Universal Thermal Corrections to Symmetry-Resolved Entanglement Entropy and Full Counting Statistics

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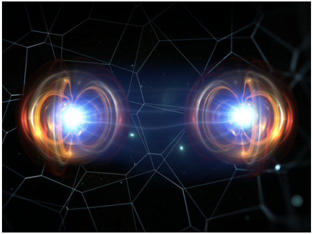
Title

- Entanglement Measures
- Symmetry- Resolved Entanglement Measures
- Universal thermal corrections to Symmetry- Resolved Entanglement Measures and Full Counting Statistics

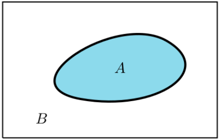
Entanglement Measures



Entanglement state: Quantum entanglement is a quantum mechanical phenomenon in which the quantum state of each subsystem of the multi-partite system cannot be described independently.

Entanglement Measures

Divide a system to A and $B = \bar{A}$: $\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$



- For bipartite pure state the well-known entanglement measures are Rényi and entanglement entropies.
- For bipartite quantum system $A \cup B$, the nth Rényi entropy (RE) is defined as

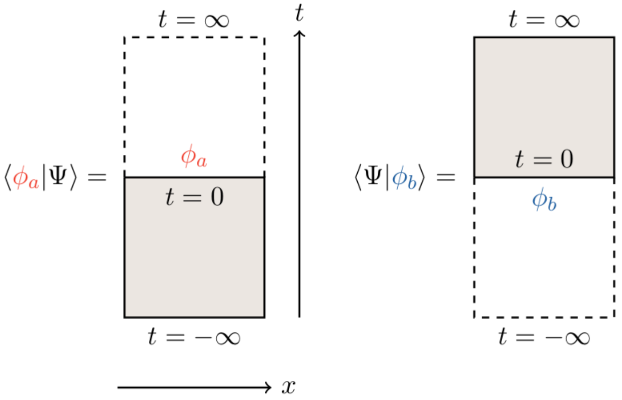
$$S_n \equiv \frac{1}{1-n} \log \mathrm{Tr}(\rho_A)^n \tag{1}$$

where $\rho_A = \mathrm{Tr}_B \rho$ is the reduced density matrix of subsystem A. EE is given by $S_E = -\mathrm{Tr}\rho_A \ln \rho_A = \lim_{n\to 1} S_n$.

Replica trick and QFTs

- dim $\mathcal{H} = \infty$ in QFT
- The computation of the EE often based on the replica trick, by introducing n copies of the system.
- We take Pure state: $\rho = |\Psi\rangle\langle\Psi|$

Replica trick and QFTs

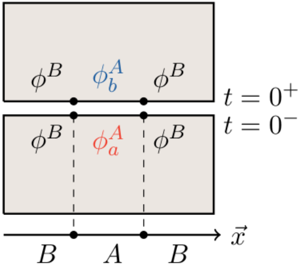


States $|\phi_{a,b}\rangle$ are the boundary conditions at $t = 0$

Replica trick and QFTs

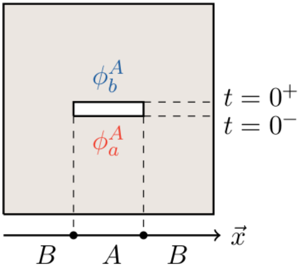
$$[\rho_A]_{ab} = \frac{1}{Z_1}\int [\mathcal{D}\phi^B(t=0,\vec{x}\in B)]\ \left(\langle\phi_a^A|\langle\phi^B|\right)|\Psi\rangle\langle\Psi|\left(|\phi_b^A\rangle|\phi^B\rangle\right)\ ,$$

