EMITTANCE GROWTH

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$$\iint d^3q_i d^3P_i = \text{const.}$$

$$A_x = \frac{1}{P}\iint dx\, dP_x = \frac{1}{\gamma\beta mc}\iint dx\, dP_x.$$

in an ideal system (linear forces, no coupling)

if there is no acceleration or deceleration ($\beta\gamma$ = const)

$$\tilde{\epsilon}_n = \beta\gamma\tilde{\epsilon}$$

An increase of the normalized emittance:
nonlinear effects causing a deterioration of beam quality

50 keV and an effective emittance: $\epsilon_1 = 200$ mm-mrad. It emerges from the accelerator with a kinetic energy of 80MeV. What is the emittance of the accelerated proton Beam ϵ_2 if no particle loss and no distortions in the phase-space volume occur?

We may treat the protons non relativistically $since\ T \ll E_0 = 938.25\ MeV$.

$vA_x = const$, or

$T^{1/2}\ A_x = const.$

$\epsilon_2 = \epsilon_1 \left(\frac{T_1}{T_2}\right)^{1/2} = \epsilon_1/40$.-------> $\epsilon_2 = 5$ mm-mrad.

while the normalized emittance $\epsilon_n = \beta\gamma\epsilon = const.$

stationary beams

The applied focusing forces : linear
--->(In stationary or quasi stationary beams)
------> the emittances associated with each direction are constant.

} ⟶ These beams: have Maxwell–Boltzmann distribution with:

$T_{\perp} \neq T_{\parallel}$

The space charge forces in stationary beams??:

in general nonlinear

except at very low temperatures: ⟶ { where the perveance K dominates over the emittance
where the transverse density profile: uniform.

in the equilibrium state,
the nonlinear space-charge forces do not cause any changes
in temperature and emittance.

any deviation from the nearly uniform density profile of
a space-charge dominated Maxwell–Boltzmann distribution -----> emittance growth

Near the source and in the low-energy part of
the accelerator system

$$\ddot{r} = v_z^2 \frac{d^2 r}{dz^2} = \beta^2 c^2 r''$$

$$\ddot{r} = \frac{\omega_p^2}{2} r.$$

generalized perveance K (dimensionless quantity):

$$K = \frac{I}{I_0} \frac{2}{\beta^3 \gamma^3} (1 - \gamma^2 f_e).$$

K:unlike the plasma frequency – does not depend on the beam radius. It is solely defined by the beam current and particle energy and, where applicable, by the charge-neutralization factor f_e .

-----> particle trajectories : $\frac{d}{dt}(\gamma m \dot{r}) = \gamma m \ddot{r} = q E_r - q \dot{z} B_\theta,$

-----> $r'' = \frac{K}{a^2} r$

If $f_e = 0$: the relationship among generalized perveance, Budker parameter, and the plasma frequency is given by:

$$K = \frac{I}{I_0} \frac{2}{\beta^3 \gamma^3} = \frac{2\nu_B}{\beta^2 \gamma^3} = \frac{\omega_p^2 a^2}{2\beta^2 c^2}$$

Beam not in equilibrium:

temperature and emittance increase

The most important causes of emittance growth:

- Nonlinearities in the applied forces
- Chromatic aberrations
- Nonlinear forces arising from nonstationary beam density profiles
- Beam mismatch causing oscillations of the rms radius
- Beam off-centering causing coherent oscillations around the optical axis or central orbit
- Misalignments of the focusing and accelerating elements
- Collisions between the beam particles (Coulomb scattering) and between the beam and a background gas or a foil
- Instabilities, including unstable interactions with applied or beam-generated electromagnetic fields
- Nonlinear single-particle resonances and nonlinear coupling between longitudinal and transverse motion (especially important in circular accelerators)
- Beam–beam effects in the interaction regions of high-energy collider
- Random kicks due to rf noise, mechanical vibrations of the magnets, and other sources of statistical fluctuations (limiting the lifetime of beams in storage rings)

1-Boltzmann distribution with a Gaussian density profile(nonuniform density profile) nonlinear space-charge forces

thermal energy → emittance growth

uniform density profile of the ideal Maxwell–Boltzmann distribution

2-emittance growth caused by nonlinear external focusing forces in beams where space charge is negligible

Beam not in equilibrium:

emittance decrease:

An example: equipartitioning,
Coulomb collisions or collective forces tend to drive a beam with an anisotropic temperature distribution toward three-dimensional thermal equilibrium. Thus, if the temperature is high in the transverse direction, it will fall, while the low temperature in the longitudinal direction will rise until both temperatures are equal. As a result, the transverse emittance in this case will become smaller while the longitudinal emittance will increase. Such equipartitioning will be discussed in connection with:

- the Boersch effect,
- intrabeam scattering,
- instabilities

beam cooling
in storage rings

Beam not in equilibrium:
(nonstationary initial beam)

at injection into the focusing channel.

