In his name

Meson Excitation Time as a Probe of Holographic Critical Point

Ali Hajilou

School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM)

hajilou@ipm.ir

Wednesday Weekly Seminar

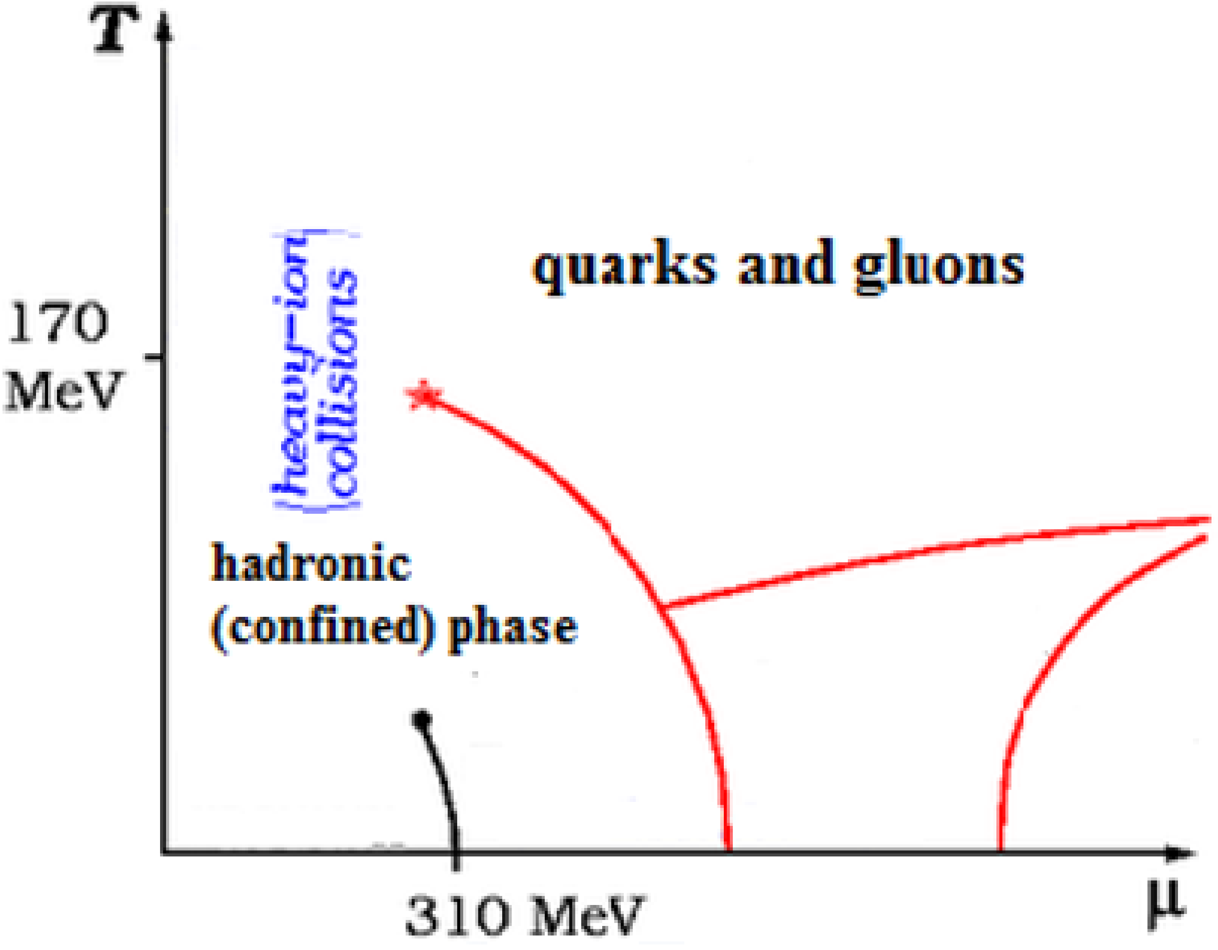
1 Jun 2022

11خرداد 1401

Outline:

- Introduction
- Set up a Question?
- Approach: AdS/CFT or Gauge/Gravity Duality
- Results
- Summary

Introduction: (QCD phase diagram)



Heavy Ion collision: (LHC)

Experimental results show that:
QGP is strongly coupled

$$\frac{\eta}{s} \sim 1/4\pi$$

Nucl. Phys. A 750, 64 (2005)

Set up a Question:

1) Local Observable

2) Non-Local Observable (Wilson loop)

Can we probe the critical point of the QCD phase diagram via Non-Local Observable ?

Approach:

QCD is strongly coupled at low energy.

So, we use Non-Perturbative approach, i.e. AdS/CFT duality

Interesting limit:

Classical gravity ⟺ Strongly coupled QFT

Ex: Ball

Adv. Theor. Math. Phys. 2 (1998)

Methods:

Top-down models: Directly constructed from string theory:

D3-D7 model

J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik , I. Kirsch,
M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters,…

D4-D8 model

T. Sakai and S. Sugimoto

Bottom-up models: (phenomenological)

Introduce a dilaton field

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov,
A. Karch, B. Batell and T. Gherghetta, U. Gursoy, E. Kiritsis,…

Duality of processes:

Gravity		Gauge
Hawking temperature of black hole	⟷	Temperature in strongly coupled QFT
Black hole formation	⟷	Thermalization process
Pure AdS → Black hole	⟷	|Vacuum> → |Thermal state>
Field	⟷	Operator

JHEP 02 (2000) 039 and JHEP 07, 086 (2014)

Our Question:

