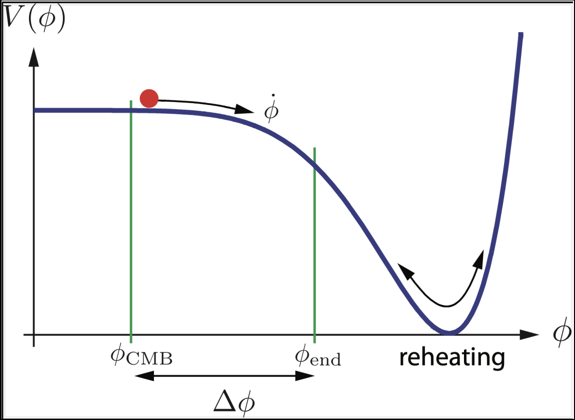
$$S = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}\mathrm{R} - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right]$$

$$\rho = \frac{1}{2}\dot\phi^2 + V(\phi) \qquad\qquad p = \frac{1}{2}\dot\phi^2 - V(\phi)$$

$$\rho + 3p = 2\left[\dot\phi^2 - V(\phi)\right] < 0 \text{ for } \dot\phi^2 \ll V(\phi)$$

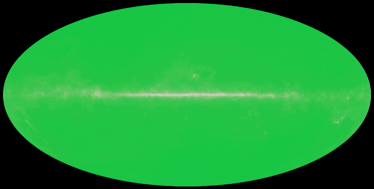
Slow-roll inflation

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{M_{\rm Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1 \qquad \eta \equiv H^{-1}(\ln \epsilon)^{\cdot} \simeq -\epsilon + M_{\rm Pl}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$



Origin of structures in the Universe?

$T \simeq 2.7$ **K**



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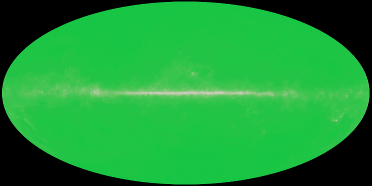

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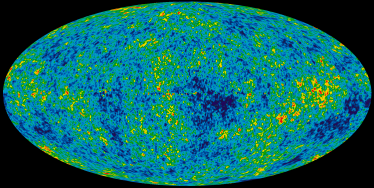
$T \simeq 2.7(1 \pm 10^{-5})$ **K**



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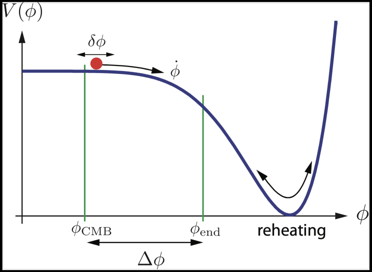
We need small inhomogeneities as a seed for the observed structures in the universe like stars, galaxies, clusters

Quantum fluctuations of the inflaton field

$$\phi(t, \mathbf{x}) = \phi(t) + \delta\phi(t, \mathbf{x})$$



The curvature perturbations $\mathcal{R} = \frac{H}{\dot{\phi}}\delta\phi$ (in spatially flat gauge). For the normalized field $u \equiv \mathcal{R}/(\sqrt{2}M_{\rm Pl}a)$ in conformal time $d\tau \equiv dt/a$

$$u_k'' + \left[(k\tau)^2 - 2 + \beta_{\mathcal{R}}\right]\frac{u_k}{\tau^2} = 0\,; \qquad \beta_{\mathcal{R}} \equiv \frac{m_{\mathcal{R}}^2}{H_{inf}^2} \simeq 6\epsilon - 3\eta \ll 1$$

Superhorizon curvature perturbations $-k\tau < -\sqrt{2}$ or $k < \sqrt{2}aH_{inf}$ are produced through interaction with gravity

