Neural network QCD analysis of charged hadron fragmentation functions in the presence of SIDIS data

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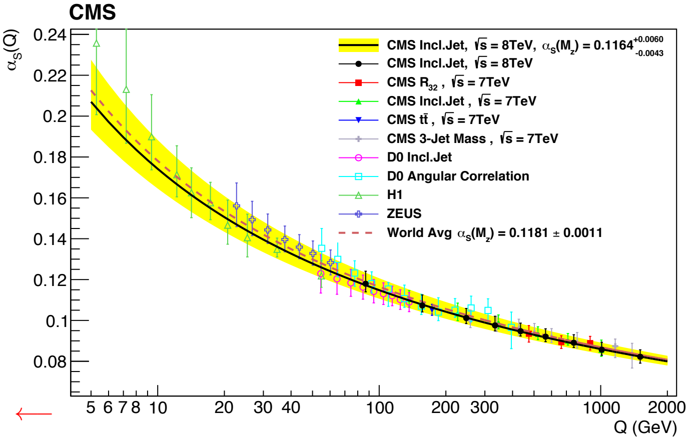
Asymptotic freedom

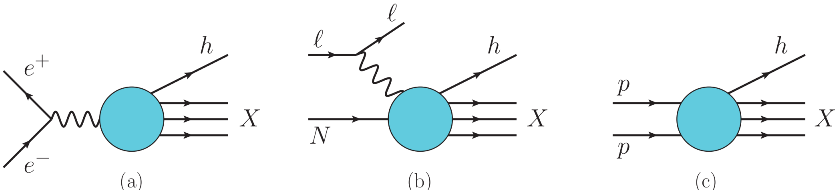


Confinement ⟵

Most strong-interaction processes can not be calculated directly with perturbative QCD, due to color confinement.

Factorization

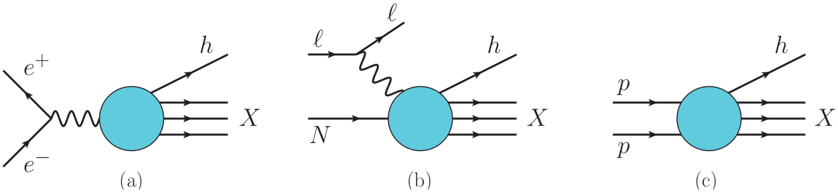
Hard and soft QCD processes contribute? they may be separable.



$$
\begin{aligned}
(a)\ \sigma^{e^+e^-\to hX} &= \hat{\sigma} \otimes FF, \\
(b)\ \sigma^{e^+e^-\to hX} &= \hat{\sigma} \otimes PDF \otimes FF, \\
(c)\ \sigma^{e^+e^-\to hX} &= \hat{\sigma} \otimes PDF \otimes PDF \otimes FF.
\end{aligned}
$$

Factorization

Hard and soft QCD processes contribute? they may be separable.



$$
\begin{aligned}
\text{(a) } \sigma^{\mathrm{SIA}} &= \hat{\sigma} \otimes FF, \\
\text{(b) } \sigma^{\mathrm{SIDIS}} &= \hat{\sigma} \otimes PDF \otimes FF, \\
\text{(c) } \sigma^{e^+e^- \to hX} &= \hat{\sigma} \otimes PDF \otimes PDF \otimes FF.
\end{aligned}
$$

Fragmentation functions in SIA

The differential cross-section for the single-inclusive e^+e^- annihilation giving a hadron,

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma^h}{dz} = \frac{1}{\sigma_{\text{tot}}}\left[F_T^h(z,Q^2) + F_L^h(z,Q^2)\right], \tag{1}$$

$$\begin{aligned} F_k^h(z,Q^2) &= \sigma_{\text{tot}}^{(0)}\left[D_S^h(z,Q^2)\otimes C_{k,q}^S(z,\alpha_s(Q)) + D_g^h(z,Q^2)\otimes C_{k,g}^S(z,\alpha_s(Q))\right] \\ &+ \sum_q \sigma_q^{(0)} D_{\text{NS,q}}^h(z,Q^2)\otimes C_{k,q}^{\text{NS}}(z,\alpha_s(Q)). \end{aligned} \tag{2}$$