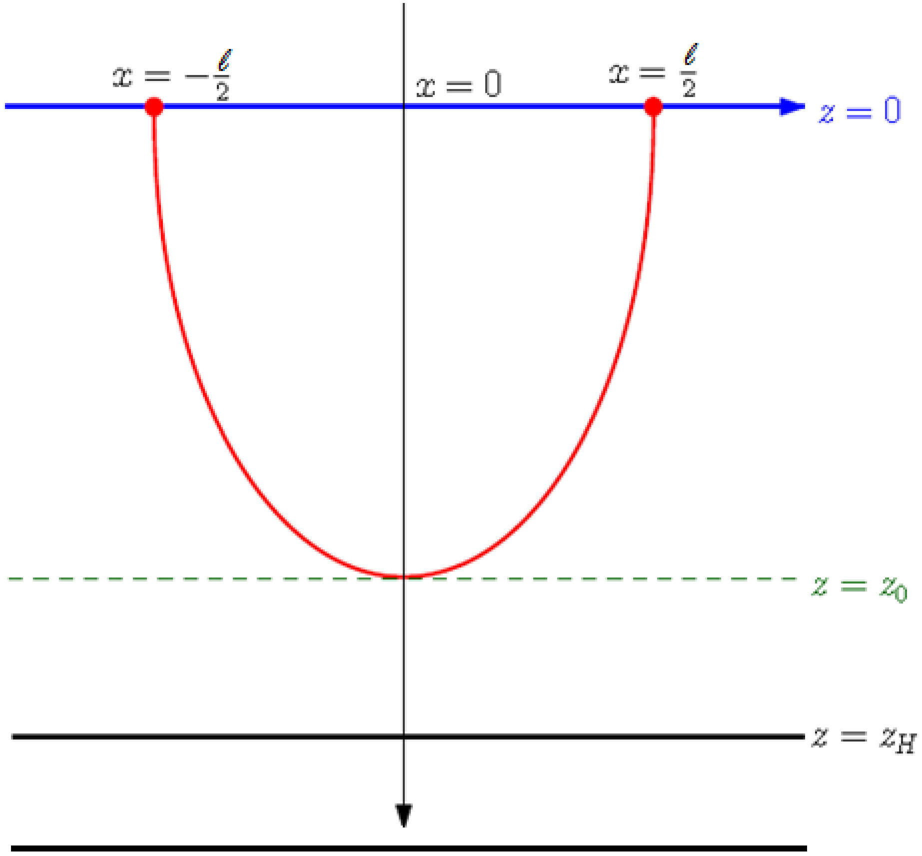
Can we probe the critical point of the QCD phase diagram via Non-Local Observable ?

Can meson excitation time, i.e. t_{ex} probe the critical point?

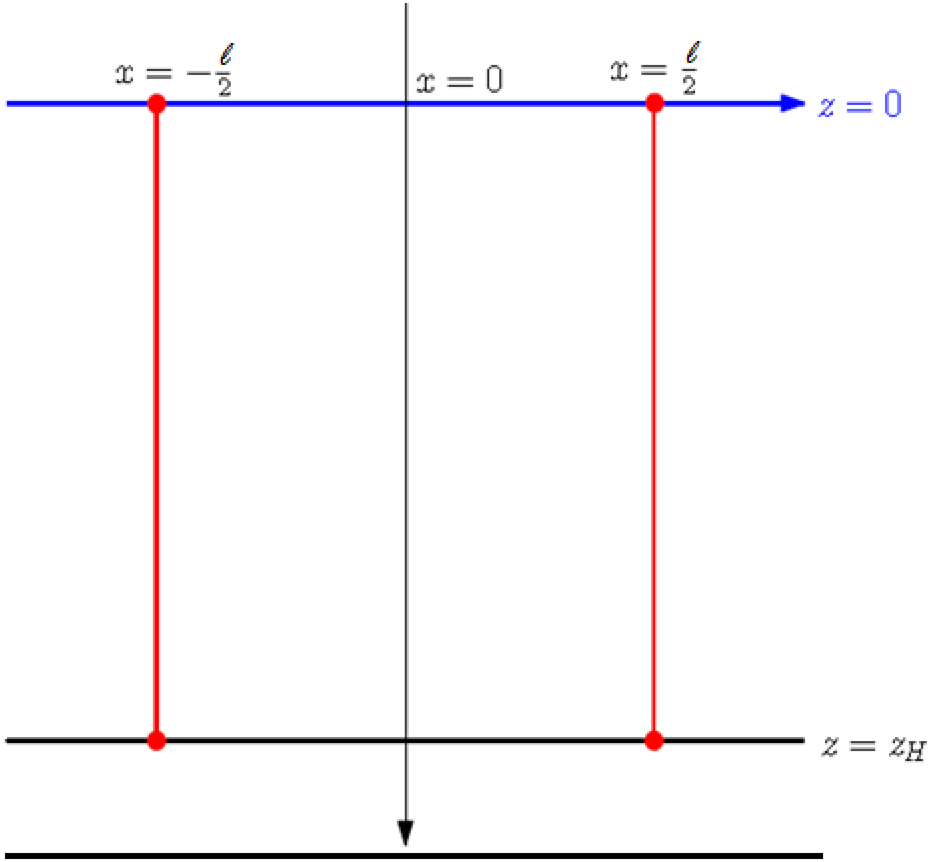
or

Can Wilson loop as a non-local observable understand the critical point?

Holographic picture (for meson):



Connected configuration
Meson bounded

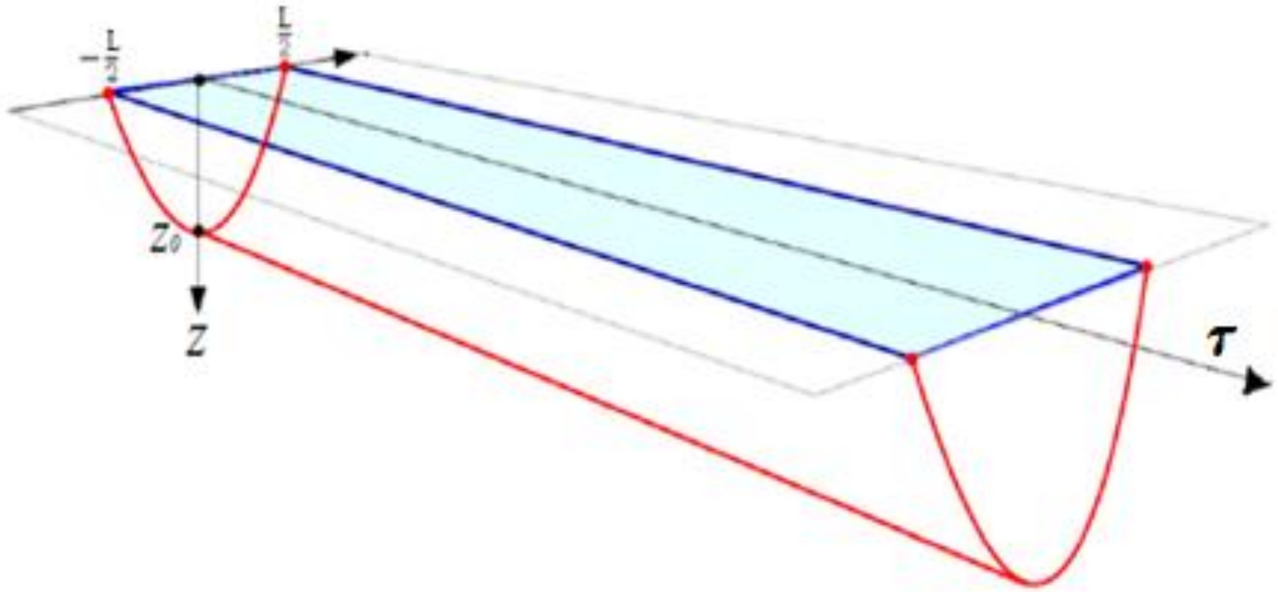


Disconnected configuration
Meson melted

Wilson loop: (static systems)