Power spectrum for the superhorizon curvature perturbations

$$\langle \mathcal{R}(\tau, \mathbf{k})\mathcal{R}(\tau, \mathbf{k}')\rangle \equiv \frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')$$

$$\mathcal{P}_{\mathcal{R}} = A_{\mathcal{R}}\left(-k\tau\right)^{n_{\mathcal{R}}-1}$$

$$A_{\mathcal{R}} = \frac{H_{inf}^2}{8\pi^2 M_{\rm Pl}^2\epsilon} \qquad n_{\mathcal{R}} - 1 = -4\epsilon + 2\eta$$

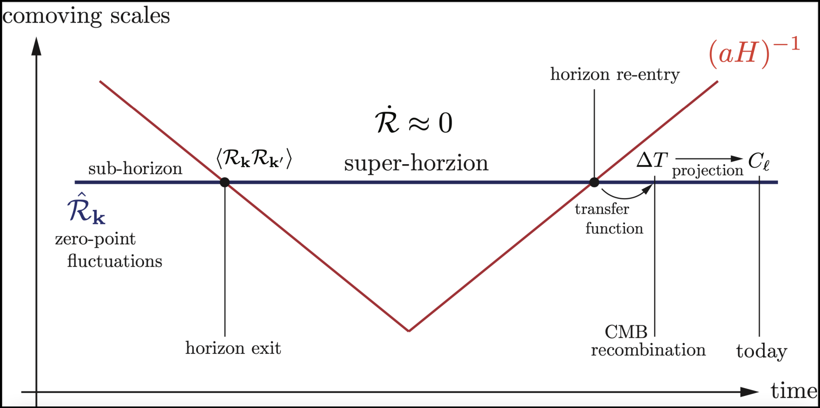
CMB observations

$$A_{\mathcal{R}} = \mathcal{O}\left(10^{-9}\right) \qquad n_{\mathcal{R}} - 1 = \mathcal{O}\left(10^{-2}\right)$$

Inflation generates, almost Gaussian, almost adiabatic, and almost scale-invariant perturbations

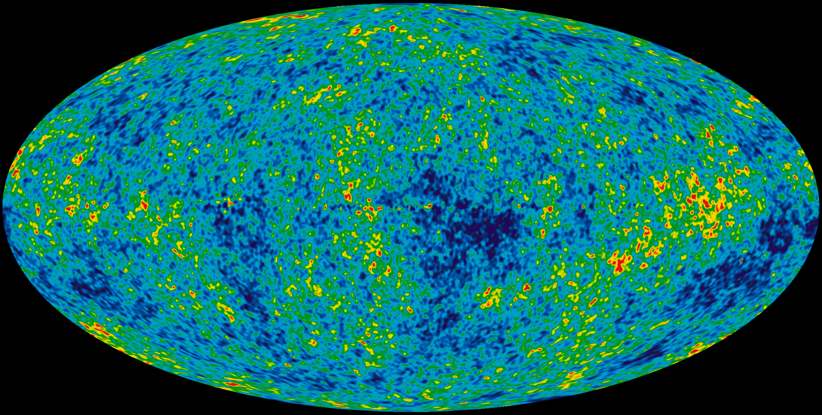
Curvature perturbations

The superhorizon curvature perturbations provide the seed for the observed structures in the universe



Inflation explains origin of structures

$$\langle \mathcal{R}\mathcal{R} \rangle \propto \left\langle \frac{\delta T}{T} \frac{\delta T}{T} \right\rangle$$



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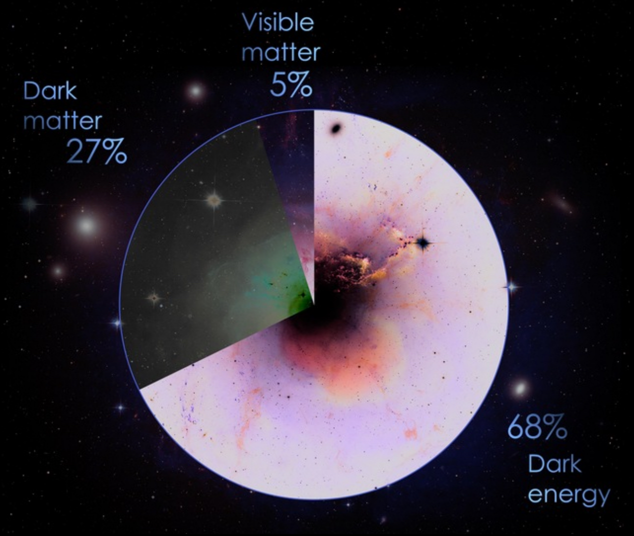
Contents of the Universe