$$= \frac{1}{Z_1}\int [\mathcal{D}\phi^B(t=0,\vec{x}\in B)]$$



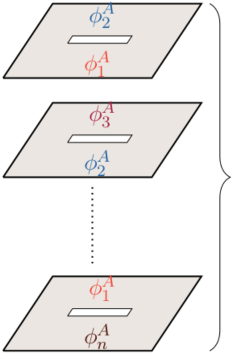
Replica trick and QFTs

$$[\rho_A]_{ab} = \frac{1}{Z_1}$$



Replica trick and QFTs

$$\mathrm{tr}_A \rho_A^n = \frac{1}{(Z_1)^n}$$



n copies $\equiv Z_n$

$$= \frac{Z_n}{(Z_1)^n}$$

Replica trick and QFTs

- dim $\mathcal{H} = \infty$ in QFT
- The computation of the EE often based on the replica trick, by introducing n copies of the system.
- $\mathrm{Tr}(\rho_A)^n = \frac{\mathcal{Z}_n}{\mathcal{Z}_1^n}$. $\mathcal{Z}_n$ is a partition function on the covering space /replicated geometry $\mathcal{R}_n$.
- Based on the path integral language, the calculation of S_n reduces to computing the partition function on a Riemann surface geometry $\mathcal{R}_n$.

Symmetry Resolved Entanglement Entropy

Symmetry Resolved Entanglement Entropy

- Suppose our system have an internal symmetry, e.g., $U(1)$ symmetry, that is generated by the charge operator $\hat{Q}$. We assume the state ρ of the system is the eigenstate of the symmetry generator $\hat{Q}$, then $[\rho, \hat{Q}] = 0$. And $\hat{Q} = \hat{Q}_A \oplus \hat{Q}_B$, where $\hat{Q}_i$ $(i = A, B)$ is the charge in the subsystem i.
- For a bipartition of the total system into two subsystems A and B, by taking the trace over the degrees of freedom of B of $[\rho, \hat{Q}] = 0$, we find that $[\rho_A, \hat{Q}_A] = 0$. Therefore, ρ_A is block-diagonal according to eigenvalue Q_A of charge operator $\hat{Q}_A$,

$$\rho_A = \bigoplus_{Q_A} \Pi_q \, \rho = \bigoplus_{Q_A} P(Q_A)\rho_A(Q_A), \tag{2}$$

where Π_{Q_A} is the projector on the sector of $\hat{Q}_A$ with eigenvalue Q_A, and $P(Q_A) \equiv \text{Tr}(\Pi_{Q_A}\rho_A)$ is the probability that measurement of the charge $\hat{Q}_A$ in region A be Q_A.

Symmetry Resolved Entanglement Entropy

By definition:

- Symmetry resolved Rényi entropies are defined as

$$S_n(q) \equiv \frac{1}{1-n} \log \mathrm{Tr} \rho_A^n(Q_A). \quad (3)$$

- Symmetry resolved entanglement entropy is defined as

$$S(Q) = -\mathrm{Tr} \rho_A(Q_A) \ln \rho_A(Q_A). \quad (4)$$

Symmetry Resolved Entanglement Entropy

Main idea behind the SRE:

- We block diagonalize the reduced density matrix of the subsystem into different sectors of fixed charge and then finds the entropy of each sector

Symmetry Resolved Entanglement Entropy

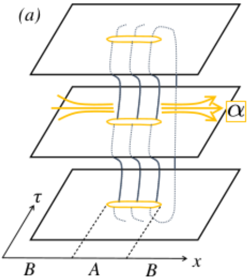
- Total entanglement entropy is the sum of the contributions of individual charge sectors,

$$S_E = \sum_{Q_A} P(Q_A) S(Q_A) - \sum_{Q_A} P(Q_A) \log(P(Q_A)) = S^c + S^n. \qquad (5)$$

- S^c is the configurational entropy, which measures the average of the entanglement in each charge sector.
- S^n is called number entropy, which quantify the entropy due to the fluctuations of the charge within subsystem A.