Free Energy and Transverse Emittance Growth in Nonstationary Beams

- mismatch in the density profile (e.g., the beam density is not uniform in the low-temperature, space-charge dominated case),
- mismatch in the rms radius,
- off-centering,
- a combination of these three effects.

free energy :ΔE------> can cause emittance growth

a nonstationary initial beam has a higher total energy per particle than that of the corresponding stationary beam

nonlinear external or self forces, instabilities, or collisions are present

external focusing force

$$E = E_k + E_p + E_s$$

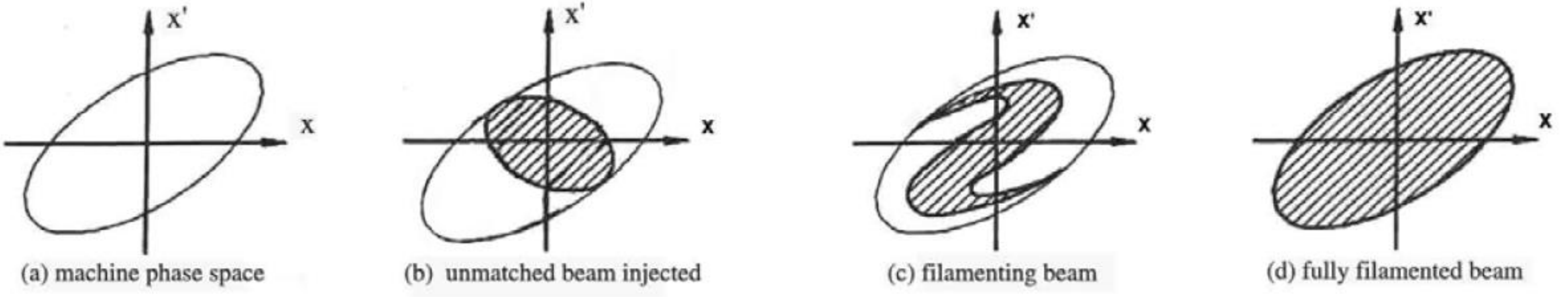
self forces

$$= \frac{\gamma m v^2}{4}\left[k^2 a^2 + k_0^2 a^2 + \frac{1}{2}[k_0^2 - k^2]a^2\left(1 + 4\ln\frac{b}{a}\right)\right].$$

linear focusing

Mismatch At Injection: *Filamentation*

dilution mechanism



Filamentation of a mismatched beam at injection

the initial correlation between amplitudes and phases of the particles has practically vanished.

Where?

Filamentation occurs in circular machines.
In a beam line or a linac, the number of betatron oscillations is usually not large enough to lead to complete 'smear-out'. Yet here also, extra aperture has to be provided to contain the phase-space rotations of a mismatched beam.

linear focusing

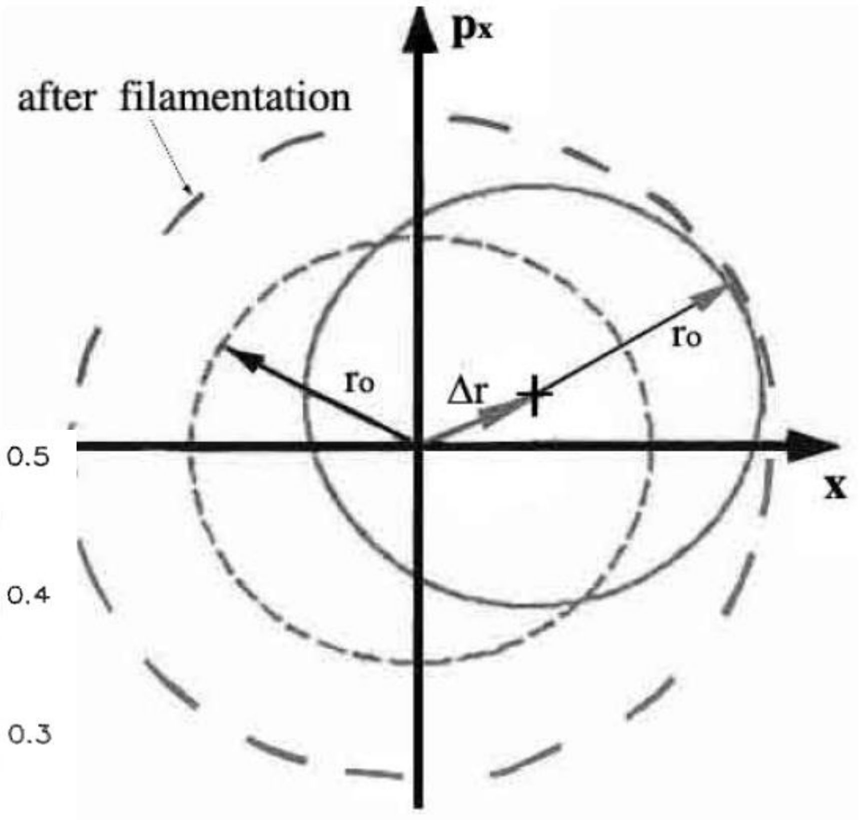
Mismatch At Transfer : ***Steering error***