https://svs.gsfc.nasa.gov/12307

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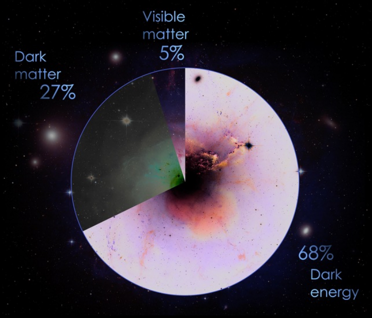
Dark Matter (DM)

Evidences:

- Rotation curves in spiral galaxies
- Gravitational lensing
- Cosmic microwave background radiation
- ...



Candidates:

- Weakly Interacting Massive Particles (WIMPs)
- Neutrino DM
- Primordial Black Holes (PBHs)
- Modified gravity

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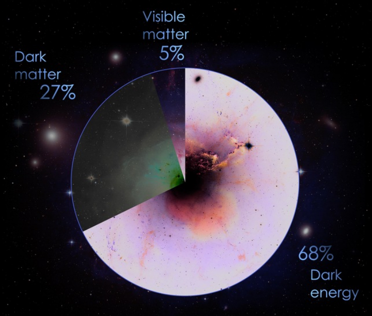

Candidates:

- Weakly Interacting Massive Particles (WIMPs)
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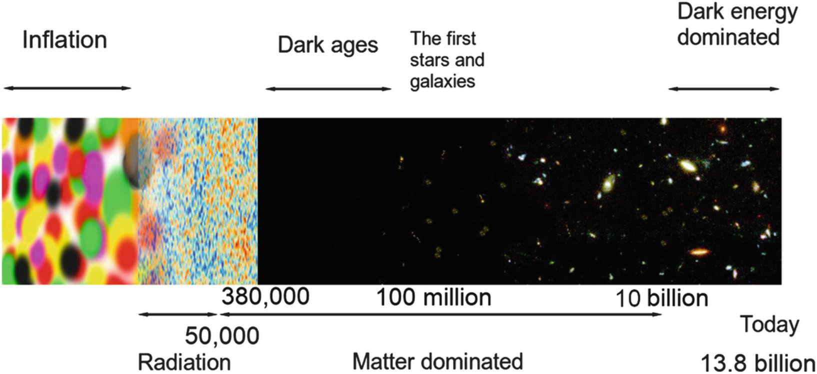
- Isocurvature fields during inflation?

Isocurvature DM

DM particles can produce during inflation



Karim A. Malik & David R. Matravers, How Cosmologists Explain the Universe to Friends and Family

Isocurvature DM: Mass

Isocurvature fields should be almost massless during inflation and massive before the time of matter and radiation equality:

- $m_i \ll H_{inf}$ to allow efficient production of entropy modes
- $m_i \gtrsim H_{M.R.}$ to make the produced entropy modes non-relativistic before the time of matter and radiation equality

$$\rho_{DM} \propto \begin{cases} a^{-4} & a < a_{\rm N} \\ a^{-3} & a > a_{\rm N} \end{cases}$$

[$a_{\rm N}$ is the solution of $H(a) = m_i$]

Roughly speaking, we can produce DM particles in the mass range 10^{-36}GeV $\ll m_i \ll 10^{14}$GeV

Backreaction: The energy density of the produced isocurvature modes should NOT change the background configuration of the gravity sector ($g_{\mu\nu}$) and inflaton (ϕ)

Isocurvature DM: Spin

Isocurvature fields can have different spins:

- Spin 0: Scalar fields like axion, ...
- Spin 1: Massive dark photon, ...
- Spin 2: Massive graviton(?), ...

