Islands in kerr- de Sitter spacetime and flat-space cosmologies

Based on:

Islands in flat-space cosmologies, 2109.04795

Islands in Kerr-de Sitter spacetime and their flat limit, 2204.08488

Information Paradox

- Black holes emit hawking radiation with Hawking temperature.
- Black hole evaporation causes information loss.
- Consider the initial state of the black hole was a pure state, at the end of the evaporation there is just thermal radiation.
- Question: how did we end up in a mixed state starting from a pure state, this cannot happen through unitary evolution.
- Thermal aspects appear because we divide the vacuum state (which is entangled) into two parts inside and outside of the horizon.

- In the initial stage of evaporation the entropy of radiation is thermal and it is okay.
- As the black hole evaporates the area of event horizon shrinks and there is a limit where the entropy of radiation would exceed the BH entropy, that is problematic!
- The entropy of the radiation is the entanglement entropy of the radiation with the black hole and so fine-grained entropy, BH is the coarse-grained entropy

- Black hole + radiation = pure state

So, for f.g. entropy

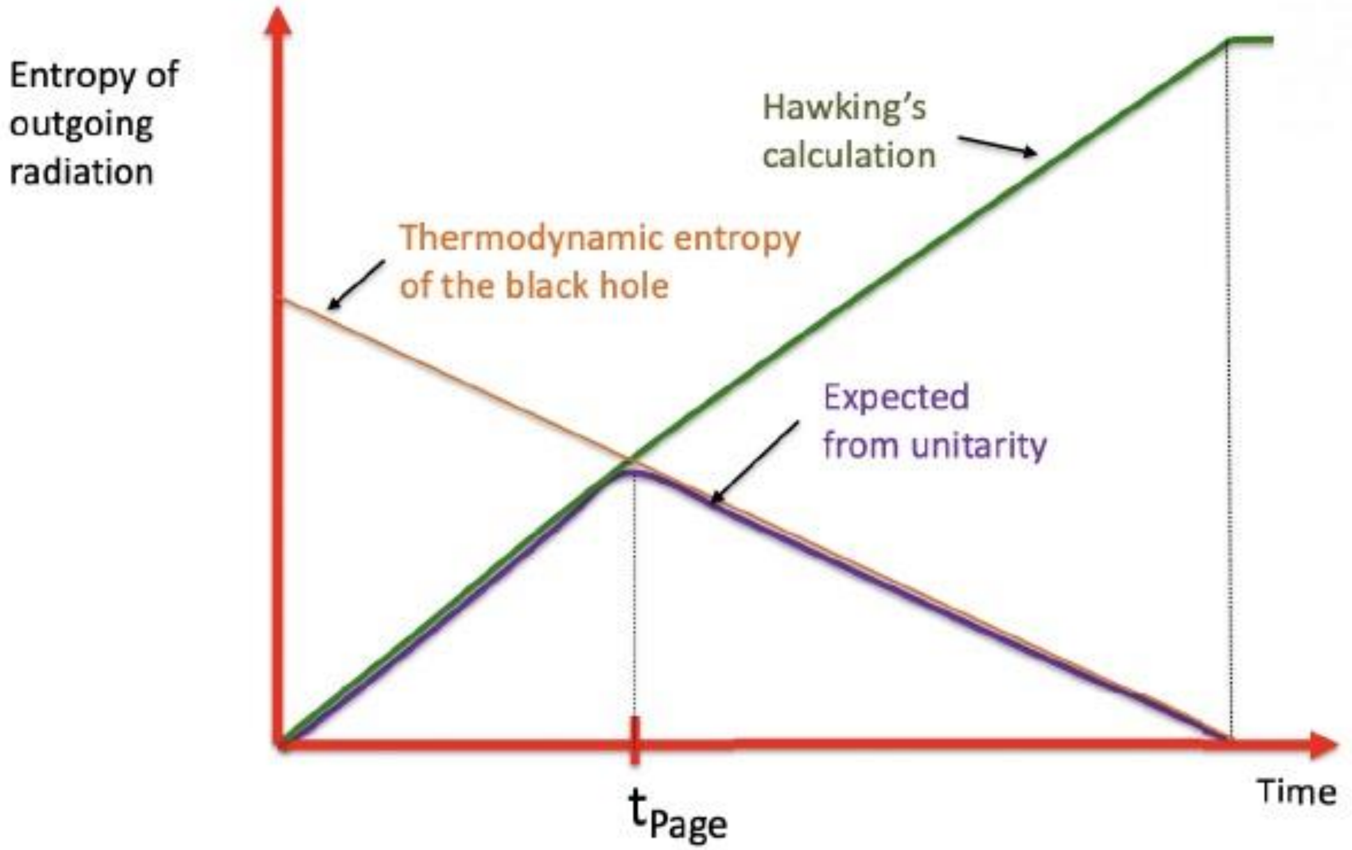
$$S_{rad} = S_{black-hole}$$

- But fine-grained entropy cannot be bigger than coarse-grained entropy:

- Increasing must stop when these two entropies are equal.

- Hawking calculation takes Hawking radiation as a thermal radiation and gives an increasing entropy with time until the total evaporation of the black hole.
- This is called Hawking information paradox or Information loss.
- To preserve the unitary evolution in black hole, entropy should follow the “Page curve” and end up in a pure state.
- Page time: the time at which entropy stops increasing.

- Page curve vs Hawking's entropy



Is there a gravitational formula for fine-grained entropy?

- Surprisingly, yes!
  - Generalized Beckenstein-Hawking formula, taking the quantum corrections into account:

$$S_{gen} = \frac{1}{4G_N} HorizonArea + S_{outside}$$

  - This formula is for the coarse-grained formula
- There is a similar formula for the fine-grained entropy with another HRT surface replacing the horizon.

- Fine-grained entropy formula:

$$S_{gen} = \min_X(ext_X[\frac{Area(X)}{4G_N} + S_{smi-classical(\Sigma_X)}]).$$

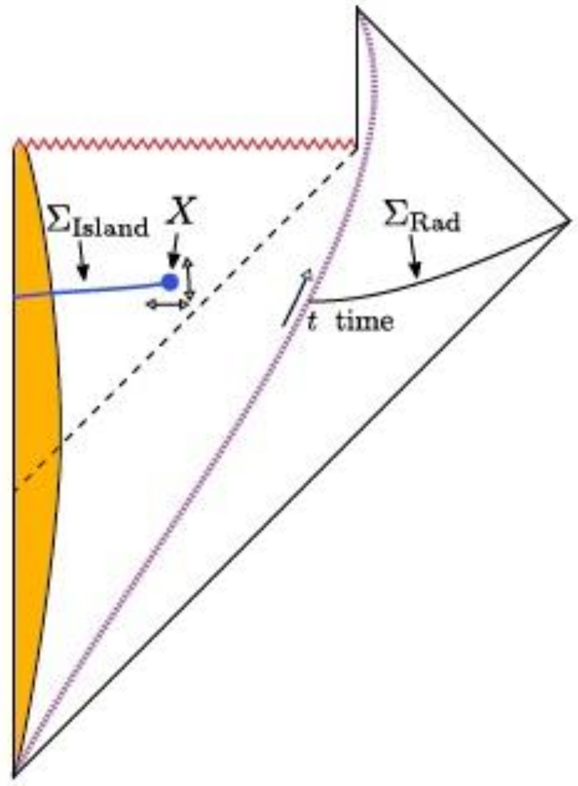
- Σ_X: the region between cut-off surface and X.

- The extremal surface minimizes the generalized entropy in spatial direction and maximizes it in the time direction.
- To calculate the entropy of radiation, we use a generalization of fine-grained entropy formula.
- We think of Σ_X as a discrete region.