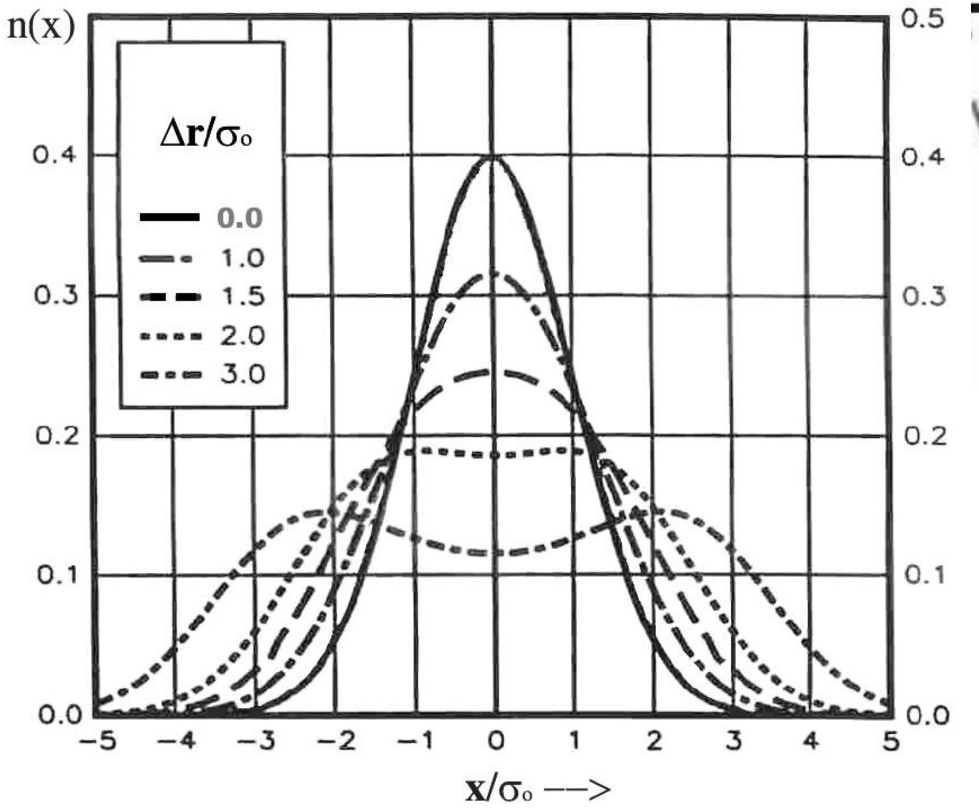
beam injected into a circular machine have:
an error in position (Δx) *and angle* $(\Delta x')$
With respect to the design orbit

$$\Delta r = \sqrt{(\Delta x)^2 + (\Delta p_x)^2} = \sqrt{(\Delta x)^2 + (\alpha_x \Delta x + \beta_x \Delta x')^2}$$

$$\varepsilon_{\%} \rightarrow \varepsilon_{\%} (1 + \Delta r / r_0)^2$$

$$\varepsilon_{k\sigma} \rightarrow \varepsilon_{k\sigma} \left\{ 1 + \frac{1}{2} \left(\Delta r^2 / \sigma_{x,0}^2 \right) \right\}$$