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Multiple field inflation

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}\mathrm{R} - \frac{1}{2}g^{\alpha\beta}\gamma_{ab}\partial_\alpha\phi^a\partial_\beta\phi^b - V(\phi^a)\right]$$

$$\phi^a(t,\mathbf{x}) = \phi^a(t) + \delta\phi^a(t,\mathbf{x}) \qquad a = 1, 2$$

$$\rho = \frac{1}{2}\left[\gamma_{ab}\dot{\phi}^a\dot{\phi}^b\right] + V(\phi^a) \qquad p = \frac{1}{2}\left[\gamma_{ab}\dot{\phi}^a\dot{\phi}^b\right] - V(\phi^a)$$

The background dynamics becomes the same as single field inflation by the following identification:

$$\left[\gamma_{ab}\dot{\phi}^a\dot{\phi}^b\right] \iff \dot{\phi}^2$$

Multiple field inflation

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{M_{\mathrm{Pl}}^2}{2} \mathrm{R} - \frac{1}{2} g^{\alpha\beta} \gamma_{ab} \partial_\alpha \phi^a \partial_\beta \phi^b - V(\phi^a) \right]$$

$$\phi^a(t, \mathbf{x}) = \phi^a(t) + \delta\phi^a(t, \mathbf{x}) \qquad a = 1, 2$$

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The background dynamics becomes the same as single field inflation by the following identification:

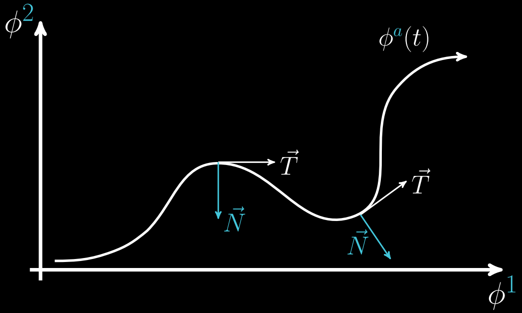
$$\left[\gamma_{ab} \dot{\phi}^a \dot{\phi}^b \right] \quad \Longleftrightarrow \quad \dot{\phi}^2$$

Which of $\delta\phi^1$ and $\delta\phi^2$ plays the role of curvature perturbations?

Adiabatic/entropy decomposition

$$\mathcal{R} = \frac{H}{\sqrt{\gamma_{ab}\dot{\phi}^a\dot{\phi}^b}} T_a \delta\phi^a$$ curvature perturbations

$$\mathcal{S} = \frac{H}{\sqrt{\gamma_{ab}\dot{\phi}^a\dot{\phi}^b}} N_a \delta\phi^a$$ entropy perturbations



Entropic DM

Defining $s \equiv a(\sqrt{\gamma_{ab}\dot{\phi}^a\dot{\phi}^b}/H)\mathcal{S}$ and considering geodesic trajectory in field space (T^a and N^a do not change in time)

$$s_k'' + \omega_k^2 s_k = 0 \qquad \omega_k^2 \equiv \left[(k\tau)^2 - 2 - \alpha + \beta_s\right]/\tau^2$$

$$\alpha \equiv -\epsilon M_{\rm Pl}^2 \mathbb{R} \qquad \beta_s \equiv \frac{m_s^2}{H_{inf}^2} = \frac{N^a N^b \nabla_a \nabla_b V}{H^2}$$

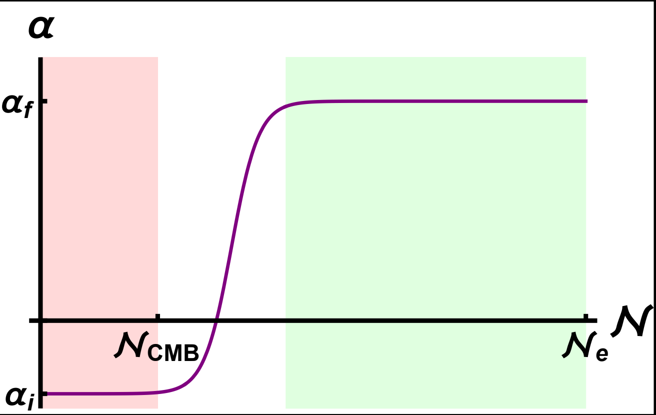
$\mathbb{R}$ is the Ricci scalar of the field space with metric γ_{ab}