If : $\tau >> L$ ➡ (QFT) $\Rightarrow \quad <W(C)> = e^{-i[2m+V(L)]\tau}$

(AdS/CFT) $\Rightarrow \quad <W(C)> = e^{-iS_{NG}(C)}$



Nambu-Goto action:

$$S_{NG} = \frac{-1}{2\pi\alpha'} \int_C d\tau d\sigma \sqrt{-\det(g_{ab})}$$

$$g_{ab} = G_{MN} \partial_a X^M \partial_b X^N$$

Phys. Rev. Lett. 80, 4859 (1998) [hep-th/9803002]

Wilson loop: (time-dependent systems)

$$(QFT) \Rightarrow \langle W(C) \rangle = e^{-i\int dt\ \mathrm{w}(\mathrm{l,t})}$$

$$(\mathrm{AdS/CFT}) \Rightarrow \langle W(C) \rangle = e^{-iS_{NG}(C)}$$

Wilson loop evolution: (Regularized function W_R (t))

$$W_R(\mathrm{l,t}) = \mathrm{w}(\mathrm{l,t}) - 2\mathrm{m}$$

$$= \int d\sigma\left(\sqrt{-\det(g_{ab})}\right)_{on-shell} - 2\mathrm{m}$$

Phys. Rev. D 93, no. 8, 086005 (2016)

JHEP 1404, 099, (2014)

Einstein-Maxwell-dilaton action:

$$S = -\frac{1}{16\pi}\int d^5x\sqrt{-g}[\mathcal{R} - \frac{4}{3}(\nabla\Phi)^2 - V(\Phi) - e^{-\frac{4\alpha}{3}\Phi}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}].$$

α : Coupling constant between dilaton and the Maxwell field.

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} - \frac{4}{3}\left(\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}(\nabla\Phi)^2 - \frac{3}{8}g_{\mu\nu}V(\Phi)\right) - 2e^{\frac{-4\alpha\Phi}{3}}\left(\hat{F}_{\mu\rho}\hat{F}^{\rho}_{\nu} - \frac{1}{4}g_{\mu\nu}\hat{F}_{\rho\sigma}\hat{F}^{\rho\sigma}\right) = 0$$

$$\nabla^2\Phi - \frac{3}{8}\frac{\partial V}{\partial\Phi} + \frac{\alpha}{2}e^{\frac{-4\alpha\Phi}{3}}\hat{F}_{\rho\sigma}\hat{F}^{\rho\sigma} = 0$$

$$\nabla_\mu\left(e^{\frac{-4\alpha\Phi}{3}}\hat{F}^{\mu\nu}\right) = 0$$

Nucl. Phys. B 896, 569-586 (2015)

Metric solution:

$$ds^2 = -N(z)f(z)dt^2 + \frac{1}{z^4}\frac{dz^2}{(1+b^2z^2)f(z)} + \frac{1+b^2z^2}{z^2}g(z)d\vec{x}^2$$

$$f(z) = \frac{1+b^2z^2}{z^2}\Gamma^{2\gamma} - M\frac{z^2}{1+b^2z^2}\Gamma^{1-\gamma},$$

$$N(z) = \Gamma^{-\gamma},\ g(z) = \Gamma^{\gamma},\ \Gamma(z) = \frac{1}{1+b^2z^2},\ \gamma = \frac{\alpha^2}{2+\alpha^2}$$

$$Q = \sqrt{\frac{6M}{2+\alpha^2}}b$$

$\alpha = 0$ → Reissner-Nordström-AdS black hole.

Nucl. Phys. B 896, 569-586 (2015)

Temperature and chemical potential:

$$\mu = \frac{b\sqrt{3M}}{(\frac{1}{{z_h}^2} + b^2)\sqrt{2(2+\alpha^2)}}$$

$$T = \frac{b\Gamma(z_h)^{\frac{3\gamma}{2}-1}}{4\pi\sqrt{1-\Gamma(z_h)}}[2(3\gamma-1)-3(2\gamma-2)\Gamma(z_h)]$$

Our choice: $\alpha = 2$ → A background that its holographic dual contains critical point

$$(\frac{\mu}{T})_{\star} = 1.11072$$

Nucl. Phys. B 896, 569-586 (2015) Phys. Lett. B783 (2018)

Einstein-Maxwell-dilaton-Vaidya metric:

$$ds^2 = -N(z)F(\bar{v},z)d\bar{v}^2 - \frac{2}{z^2}\sqrt{\frac{N(z)}{1+b^2z^2}}d\bar{v}dz + \frac{1+b^2z^2}{z^2}g(z)d\vec{x}^2.$$

$$F(\bar{v},z) = \frac{1+b^2z^2}{z^2}\Gamma^{2\gamma} - M(\bar{v})\frac{z^2}{1+b^2z^2}\Gamma^{1-\gamma},$$

$$\zeta(\bar{v}) = \zeta_f \begin{cases} 0 & \bar{v} < 0, \\ k^{-1}\left[\bar{v} - \frac{k}{2\pi}\sin(\frac{2\pi\bar{v}}{k})\right] & 0 \leqslant \bar{v} \leqslant k. \\ 1 & \bar{v} > k, \end{cases}$$

$$\zeta \in (M, Q)$$

k <<1 (fast quench)

k >>1 (slow quench)

Nucl. Phys. B 896, 569-586 (2015)

Equations of motion:

$$X_{,uv} = (Z_{,u}X_{,v} + Z_{,v}X_{,u})\,\frac{3+2b^2Z^2}{3Z(1+b^2Z^2)}\ ,$$

$$V_{,uv} = \left(\frac{2b^2Z^3F}{3}(1+b^2Z^2)^{\frac{-1}{6}} + \frac{Z^2F_{,Z}}{2}(1+b^2Z^2)^{\frac{5}{6}}\right)V_{,u}V_{,v}$$
$$+\left(\frac{1}{Z}(1+b^2Z^2)^{\frac{1}{2}} - \frac{b^2Z}{3}(1+b^2Z^2)^{\frac{-1}{2}}\right)X_{,u}X_{,v}\ ,$$