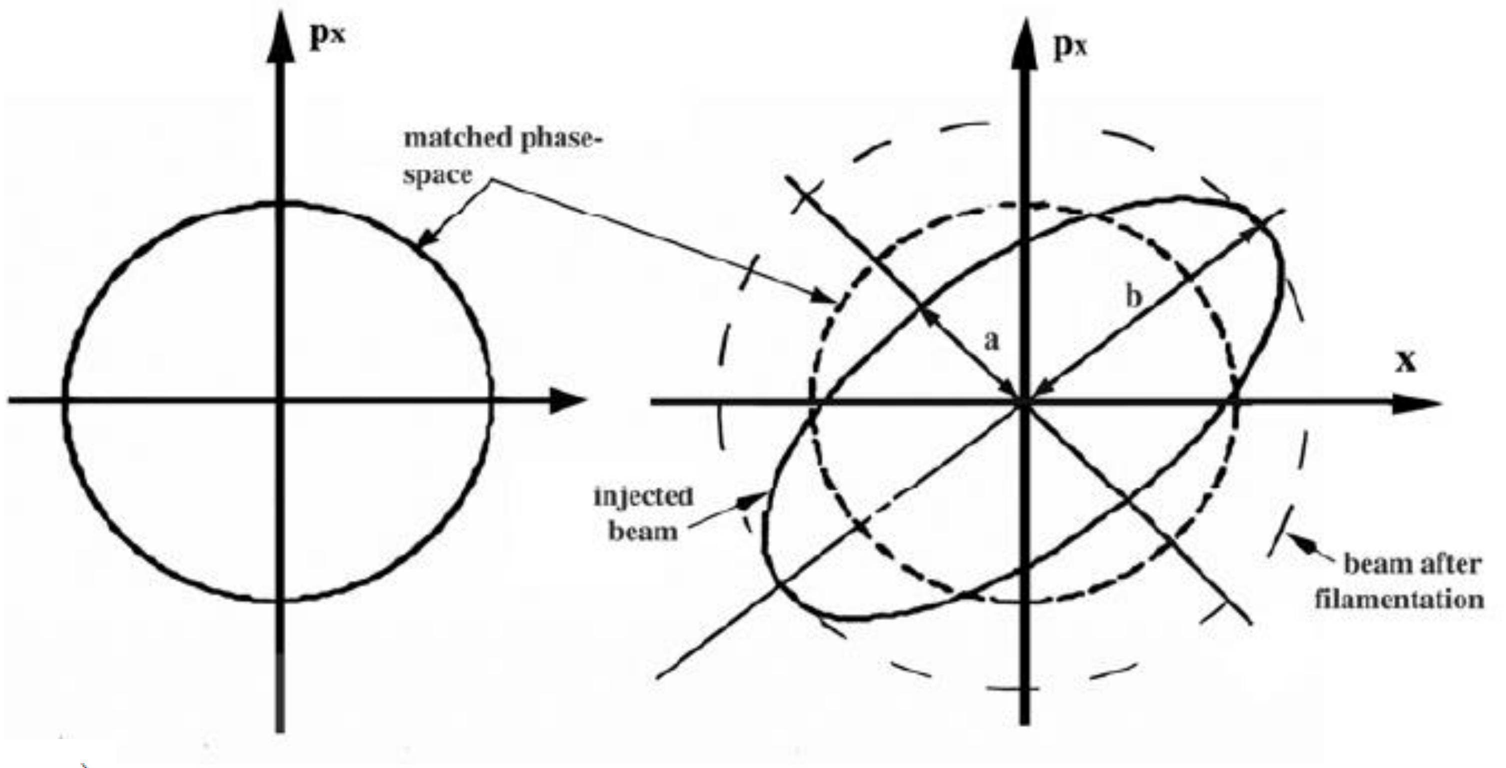




the shape of the final distribution depends strongly on the magnitude of the error.

For large errors (compared to the standard deviation of the original distribution), the beam centre becomes depopulated and double-humped distributions emerge .

Mismatch At Transfer : **Focusing errors (Gradient errors)**



$$\varepsilon_{k\sigma} \to \varepsilon_{k\sigma s} \frac{(b/a + a/b)}{2}$$

non-linearity

MISMATCH AT TRANSFER

Mismatch due to:

- **Nonlinear external focusing forces**
- **space charge**

The area enclosed: constant
in agreement with Liouville's theorem

But, $\epsilon \neq Const$ ——→ correlation

$$\Delta r' = -a_1 r - a_3 r^3$$

- Progressive distortion of trace-space ellipse during beam propagation through a periodic channel of thin lenses with spherical aberrations.
- The numbers: the lens periods that have been traversed.