There is an upper bound on the power spectrum of the superhorizon entropy perturbations

$$\mathcal{P}_{\mathcal{S}}/\mathcal{P}_{\mathcal{R}} \lesssim 10^{-3} \quad \Rightarrow \quad \alpha - \beta_s \lesssim -0.1$$

For $\mathcal{N} > \mathcal{N}_{CMB}$, there is NO constraint on α and we can have

$$\alpha > 0\,(\mathbb{R} < 0) \quad \text{with} \quad |\alpha| \gg 1$$



For negative curvature of the field space we can achieve

$$\omega_k^2 = \left[(k\tau)^2 - 2 - \alpha + \beta_s\right]/\tau^2 < 0 \qquad \beta_s = m_s^2/H_{inf}^2$$

Observables

The accumulated energy density of the excited entropy modes with $\omega_k^2 < 0$ at the end of inflation $(H_{\rm e} \sim H_{inf})$

$$\Omega_{s,\rm e} = \frac{\rho_{s,\rm e}}{3M_{\rm Pl}^2 H_{\rm e}^2} \qquad \Omega_{s,\rm e} \ll 1 \quad \text{(To avoid backreaction)}$$

The relic density for the DM today is

$$\Omega_{s,0} = \mathcal{O}(10^{20}) \left(\frac{m_s}{H_{\rm e}}\right)^{1/2} \left(\frac{T_{\rm r}}{10^{12}\rm GeV}\right) \Omega_{s,\rm e}$$

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The spectrum and tilt are characterized by

$$\Omega_s = \int d \ln k \, P_s(k)$$

$$n_s(k) - 1 = \frac{d \ln P_s}{d \ln k}$$

A model

$$\alpha = \alpha_{\rm e} \left(\frac{\tau_{\rm e}}{\tau} \right)^{2p} \qquad p > 0\,, \qquad \alpha_{\rm e} > 0$$

$$s_k'' + \omega_k^2\, s_k = 0 \qquad \omega_k^2 = \left[(k\tau)^2 - 2 - \alpha_{\rm e} \left(\frac{\tau_{\rm e}}{\tau} \right)^{2p} + \beta_s \right] / \tau^2$$

- Interaction with gravity gives a NEGATIVE contribution
- Curvature of field space gives a NEGATIVE contribution
- The mass term gives a POSITIVE contribution

A model

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- The mass term gives a POSITIVE contribution

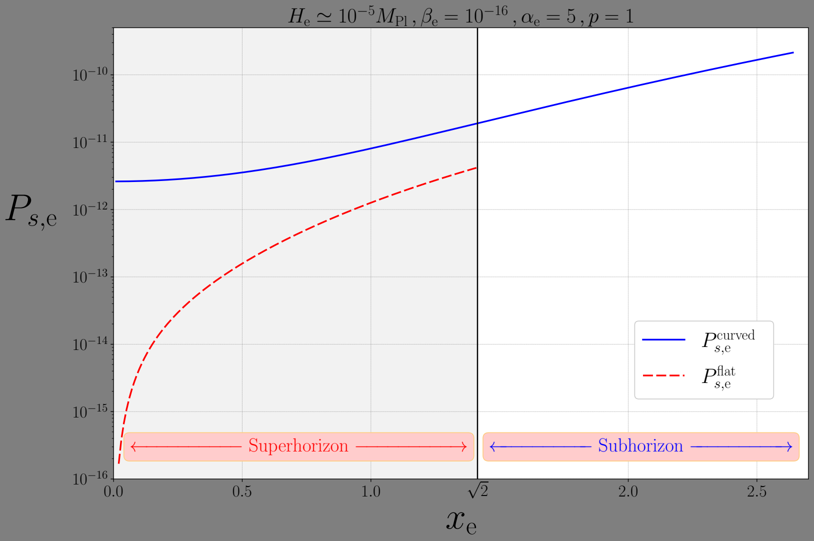
Gravity excites *light* $\beta_s \ll 1$ *superhorizon* $-k\tau < \sqrt{2}$ modes