Symmetry Resolved Entanglement Entropy

- Theoretical framework to evaluating the contribution of each symmetry sector by relating the symmetry-resolved entanglement entropy to the Fourier transform of partition function on the n-sheet Riemann surface with generalized Aharonov-Bohm flux:



Charged moments:

$$\mathcal{Z}_n(\alpha) = \text{tr}\rho_A^n e^{-i\alpha \hat{Q}_A}, \quad (6)$$

Symmetry Resolved Entanglement Entropy

- Theoretical framework to evaluating the contribution of each symmetry sector by relating the symmetry-resolved entanglement entropy to the Fourier transform of partition function on the n-sheet Riemann surface with generalized Aharonov-Bohm flux:

$$\mathcal{Z}_n(Q_A) = \int_{-\pi}^{\pi} \frac{\mathrm{d}\alpha}{2\pi} \mathcal{Z}_n(\alpha) e^{-i\alpha Q_A}, \tag{7}$$

- Symmetry-resolved Rényi and entanglement entropies can be obtained as :

$$S_n(Q_A) = \frac{1}{1-n} \log \left[\frac{\mathcal{Z}_n(Q_A)}{\mathcal{Z}_1^n(Q_A)} \right], \tag{8}$$

$S(Q_A) = \lim_{n\to 1} S_n(Q_A)$. Probability is then $P(Q_A) = \mathcal{Z}_1(Q_A)$.

Universal Thermal correction

- It is well known that entanglement entropy is a good entanglement measure for quantum systems in their ground state. However, the real world not lying at a zero temperature regime, and hence, the entanglement entropy is no longer proper for thermal states.
- At finite temperature, the entanglement entropy of subsystem A is contaminated by thermal fluctuation and, in fact, in the high-temperature limit becomes dominated by thermal entropy.
- To determine the quantum entanglement of the thermal systems, one should subtract off the thermal contribution to entanglement entropy.
- For the systems with the mass gap m_{gap}, it is conjectured that, in the limit $\beta m_{\text{gap}} \gg 1$, these corrections scale as $e^{-\beta m_{\text{gap}}}$.
- By putting the conformal field theory on the cylinder and introducing the mass gap between the ground state and the first excited state through the finite size of the system, the coefficient of the Boltzmann factor can be computed, which are universal and depend on the size of the mass gap and the degeneracy of the first excited state.

Universal Thermal correction

The interesting questions that motivated this work are:

- What are the thermal corrections to the contribution of individual system charge sectors? How are these corrections scale?
- If these corrections are universal, what are their physical meaning?

In this work, we are addressing these questions. We introduce *the thermal charged moments*, we derive the low-temperature expansion of it. We find that these thermal corrections are encoded in the four-point function of primary fields, the scaling dimension of the lowest weight primary field, and its degeneracy. Consequently, we can find the thermal corrections to the symmetry-resolved Rényi and entanglement entropies. We also obtain thermal corrections to the full counting statistics of the ground state (FCS) and find that the fluctuations of probabilities scale as $e^{-2\pi\Delta_\psi \beta/L}$.

Thermal charged moments and universal corrections

The main quantity for computing the symmetry-resolved entanglement entropy is charge moments $\mathcal{Z}_n(\alpha)$. So to calculate the thermal corrections, we introduce *the thermal charged moments*.

- The thermal density matrix $\rho = \frac{e^{-\beta H}}{\text{tr} e^{-\beta H}}$, where β and H are the inverse temperature and the Hamiltonian, respectively. The thermal density matrix can be written as a Boltzmann sum via introducing a complete set of states: $\rho = \frac{1}{\text{tr} e^{-\beta H}} \sum_{|\phi\rangle} |\phi\rangle\langle\phi| \, e^{-\beta E_\phi}$.
- The low-temperature expansion of thermal density matrix becomes as

$$\rho \sim \rho_0 + e^{-2\pi\Delta_\psi \beta/L} \left(\rho_\psi - \rho_0\right) \ , \tag{9}$$

- Accordingly, the low-temperature expansion of reduced density matrix becomes as

$$\rho_A \sim \rho_{0,A} + e^{-2\pi\Delta_\psi \beta/L} \left(\rho_{\psi,A} - \rho_{0,A}\right), \tag{10}$$

  where $\rho_{0,A} = \mathrm{Tr}_B(|0\rangle\langle 0|)$ and $\rho_{\psi,A} = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$. The Δ_ψ is the smallest scaling dimension of primary operator, $\psi\,(w)$.