In this equation, symbol $\otimes$ also denotes the standard convolution integral defined as

$$D(z)\otimes C(z) \equiv \int_z^1 dy D(y)C(z/y). \tag{3}$$

Fragmentation function in SIDIS

The basic cross-section for the charged hadron production in deep inelastic scattering of a lepton from a nucleon can be written as,

$$\frac{d\sigma^h}{dx\,dy\,dz_h} = \frac{2\,\pi\alpha^2}{Q^2}\left[\frac{(1+(1-y)^2)}{y}2\,F_1^h(x,z_h,Q^2) + \frac{2(1-y)}{y}F_L^h(x,z_h,Q^2)\right]. \quad (4)$$

The structure functions F_1^h and F_L^h in Eq. 4 are the relevant inclusive DIS structure functions in which at NLO accuracy are given by,

$$F_1^h(x,z_h,Q^2) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2 \Big\{ q(x,Q^2)D_q^h(z_h,Q^2) + \frac{\alpha_s(Q^2)}{2\pi}\Big[q\otimes C_{qq}^1\otimes D_q^h + q\otimes C_{gq}^1\otimes D_g^h + g\otimes C_{qg}^1\otimes D_q^h\Big](x,z_h,Q^2)\Big\},$$

and a similar relation for F_L.

The convolution symbol $\otimes$ in equations above is defined as,

$$q(x) \otimes C(x, z_h) \otimes D^h(z_h) \equiv \int_x^1 \frac{dx'}{x'} \int_{z_h}^1 \frac{dz'_h}{z'_h} q(\frac{x}{x'}) c(x', z'_h) D(\frac{z_h}{z'_h}) \,.$$

Evolution

The fragmentation functions are not perturbatively calculable, but their scale dependence (scaling violation) is given by the DGLAP equation:

$$\mu \frac{\partial}{\partial \mu} D_i^h(x, \mu) = \sum_j P_{ij}(z, \alpha_s(\mu)) \otimes D_j^h(x/z, \mu) \tag{5}$$

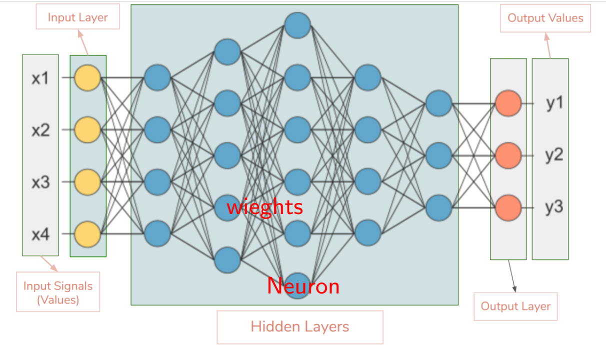
What is Machine learning?

Artificial intelligence (AI) is intelligence demonstrated by machines, as opposed to the natural intelligence displayed by animals including humans.

Machine learning (ML) is a field of inquiry devoted to understanding and building methods that 'learn', It is often seen as a part of artificial intelligence.

An artificial neural network(NN) is a simulation of a biological brain. In other words, they're a way that a machine can process data, that's inspired by human and animal brains.

Artificial Neural Network



$$x_i^{(l)} = \sum_{j=1}^{N_l-1} \omega_{ij}^{(l)} y_j^{(l-1)} + \theta_i^{(l)} \quad (6)$$

the output of each layer is then fed into an activation function to simulate non-linear behavior.

Why NNs?

There are a number of activation functions to use we have used `ReLU`.

Why?

- NNs are flexible, one can fit any function using just simple feed forward NNs.
- They are redundant (by construction). They have many adjustable parameters.
- The above features together result in NN being unbiased.
- NNs have the ability to learn and model non-linear and complex relationships.

Structure of our NNs

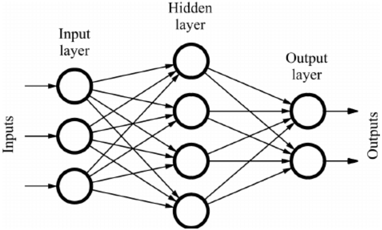


Figure: The feed forward model is the simplest form of neural network as information is only processed in one direction.