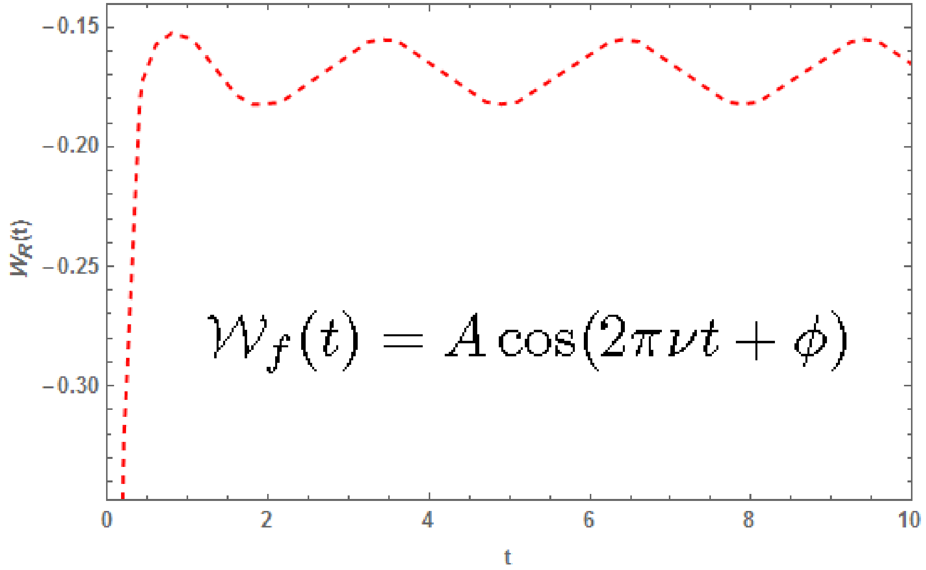
$$Z_{,uv} = \left(-\frac{2b^2F^2Z^5}{3}(1+b^2Z^2)^{\frac{2}{3}} - \frac{FZ^4F_{,Z}}{2}(1+b^2Z^2)^{\frac{5}{3}} - \frac{Z^2F_{,V}}{2}(1+b^2Z^2)^{\frac{5}{6}}\right)V_{,u}V_{,v}$$
$$+\left(-\frac{2b^2Z^3F}{3}(1+b^2Z^2)^{\frac{-1}{6}} - \frac{Z^2F_{,Z}}{2}(1+b^2Z^2)^{\frac{5}{6}}\right)(Z_{,u}V_{,v}+Z_{,v}V_{,u})$$
$$+\left(-ZF(1+b^2Z^2)^{\frac{4}{3}} + \frac{b^2Z^3F}{3}(1+b^2Z^2)^{\frac{1}{3}}\right)X_{,u}X_{,v} + \left(\frac{2}{Z} + \frac{b^2Z}{3}(1+b^2Z^2)^{-1}\right)Z_{,u}Z_{,v}$$

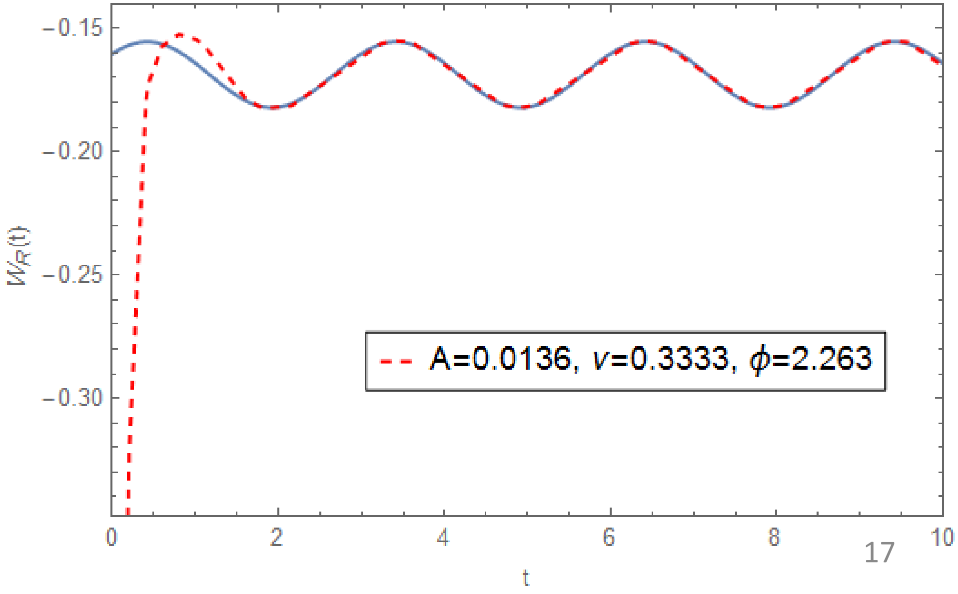
Meson excitation time, t_{ex}: (RN-AdS-Vaidya background)

$$\epsilon(t) = \left| \frac{\mathcal{W}_R(t) - \mathcal{W}_f(t)}{\mathcal{W}_R(t)} \right|$$