Thermal charged moments and universal corrections

- We define *the thermal charged moments* as:

Thermal charged moments

$$\mathcal{Z}_n^{(\text{th})}(\alpha) = \text{Tr}\left(\rho_A^n e^{i\alpha \hat{Q}_A}\right)$$
$$= \mathcal{Z}_n^{(0)}(\alpha) + n\mathcal{Z}_n^{(0)}(\alpha) e^{-2\pi\Delta_\psi \beta/L} \mathcal{F}_n(\alpha) \quad (11)$$

- The first term is nothing but the charged moments at zero temperature that gives the resolved-symmetry entanglement entropy for a finite system

$$\mathcal{Z}_n^{(0)}(\alpha) = c_{n,\alpha}\left[\frac{L}{\pi}\sin\left(\frac{\pi\ell}{L}\right)\right]^{\frac{1-n^2}{6n} - 2\frac{\Delta_{\mathcal{V}}}{n}}, \quad (12)$$

where $c_{n,\alpha}$ is non-universal constant, ℓ is the length of A, and $\Delta_{\mathcal{V}}$ denotes the scaling dimension of the operator $\mathcal{V}$ generating the Aharonov-Bohm flux.

Thermal charged moments and universal corrections

$\mathcal{F}_n(\alpha)$ is defined as:

$$\mathcal{F}_n(\alpha) = \mathcal{M}_n(\alpha) - 1, \qquad \text{where} \qquad \mathcal{M}_n(\alpha) = \frac{\mathrm{tr}\left(\rho_{0,A}^{n-1}\rho_{\psi,A}e^{i\alpha\hat{Q}_A}\right)}{\mathrm{tr}\left(\rho_{0,A}^{n}e^{i\alpha\hat{Q}_A}\right)}. \tag{13}$$

The denominator is the charged moments at zero temperature. The nominator is a new term. It can be interpreted as a correlation function in the presence Aharanon-Bohm flux, which then takes the following form,

$\mathcal{M}_n$ expression

$$\mathcal{M}_n = \frac{\langle \mathcal{V}_\alpha \mathcal{V}_{-\alpha} \psi\psi \rangle_{\mathcal{R}_n}}{\langle \mathcal{V}_\alpha \mathcal{V}_{-\alpha} \rangle_{\mathcal{R}_n} \langle \psi\psi \rangle_{\mathcal{R}_1}} \tag{14}$$

The numerator term is a four-point function of two $\mathcal{V}_\alpha$ and two ψ in the replicated geometry $\mathcal{R}_n$. The denominator terms are two two-point functions of the $\mathcal{V}_\alpha$ in the replicated geometry $\mathcal{R}_n$ and ψ in the original geometry $\mathcal{R}_1$, cylinder.

Thermal charged moments and universal corrections

- Through the uniformizing map which takes the multisheeted cylinder to the plane, we can evaluate these correlation functions on the plane.

$$\zeta^{(n)} = \left(\frac{e^{2\pi i w/L} - e^{i\theta_2}}{e^{2\pi i w/L} - e^{i\theta_1}} \right)^{1/n} . \quad (15)$$

- This map takes the multisheeted cylinder to the plane. The parameters θ_1 and θ_2 are selected so that $\theta_2 - \theta_1 = \frac{2\pi\ell}{L}$. Subsequently, the insertion points of the primary fields on the jth cylinder at the points $t = -\infty$ and $t = \infty$ are transformed to the points $\zeta_4^{(n)} \equiv \zeta_{-\infty}^{(n)} = e^{i(\theta_2-\theta_1)/n+2\pi ij/n}$ and $\zeta_3^{(n)} \equiv \zeta_{\infty}^{(n)} = e^{2\pi ij/n}$, respectively, on the $\zeta^{(n)}$ plane. While the operators on the end points of the interval are mapped to the points $\zeta_1^{(n)} = \infty$ and $\zeta_2^{(n)} = 0$ on the $\zeta^{(n)}$ plane.