- We chose {1-20-7} as our NN architecture.
- Number of adjustable parameters in this configuration is 1*20+20+20*7+7=187, compared to 30-50 in conventional analyses.
- The FF at $Q_0 = 5$ GeV is given as,

$$zD_i^{h^+}(z, Q_0) = \left(N_i(z,\theta) - N_i(1,\theta)\right)^2 . \tag{7}$$

Data
a glimpse

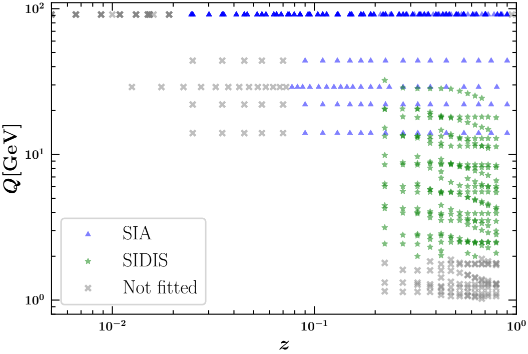


Figure: Kinematic coverage of the data sets analyzed in SHK22.h study.

Data points shown by a cross were excluded from the fit using well established kinematic cuts.

Data set

Dataset facts

- a total of 18 data sets including tagged and untagged SIA and SIDIS for $h^{\pm}$ and h^{+} and h^{-} separately.
- These data sets include the TASSO experiment at DESY; the TPC and SLD experiment at SLAC, ALEPH, DELPHI, and OPAL experiments at CERN.
- number of data points in the fit,
  SIA: 370 data, SIDIS: 314 data, TOTAL: 684
- These data include all of SIDIS and all SIA data sets except for the `TASSO 35` data set, that was left out because of tension with `COMPASS` data sets.
- We checked an analysis including the `TASSO 35` and the results were practically unchanged, except for larger error bands and worse fit quality(larger $\chi^2/N_{\rm dat}$).
- Other TASSO data sets also show a little bit of tension with `COMPASS`, but they were kept in the analysis.

Fitting procedure

Flavor decomposition

- It has been shown that adding the SIDIS datasets to the data sample could provide a direct constraint on the individual q and $\bar{q}$ FFs for light quarks.

$$D_u^{h^+},\ D_{\bar{u}}^{h^+},\ D_{d+s}^{h^+},\ D_{\bar{d}+\bar{s}}^{h^+},\ D_{c^+}^{h^+},\ D_{b^+}^{h^+},\ D_g^{h^+}\ . \quad (8)$$

χ^2 function

We seek to minimize a χ^2 function of form:

$$\chi^{2(k)} \equiv \left(\mathbf{T}(\boldsymbol{\theta}^{(k)}) - \boldsymbol{x}^{(k)}\right)^{\mathrm{T}} \cdot \mathbf{C}^{-1} \cdot \left(\mathbf{T}(\boldsymbol{\theta}^{(k)}) - \boldsymbol{x}^{(k)}\right)\ . \quad (9)$$

Here k is the replica number, we get back to this in unc. propagation.

Minimization

Minimization techniques

- Explicit differentiation
- Genetic algorithm
- Stochastic Gradient Descent methods (SGD)
- Trust-region methods
  implemented in MontBlanc

Uncertainty propagation

Monte Carlo method

- The Monte Carlo approach assumes: the experimental data follow a multivariate Gaussian distribution,

$$\mathcal{G}(\boldsymbol{x}^k) \propto \exp\left(\left(\boldsymbol{x}^k - \boldsymbol{\mu}\right)^T \cdot \boldsymbol{C}^{-1}\left(\boldsymbol{x}^k - \boldsymbol{\mu}\right)\right). \quad (10)$$

where $\boldsymbol{x}^k = \left\{x_1^k, x_2^k, \cdots, x_{N_{\mathrm{dat}}}^k\right\}$ are so called "replicas".