$$\epsilon(t_{ex}) < 5 \times 10^{-6}$$

Excitation time





Eur. Phys. J. C **79 (2019) 3, 254**

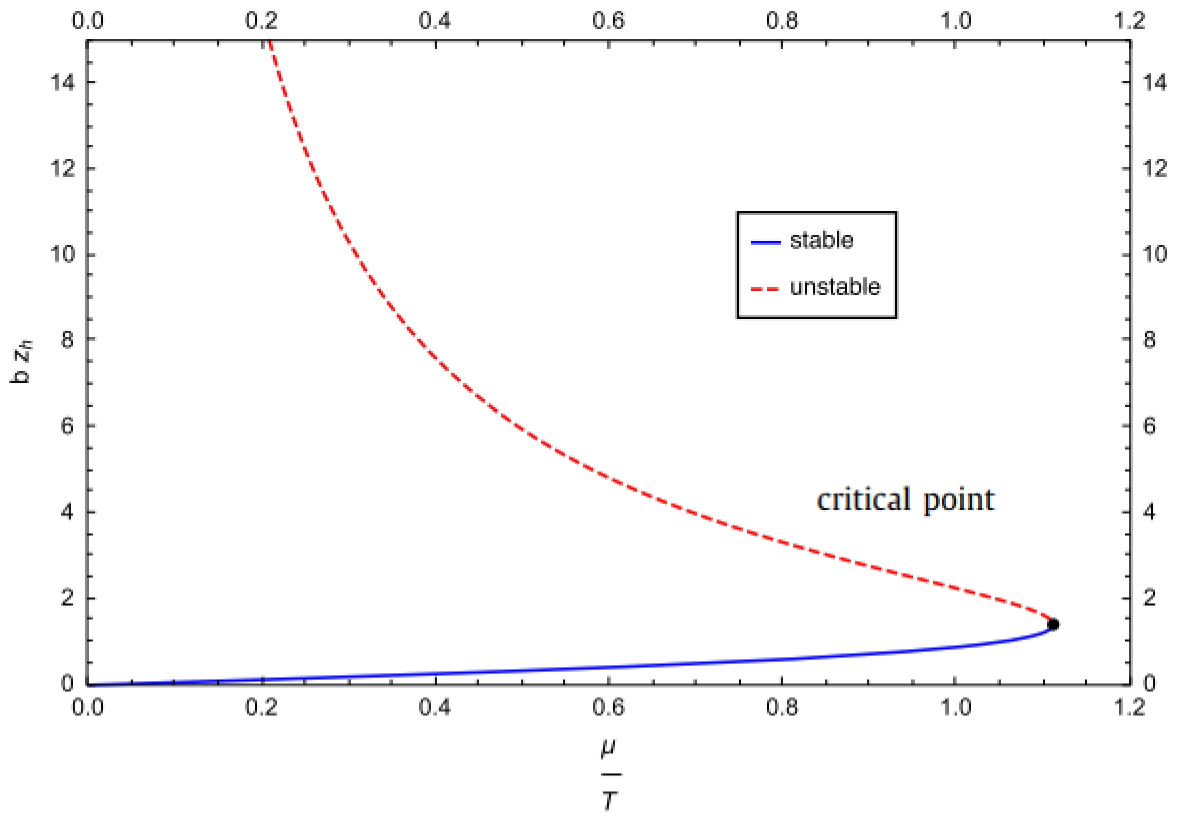
Choosing thermodynamically stable solutions:

$$bz_h = \frac{1 \pm \sqrt{1 - \frac{8\mu^2}{\pi^2 T^2}}}{\frac{2\mu}{\pi T}}$$

$$\mathcal{J} = \frac{\partial(s,\rho)}{\partial(T,\mu)}$$: Jacobian is positive

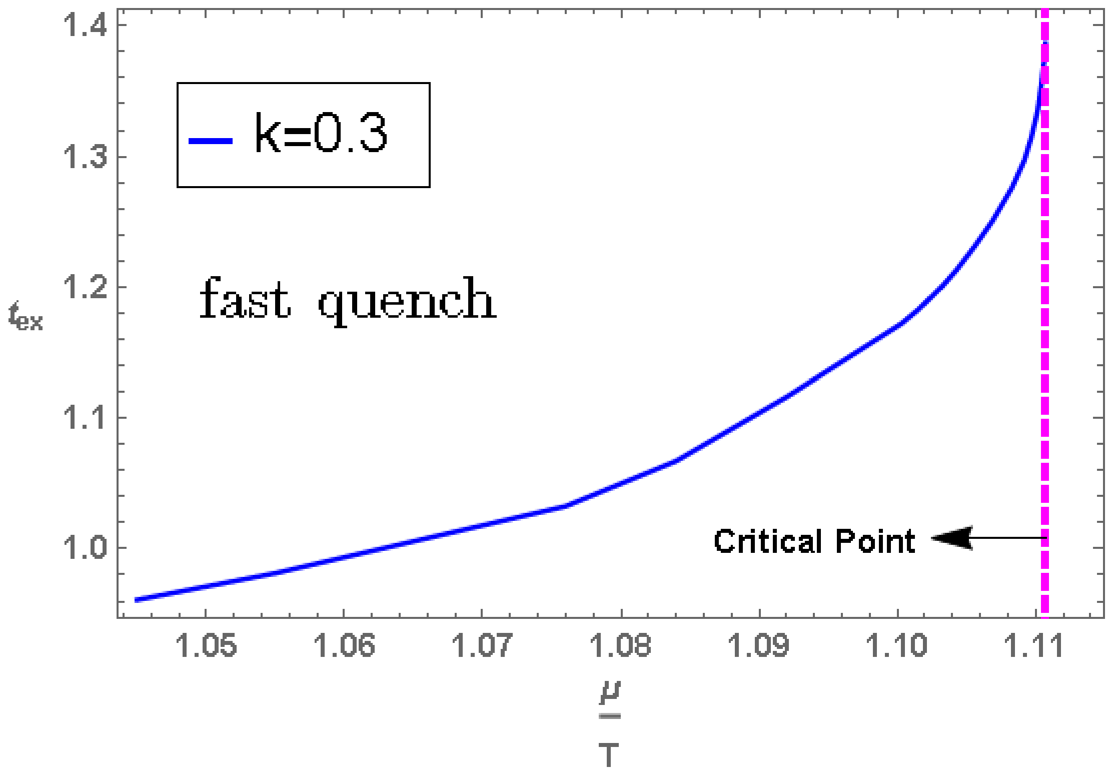
$$s \propto \frac{T^3(1+b^2 z_h^2)^2}{(2+b^2 z_h^2)^3}$$