Thermal charged moments and universal corrections

Main relation:

$\mathcal{M}_n$ expression

$$\mathcal{M}_n(\alpha) = \frac{1}{n^{2\Delta_\psi}} \frac{\sin^{2\Delta_\psi}(\pi x)}{\sin^{2\Delta_\psi}(\pi x/n)} G_{n,\alpha}(z,\bar{z}) \tag{16}$$

- where $x = \frac{l}{L}$, $z = \frac{\zeta_{12}^{(n)}\zeta_{34}^{(n)}}{\zeta_{14}^{(n)}\zeta_{23}^{(n)}}$, and $\bar{z} = \frac{\bar{\zeta}_{12}^{(n)}\bar{\zeta}_{34}^{(n)}}{\bar{\zeta}_{14}^{(n)}\bar{\zeta}_{23}^{(n)}}$.
- All information about the thermal corrections to the charged moments and the symmetry-resolved entanglement entropy, is encoded in the $G_{n,\alpha}(z,\bar{z})$. It can be regarded as the building block of our computations.

Full Counting Statistics

- Thermal charged moments can be defined as a generating function of FCS at finite temperature. FCS defines the distribution probability of conserved charge in the subsystem A with length l.
- It can be defined via generating function

$$\mathcal{Z}_1(\alpha) \equiv \chi(\alpha) = \sum_{Q_A=-\infty}^{\infty} P(Q_A) e^{i\alpha Q_A} = \left\langle e^{i\alpha \hat{Q}_A} \right\rangle, \tag{17}$$

which encodes all the cumulants C_m,

$$\ln \chi(\alpha) = \sum_{Q_A=1}^{\infty} \frac{(i\alpha)^m C_m}{m!}, \tag{18}$$

where $C_m = (-i\partial_\alpha)^m \ln \chi(\alpha)$. C_m describe properties of the distribution probability $P(Q_A)$. For example, the mean $C_1 = \left\langle \hat{Q}_A \right\rangle$, the fluctuations $C_2 = \left\langle \left(\hat{Q}_A - \left\langle \hat{Q}_A \right\rangle \right)^2 \right\rangle$, and so on.

- We derive the thermal corrections to the FCS. Let us first define the quantity:

$$f_n(\alpha, T) = \frac{\mathcal{Z}_n^{(\text{th})}(\alpha)}{\mathcal{Z}_n^{(0)}(\alpha)} = 1 + n\mathcal{F}_n(\alpha)e^{-2\pi\Delta_\psi \beta/L}. \tag{19}$$

- If we take a logarithm of $f_n(\alpha, T)$, we reach the universal quantity $g_n(\alpha, T) = \log f_n(\alpha, T)$ that can be used to define the excess-cumulant generating function, such that, its different derivative in $\alpha = 0$ gives the excess of various moments of $\hat{Q}_A$

Therefore,

FCS

$$\begin{aligned}\Delta C_{n,m} &= (-i\partial_\alpha)^m \, g_n(\alpha, T)|_{\alpha=0} \\ &= \frac{g e^{-2\pi\Delta_\psi \beta/L}}{n^{2\Delta_\psi - 1}} \frac{\sin^{2\Delta_\psi}(\pi x)}{\sin^{2\Delta_\psi}\left(\frac{\pi x}{n}\right)} (-i\partial_\alpha)^m \, G_{n,\alpha}(z, \bar{z})|_{\alpha=0}. \end{aligned} \quad (20)$$

The above relation, for $\Delta C_{1,m} = \Delta(\Delta Q_A^m)$ with $\Delta Q_A = Q_A - \langle \hat{Q}_A \rangle$, denotes that the exceed-FCS depend on the mass gap, degeneracy, and the field content of the theory.

The symmetry-resolved thermal partition function becomes

symmetry-resolved thermal partition function

For a given scale μ_0^2,

$$\mathcal{Z}_n^{(\mathrm{th})}(Q_A) = \mathcal{Z}_n^{(0)}(Q_A)\left[1 + gn\mathcal{F}_n(Q_A)e^{-2\pi\Delta_\psi\beta/L}\right], \tag{21}$$

$$\mathcal{F}_n(Q_A) = \frac{1}{n^{2\Delta_\psi}}\frac{\sin^{2\Delta_\psi}(\pi x)}{\sin^{2\Delta_\psi}\left(\frac{\pi x}{n}\right)}\frac{\mathcal{X}_n(Q_A)}{\mathcal{Z}_n^{(0)}(Q_A)} - 1, \tag{22}$$

and

$$\mathcal{X}_n(Q_A) = \int_{-\pi}^{\pi}\frac{\mathrm{d}\alpha}{2\pi}\mathcal{Z}_n^{(0)}(\alpha)G_{n,\alpha}(z,\bar{z})e^{-i\alpha Q_A}. \tag{23}$$