The elements of the covariance matrix are defined as in Ref. [**?**],

$$C_{ij} = \delta_{ij}\sigma_{i,\mathrm{unc}}^2 + \sum_\beta \sigma_{i,\mathrm{corr}}^{(\beta)}\sigma_{j,\mathrm{corr}}^{(\beta)}, \quad (11)$$

Monte Carlo method

Statistical framework

1. The covariance matrix $\boldsymbol{C}$ is decomposed to a lower triangular $\boldsymbol{L}$ using Cholesky method ($\boldsymbol{C} = \boldsymbol{L} \cdot \boldsymbol{L}^T$).
2. Apply this matrix to an N_{dat}-dimensional Gaussian random vector, $\boldsymbol{r}^k$, a vector with the covariance properties of the experimental data is obtained.
3. Accordingly the pseudo dataset is constructed as follows,

$$\boldsymbol{x}^k = \boldsymbol{\mu} + \boldsymbol{L} \cdot \boldsymbol{r}^k \tag{12}$$

Monte Carlo method

Statistical framework

5. for sufficiently large N_{rep}, the set of replicas satisfies the following conditions,

$$\frac{1}{N_{\text{rep}}}\sum_{k}^{N_{\text{rep}}} x_i^k \simeq \mu_i\,, \quad \frac{1}{N_{\text{rep}}}\sum_{k}^{N_{\text{rep}}} x_i^k x_j^k \simeq \mu_i\mu_j + C_{ij}\,. \tag{13}$$

6. Perform the QCD analysis, N_{rep} times, trained NNs are gained,

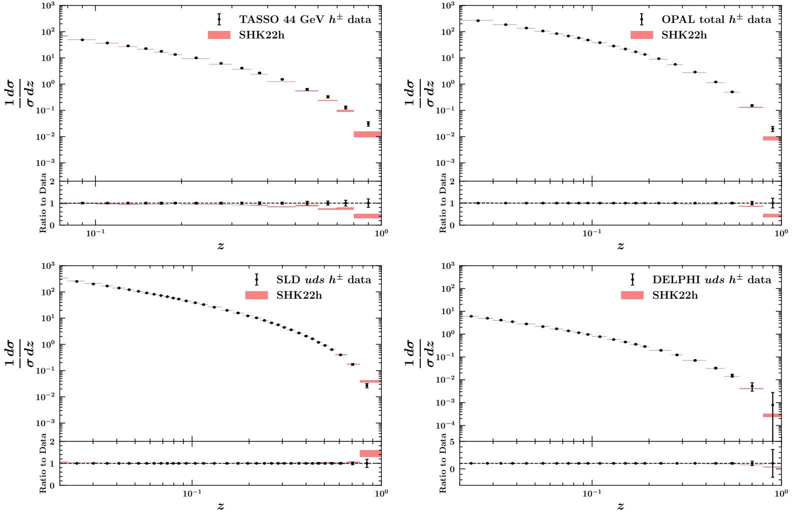
$$\begin{aligned}\langle\mathcal{D}(x)\rangle &= \frac{1}{N_{\text{rep}}}\sum_{k=1}^{N_{\text{rep}}} \mathcal{D}^k(x)\,,\\ \sigma_{\mathcal{D}} &= \sqrt{\langle\mathcal{D}^2\rangle - \langle\mathcal{D}\rangle^2}\,,\\ c_{\mathcal{D}}^{ij} &= \frac{\langle\mathcal{D}_i\mathcal{D}_j\rangle - \langle\mathcal{D}_i\rangle\langle\mathcal{D}_j\rangle}{\sigma_i\sigma_j}\,.\end{aligned} \tag{14}$$

Fit quality

Experiment	Observable	$\chi^2/N_{\rm dat}$	$N_{\rm dat}$
TASSO 14 GeV $h^\pm$	$\frac{1}{\sigma}\frac{d\sigma}{dz}$	1.791	14
TASSO 22 GeV $h^\pm$		1.254	14
TASSO 44 GeV $h^\pm$		2.912	14
TPC $h^\pm$		0.659	21
ALEPH $h^\pm$		0.825	32
DELPHI total $h^\pm$		0.610	21
DELPHI *uds* $h^\pm$		0.380	21
DELPHI bottom $h^\pm$		1.028	21
OPAL total $h^\pm$		1.821	19
OPAL *uds* $h^\pm$		0.794	19
OPAL charm $h^\pm$		0.599	19
OPAL bottom $h^\pm$		0.299	19
SLD total $h^\pm$		1.047	34
SLD *uds* $h^\pm$		0.946	34
SLD charm $h^\pm$		1.034	34
SLD bottom $h^\pm$		1.102	34
COMPASS h^-	$\frac{dM^h}{dz}$	0.907	157
COMPASS h^+		1.338	157
Global dataset		**1.079**	**684**