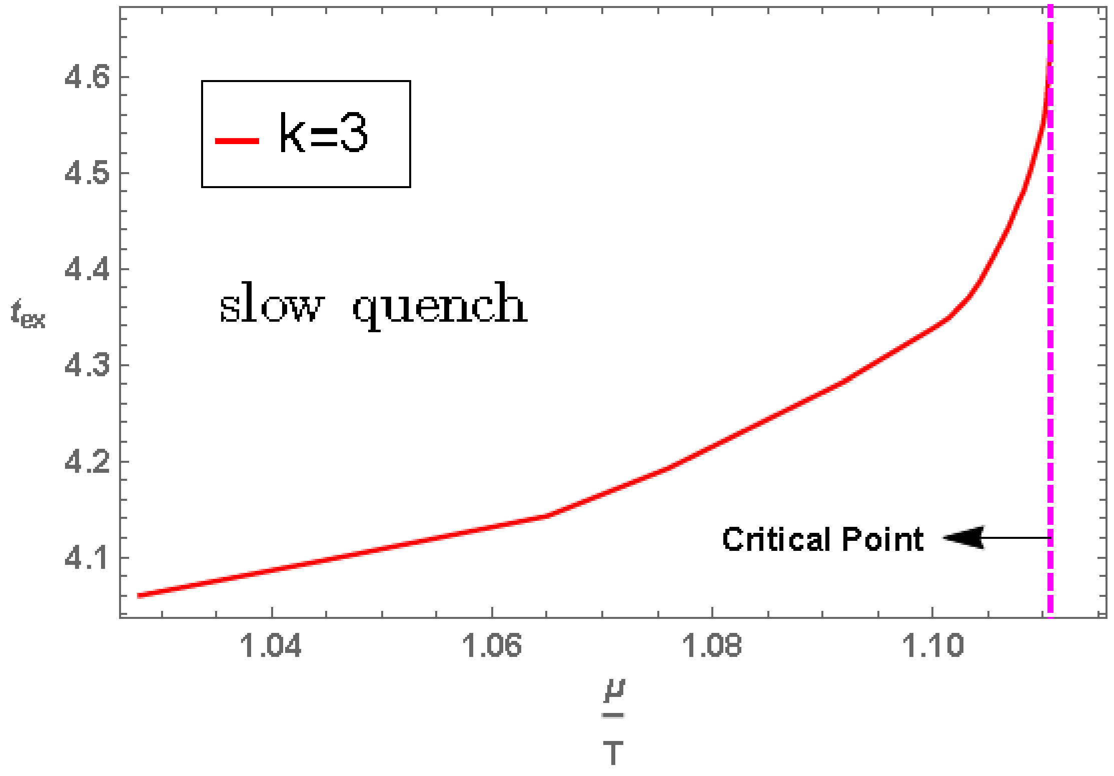
$$\rho \propto \frac{\mu}{T}(2+b^2 z_h^2)\sqrt{1+b^2 z_h^2}$$



Phys. Lett. B 783 (2018) , JHEP 1701 (2017) 137

Towards the critical point:





The slope: $\frac{dt_{ex}}{d\frac{\mu}{T}} = (\frac{\pi}{2\sqrt{2}} - \frac{\mu}{T})^{-\theta}$ → $\frac{dt_{ex}}{d\frac{\mu}{T}}(i) = \frac{t_{ex}(i+1) - t_{ex}(i)}{\frac{\mu}{T}(i+1) - \frac{\mu}{T}(i)}$

Our Question:

Can meson excitation time, i.e. t_{ex} probe the critical point?

or

Can Wilson loop as a non-local observable understand the critical point?

$$\frac{dt_{ex}}{d\frac{\mu}{T}} = \left(\frac{\pi}{2\sqrt{2}} - \frac{\mu}{T}\right)^{-\theta} \longrightarrow \theta = ?$$

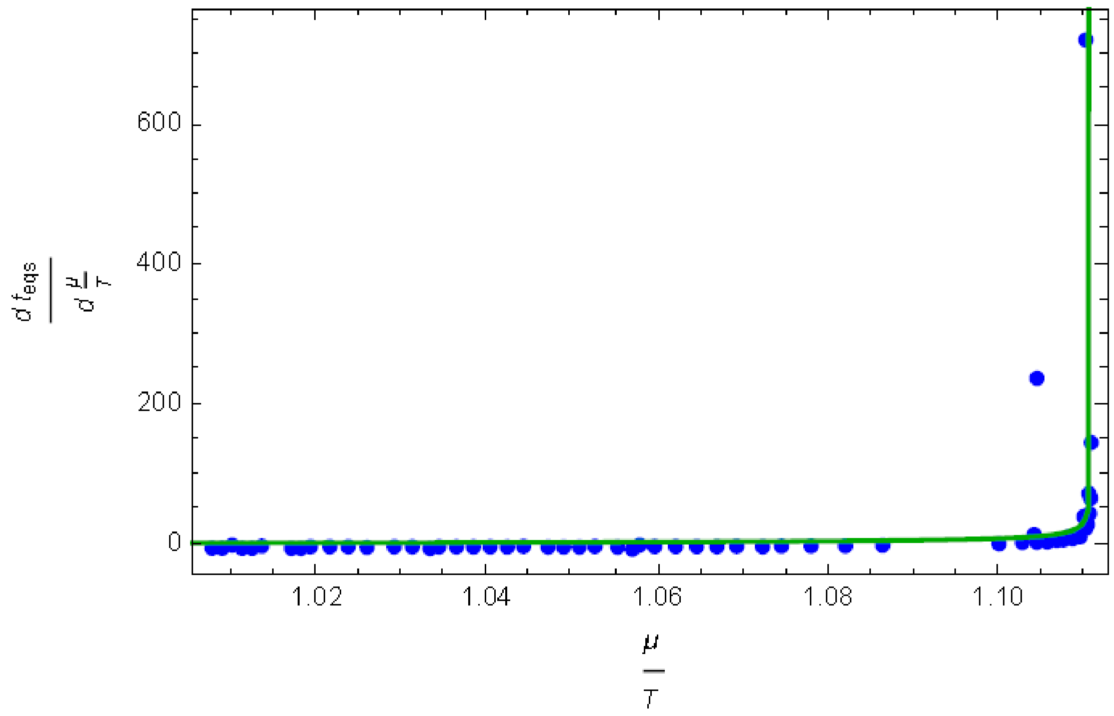
What is the effect of ℓ and k **on** θ ?

Consider the paper: Phys .Lett. B 783 (2018)

Does the equilibration time know the critical point in QGP using dynamical scalar operator as a probe?