The leading correction term to the probability distribution of charge in each sector can be obtained by $P^{(\mathrm{th})}(Q_A) = \mathcal{Z}_1^{(\mathrm{th})}(Q_A)$. FCS was previously calculated for the ground state. It was also calculated for the excited state in free compact boson. Here, we derived thermal corrections for the FCS for any two dimensional conformal field theory.

Probability Fluctuation

We define the quantity:

$$g_n(Q_A,T) = \log f_n(Q_A,T) = ng\mathcal{F}_n(Q_A)e^{-2\pi\Delta_\psi\beta/L}. \tag{24}$$

where $f_n(Q_A,T) = \frac{\mathcal{Z}_n^{(\text{th})}(Q_A)}{\mathcal{Z}_n^{(0)}(Q_A)}$.

symmetry-resolved thermal partition function

By choosing $n=1$, we find ,

$$g_1(Q_A,T) = \left(\frac{\mathcal{X}_1(Q_A)}{\mathcal{Z}_1^{(0)}(Q_A)} - 1\right) ge^{-2\pi\Delta_\psi\beta/L}., \tag{25}$$

This quantity represent the probability fluctuations. In other words, it expresses that the ratio of the probability of finding a charge Q_A at inverse temperature β to it's value at zero temperature scales as $e^{-2\pi\Delta_\psi\beta/L}$. Its coefficient depends on the degeneracy of the first excited state, charge of the sector, and field content of the theory.

Entanglement Measures

In general, the thermal correction to the symmetry-resolved Rényi and entanglement entropies take the following forms:

Symmetry-Resolved Entanglement Measures

$$\delta S_n(Q_A) = \frac{ng}{1-n}\left[\frac{1}{n^{2\Delta_\psi}}\frac{\sin^{2\Delta_\psi}(\pi x)}{\sin^{2\Delta_\psi}\left(\frac{\pi x}{n}\right)}\frac{\mathcal{X}_n(Q_A)}{\mathcal{Z}_n^{(0)}(Q_A)} - \frac{\mathcal{X}_1(Q_A)}{\mathcal{Z}_1^{(0)}(Q_A)}\right] e^{-2\pi\Delta_\psi\beta/L}, \tag{26}$$

$$\delta S_E(Q_A) = g\left[2\Delta_\psi\left(1-\pi x\cot(\pi x)\right)\frac{\mathcal{X}_1(Q_A)}{\mathcal{Z}_1^{(0)}(Q_A)} + \partial_n\left(\frac{\mathcal{X}_n(Q_A)}{\mathcal{Z}_n^{(0)}(Q_A)}\right)\Big|_{n=1}\right] e^{-2\pi\Delta_\psi\beta/L} \tag{27}$$

Entanglement Measures

- Symmetry sector corrections scales as $e^{-2\pi\Delta_\psi \beta/L}$.
- Their coefficients, besides the size of the mass gap and degeneracy of the first excited state, depending on the four-point correlation function of primary fields.
- Compared to the total entanglement entropy, the scaling of these sectors is similar to the scaling of the total entanglement entropy, except that the correction scaling coefficients depend only on the mass gap and the degeneracy of the excited state and are independent of the charge of the sector

Example

- As a example, we specialize to compactified massless bosons with $c = 1$.
- In this CFT, there are two holomorphic primary fields, vertex operator $V_\beta = e^{i\tilde{\beta}\phi_j}$ and derivative operator $i\partial\phi$, with scaling dimensions $h_V = K\frac{\tilde{\beta}^2}{2}$ and $h_{i\partial\phi} = 1$, respectively. The Luttinger parameter K is related to the compactification radius via the bosonization relation.
- The lowest scaling primary field depends on range $\tilde{\beta}$ that is determined via Luttinger parameter K.