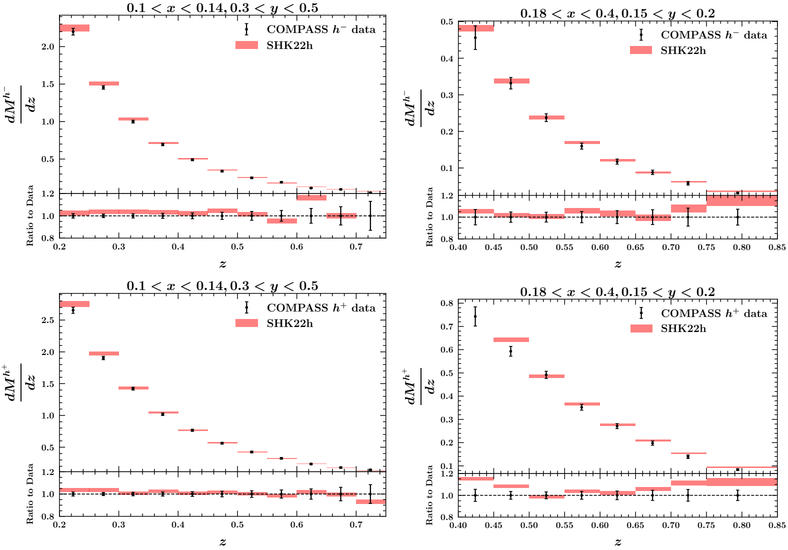
The χ^2 per data point after the cut.

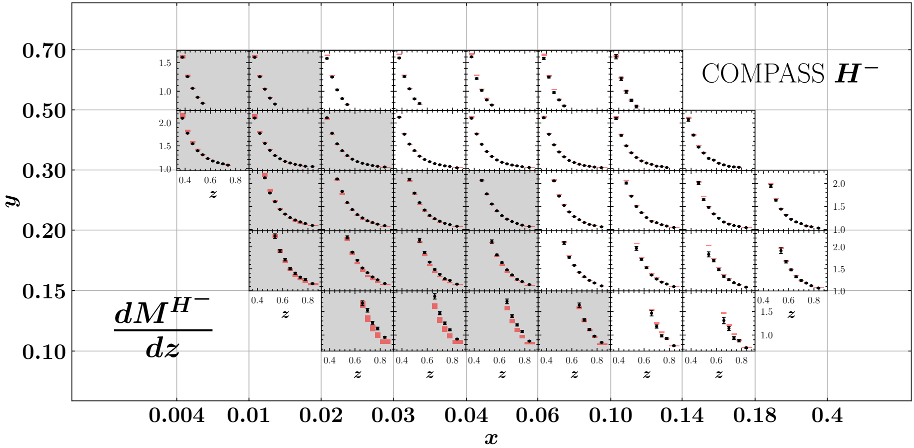
Fit quality - SIA experiments



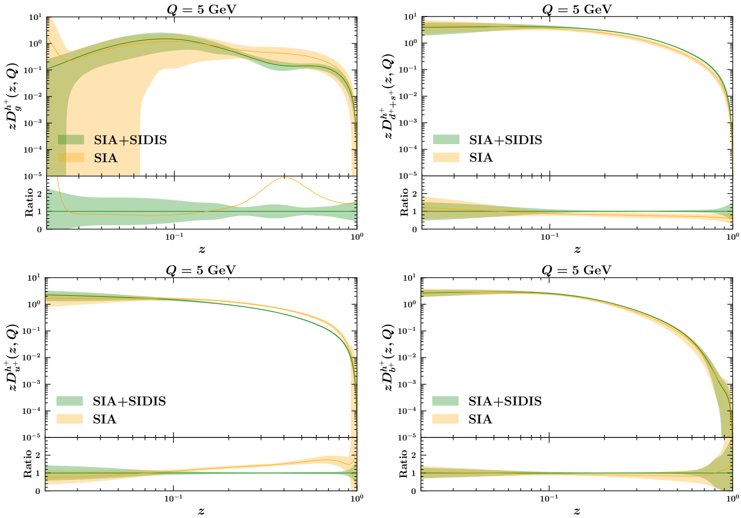
From top left very high to right bottom very good χ^2.