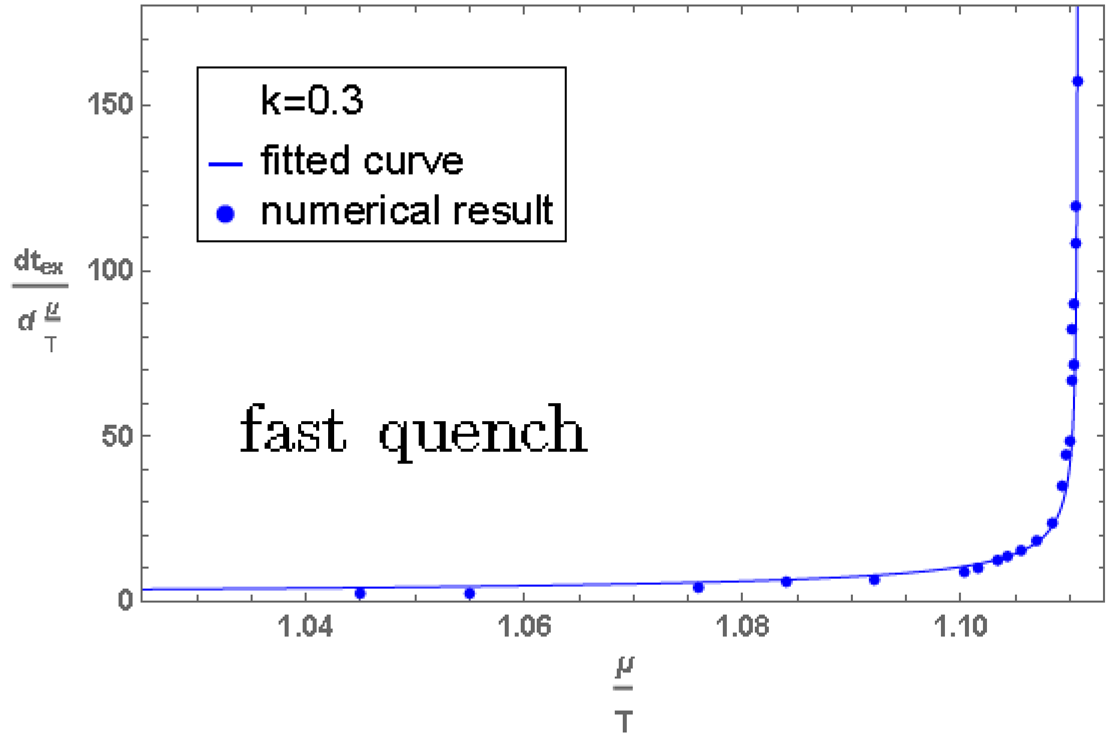
fast quench → $\theta = 0.489682$

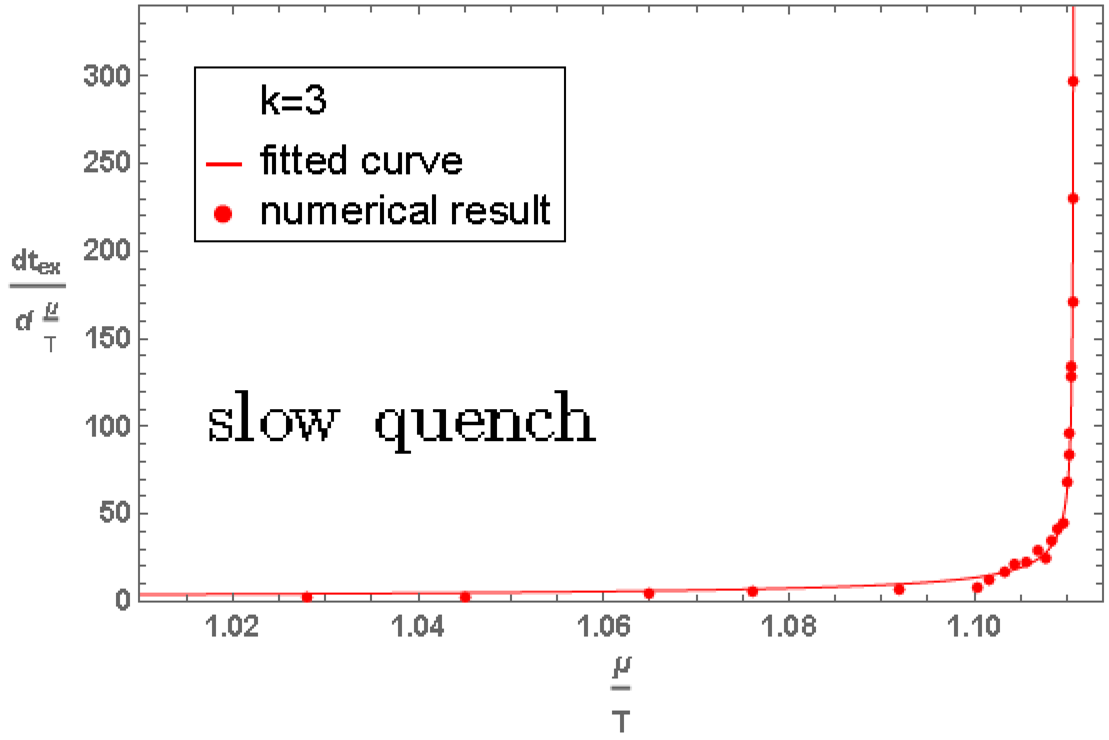
slow quench → $\theta = 0.33901$



Numerical results: (different quenches)