Example

- If the excited state generated by the vertex operator $V_{\tilde{\beta}} = e^{i\tilde{\beta}\phi_j}$, we find that $G_{n,\alpha}(z,\bar{z}) = e^{-iK\frac{\alpha\tilde{\beta}x}{n}}$ It follows that, the exceed-cumulant generating function is $g_n(\alpha, T, x) = -iK\alpha\tilde{\beta}x/n$), which is universal.

$$\Delta C_{1,m} = g(-K\tilde{\beta}x)^m e^{-K\pi\tilde{\beta}^2\beta/L}. \tag{28}$$

- The fluctuations (and all the other cumulants) are derived by putting $n = 1$. With $n = 1, m = 2$ and $\Delta Q_A = Q_A - \langle\hat{Q}_A\rangle$, implies that the excced in the variance in the conserved charge(number of particles) is $(\Delta Q_A)^2 = g(K\tilde{\beta}x)^2 e^{-K\pi\tilde{\beta}^2\beta/L}$.

Example

- If the excited state is induced by the derivative primary operator $i\partial\phi$, as a generator of low-dimensional primary state, the conformal block is

Derivative Operator $i\partial\phi$

$$G_{n,\alpha}(z,\bar{z}) = 1 - K\left(\frac{\alpha}{\pi}\right)^2 \sin^2\left(\frac{\pi x}{n}\right) \tag{29}$$

The only non-zero exceed- cumulant is second cumulants, which is specify the charge fluctuation

Cumulant

$$\Delta C_{n,2} = \frac{2gK}{n}\frac{\sin^2(\pi x)}{\pi^2} e^{-2\pi\beta/L}, \tag{30}$$

Probability Fluctuations

$$g_1(Q_A) = \frac{K \sin^2(\pi x)}{\pi^2 \sigma_1^2} \left(\left(\frac{Q_A}{\sigma_1} \right)^2 - 1 \right) e^{-2\pi\beta/L} \tag{31}$$

It denotes the probability fluctuations. And

Symmetry-Resolved Entanglement Entropy

$$\begin{aligned} \delta S_E(Q_A) &= \delta S_E + \mathcal{B}(Q_A) e^{-2\pi\beta/L}, \\ \mathcal{B}(Q_A) &= \left(3\sin^2(\pi x) - \pi(1+x)\sin(2\pi x)\right) \frac{1}{\ln(l)} \\ &+ \left(-4\pi \sin^2(\pi x) + \pi(1+x)\sin(2\pi x)\right) \frac{\pi Q_A^2}{K \ln(l)^2}. \end{aligned} \tag{32}$$

We see that the thermal corrections of the charge-sector contributions, at order $\ln(l)^{-2}$, are charge-dependent.

Conclusion

- In this work, we have derived a formula for thermal corrections to the symmetry-resolved Rényi and entanglement entropies for general two-dimensional conformal field theories on a circle.
- Besides the size of the mass gap and the degeneracy of the first excited state, these terms depend only on the four-point function of primary fields. It is worth noting that, until now, these thermal corrections have only been studied in the free case, whereas we found it for any two-dimensional CFT.
- Specially, we have derived the thermal corrections to full counting statistics, and the excess-cumulant generating function.

Conclusion

- We also have obtained the probability fluctuation, which is defined as the ratio of the probability of finding a charge Q_A at inverse temperature β to it's value at zero temperature scales as $e^{-2\pi\Delta_\psi\beta/L}$. Its coefficient depends on the degeneracy of the first excited state, charge of the sector, and field content of the theory.
- We have explicitly evaluated thermal corrections for the entanglement entropy and FCS in the free compact boson theory for both derivative and vertex operators. We have found that in the first case, the entanglement equipartition break at order of $\ln(l)^{-2}$.
- We also obtain thermal corrections to the full counting statistics of the ground state and find that the fluctuations of probabilities scale as $e^{-2\pi\Delta_\psi\beta/L}$, where Δ_ψ is the scaling dimension of the lowest weight states.

Thank You