Fit quality - COMPASS - SIDIS

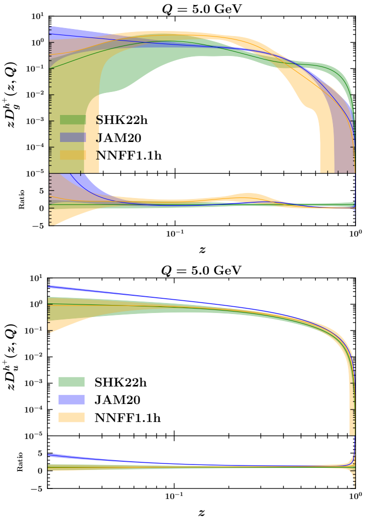



COMPASS H^-
$\frac{dM^{H^-}}{dz}$
y
0.70
0.50
0.30
0.20
0.15
0.10
x
0.004
0.01
0.02
0.03
0.04
0.06
0.10
0.14
0.18
0.4
z
0.4
0.6
0.8
1.0
1.5
2.0

Effect of SIDIS data

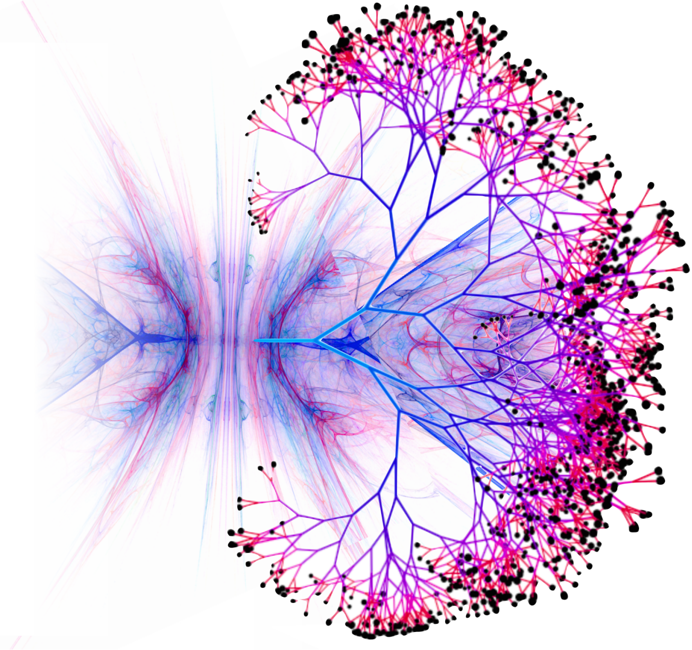


Comparison with other groups



Summary

- In summary, we have presented a new global QCD analysis of light-charged hadron FFs, `SHK22.h`, with several new features and some methodological improvements.
- We have used the machine learning framework to extract the `SHK22.h` FFs sets, along with the Monte Carlo uncertainty analysis.
- In addition to the comprehensive set of high-energy lepton-lepton annihilation (SIA), we have added the lepton-hadron scattering (SIDIS) data sets.
- We have shown that SIDIS data sets have significant effect on the FFs, and more specifically on the gluon FFs and the reduction of its uncertainty.
- The detailed comparisons to the existing FFs sets (NNFF1.1h and JAM20) demonstrate a reasonable agreement.
- Overall, the resulting NLO theory predictions for the SIA and SIDIS cross sections show very good agreement with the corresponding analyzed experimental datasets, as confirmed by the reported total χ^2 per data point.



Thank You!