D. Polarski, A. A. Starobinsky, PRD (1994), T. Tenkanen, PRL (2019)

Curvature of field space excites *semiheavy* $\beta_s = \mathcal{O}(1-10)$ *subhorizon* $-k\tau < \sqrt{2+\alpha}$ modes

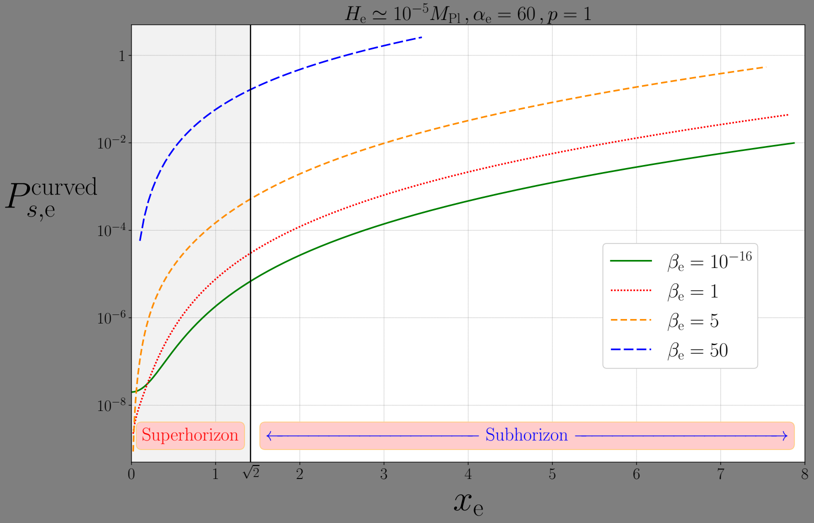
H. Firouzjahi, MAG, S. Mukohyama, A. Talebian (2021)

Flat vs curved field spaces: Spectrum

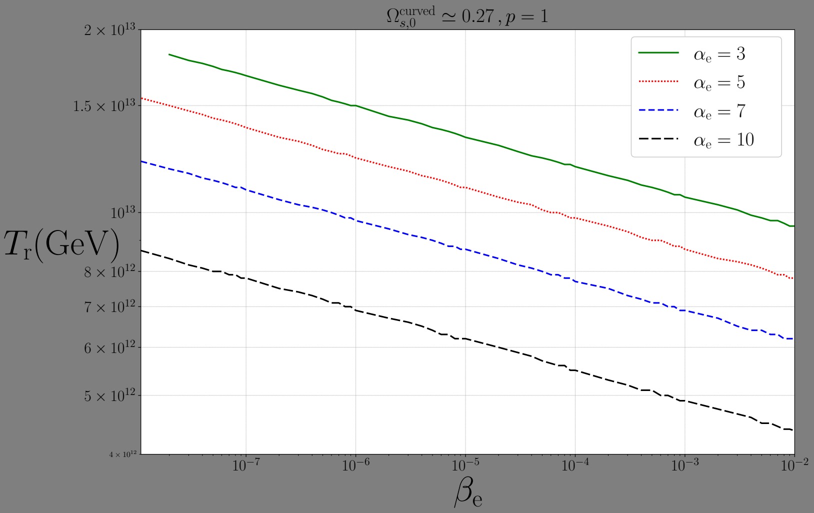


The location of the peaks for the spectral densities in flat and curved field spaces are different

Spectrum for different masses



Relic density



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Vector Dark Matter (VDM)