$$lT = 0.10$$





$$\theta = 0.515667$$

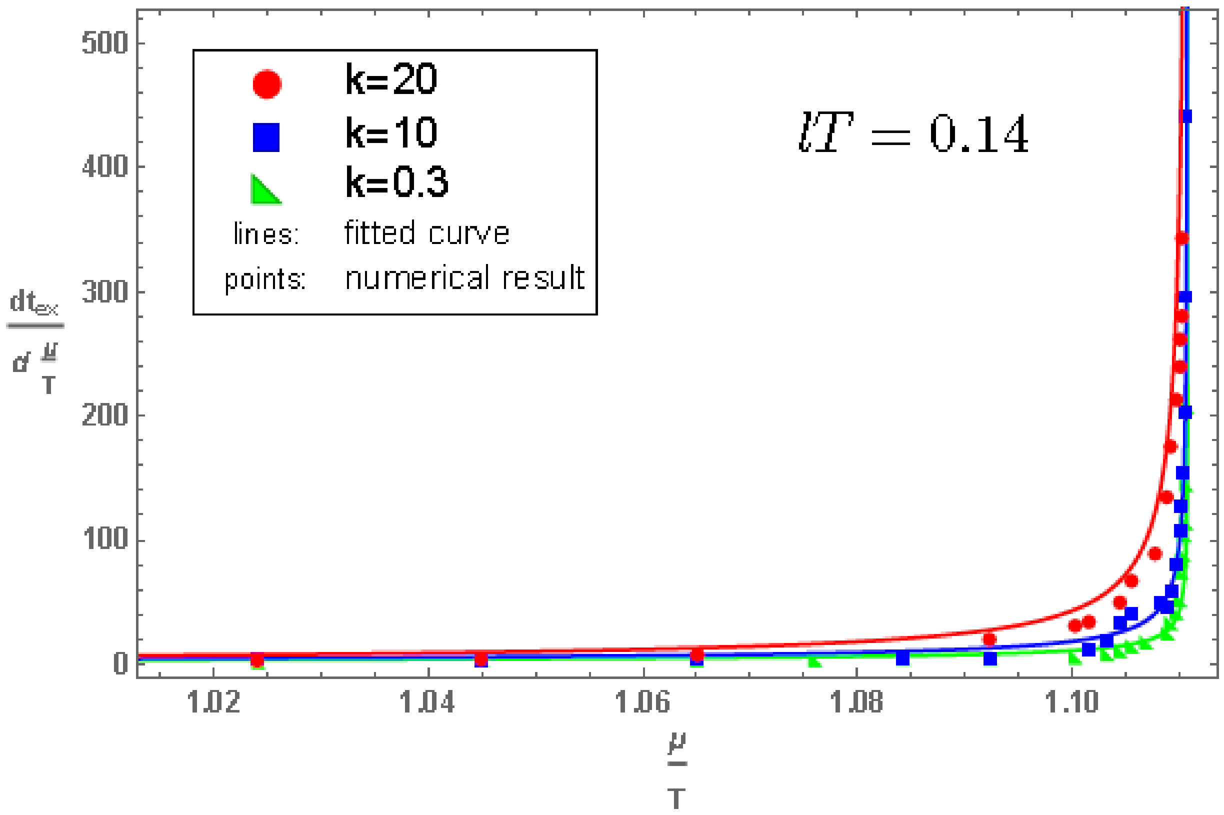
$$\theta = 0.572473$$

Different quenches: (more concentration)

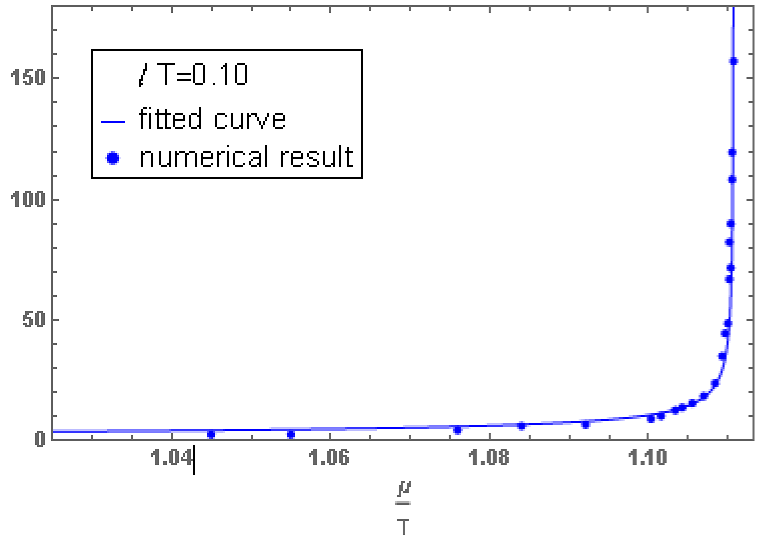
k=20 $\theta = 0.830707$

k=10 $\theta = 0.641450$

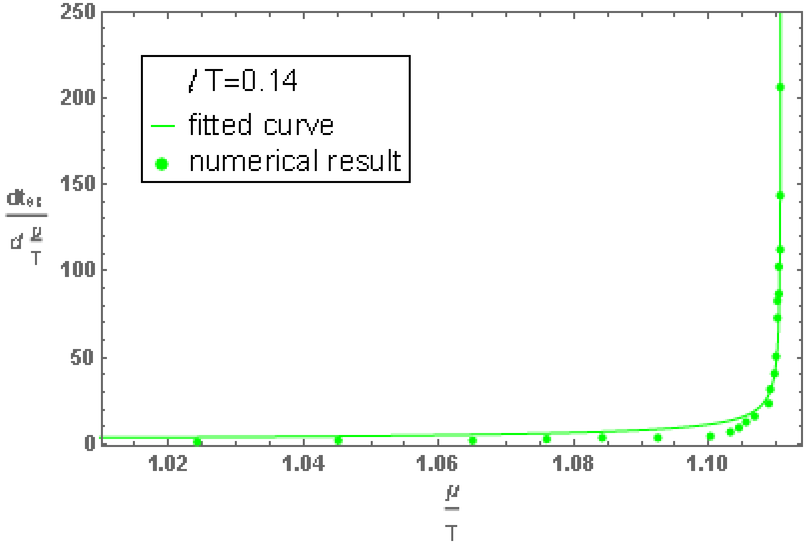
k=0.3 $\theta = 0.529178$



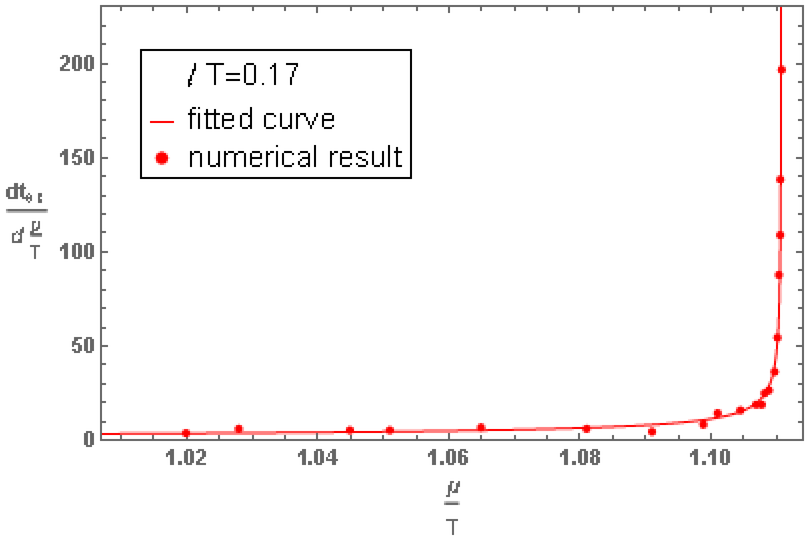
Different interquark distances: $k = 0.3$



$\theta = 0.515667$



$\theta = 0.529178$



$\theta = 0.535088$

Consider the paper: *JHEP* 01 (2017) 137,
Studying the behavior of scalar
quasi-normal modes near the critical point

$\theta = 0.5$

Summary:

- The value of θ depends on the interquark distance and the injection of energy.

- We observed that increasing both values of k and ℓ the value of θ increases.

- For fast quenches, $\theta \sim 1/2$ that is in good agreement with the result that is obtained from the behavior of scalar quasi-normal modes and local observable near the critical point

- The Wilson loop as a non-local observable knows about the phase structure of the gauge theory.

Thanks…