Massive vector field A_μ *minimally* coupled with gravity $g_{\mu\nu}$ and inflaton ϕ during inflation

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}{\rm R} - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)$$
$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_A^2 A_\mu A^\mu + \mathcal{L}_{int}(A_\mu, \phi)$$

One longitudinal dof (scalar dark matter):

$$A''_{k,L} + \omega_k^2 A_{k,L} = 0 \quad \omega_k^2 = \left[(k\tau)^2 - 2 - \alpha_{int} + \beta_A\right]/\tau^2; \qquad \beta_A \equiv \frac{m_A^2}{H_{inf}^2}$$

It CAN be excited even for $\alpha_{int} = 0$

P. W. Graham, J. Mardon, S. Rajendran, PRD (2016)

Two transverse dofs (VDM):

$$A''_{k,\lambda} + \omega_{k,\lambda}^2 A_{k,\lambda} = 0 \quad \omega_{k,\lambda}^2 = \left[(k\tau)^2 - \alpha_{int} + \beta_A\right]/\tau^2; \qquad \lambda = (+,-)$$

It CANNOT be excited for $\alpha_{int} = 0$ as always $\omega_k^2 > 0$

The **transverse modes** can be excited through interactions with inflaton which breaks the conformal symmetry

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\mathrm{R} - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)$$
$$- \frac{f(\phi)^2}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_A^2 A_\mu A^\mu - \frac{h(\phi)^2}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

The mode functions $\nu_{k,\lambda} = f(\phi)A_{k,\lambda}$ satisfy

$$\nu''_{k,\lambda} + \omega_{k,\lambda}^2\nu_{k,\lambda} = 0 \qquad \omega_{k,\lambda}^2 = \left[(k\tau)^2 - \alpha_{int} + \frac{\beta_A}{f^2}\right]/\tau^2$$

$$\alpha_{int} = \frac{\tau^2 f''}{f} + 2\lambda(k\tau)\frac{\tau h' h}{f^2}$$

It is NOT possible to produce VDM with $m_A \sim H_{inf}$

M. Bastero-Gil, J. Santiago, L. Ubaldi, R. Vega-Morales, JCAP (2019)

K. Nakayama, JCAP (2020)

Y. Nakai, R. Namba, Z. Wang, JHEP (2020)

Heavy VDM via inflation with symmetry breaking

Vector field is massless during inflation while it acquires mass through the **symmetry breaking mechanism** toward the end of inflation:

$$\omega_k^2 = \begin{cases} \left[(k\tau)^2 - \alpha_{int}\right]/\tau^2 & m_A = 0 \text{ during inflation} \\ \left[(k\tau)^2 - \alpha_{int} + \beta_A\right]/\tau^2 & m_A \neq 0 \text{ at the end of inflation} \end{cases}$$

Vector field particles produce very efficiently during inflation since $m_A = 0$. After the production, they acquire masses $m_A \neq 0$

It is possible to produce VDM with mass $m_A \sim H_{inf}$ $(\beta_A = m_A^2/H_{inf}^2 \sim \mathcal{O}(1))$

B. Salehian, MAG, H. Firouzjahi, S. Mukohyama, PRD (2021)

H. Firouzjahi, MAG, S. Mukohyama, B. Salehian, JHEP (2021)

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Summary

- Inflation generates perturbations on CMB and seeds the observed structures like stars, galaxies, and clusters in the universe
- Massive isocurvature fields with different spins can be also excited during inflation which can play the role of dark matter
- Not only superhorizon but also subhorizon entropy perturbations can be excited in multiple field inflationary models
- “Entropic dark matter” has its own observational signature which makes it distinguishable, e. g., from ”vector dark matter” models

Thank You

References

- Dark matter from entropy perturbations in curved field space [arXiv:2110.09538 [gr-qc]]
- Dark photon dark matter from charged inflaton, JHEP (2021) [arXiv:2011.06324 [hep-ph]]
- Vector dark matter production from inflation with symmetry breaking, PRD (2021) [arXiv:2010.04491 [hep-ph]]

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