- The discrete part covers part of black hole interior and is called "Island".
- Finally the fined-grained entropy of radiation is given by:

$$S_{Rad} = \min_X (ext_X [\frac{Area(X)}{4G_N} + S_{semi-cl}[\Sigma_{rad} \cup \Sigma_{Island}]]).$$

- The island conjecture gives us the mathematical tools needed to decrease the entropy as shown on the Page curve.



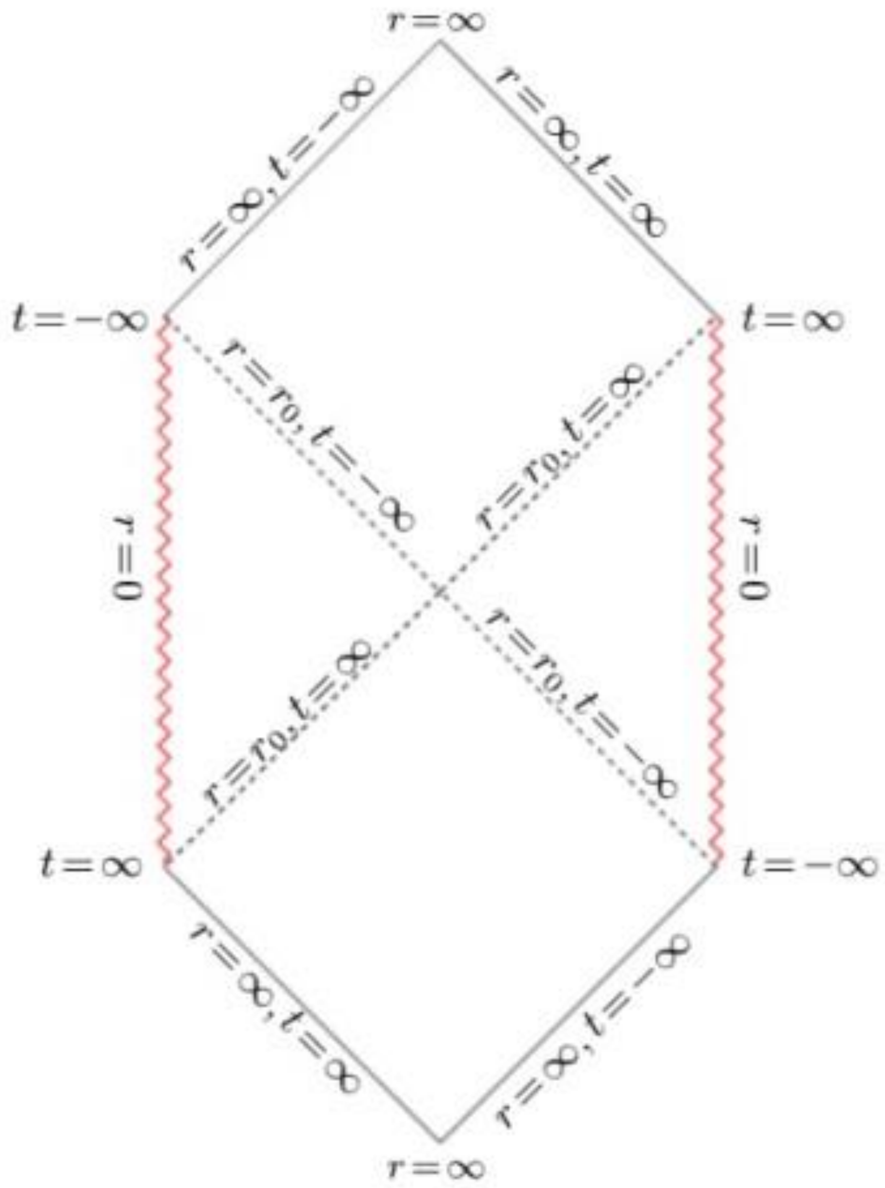
- Flat-space cosmologies (FSCs) are solutions to the Einstein equations of motion for vanishing cosmological constants in three dimensions.

$$ds^2 = -\frac{dr^2}{f(r)} + f(r)dt^2 + r^2\left(d\phi - N_\phi(r)dt\right)^2$$

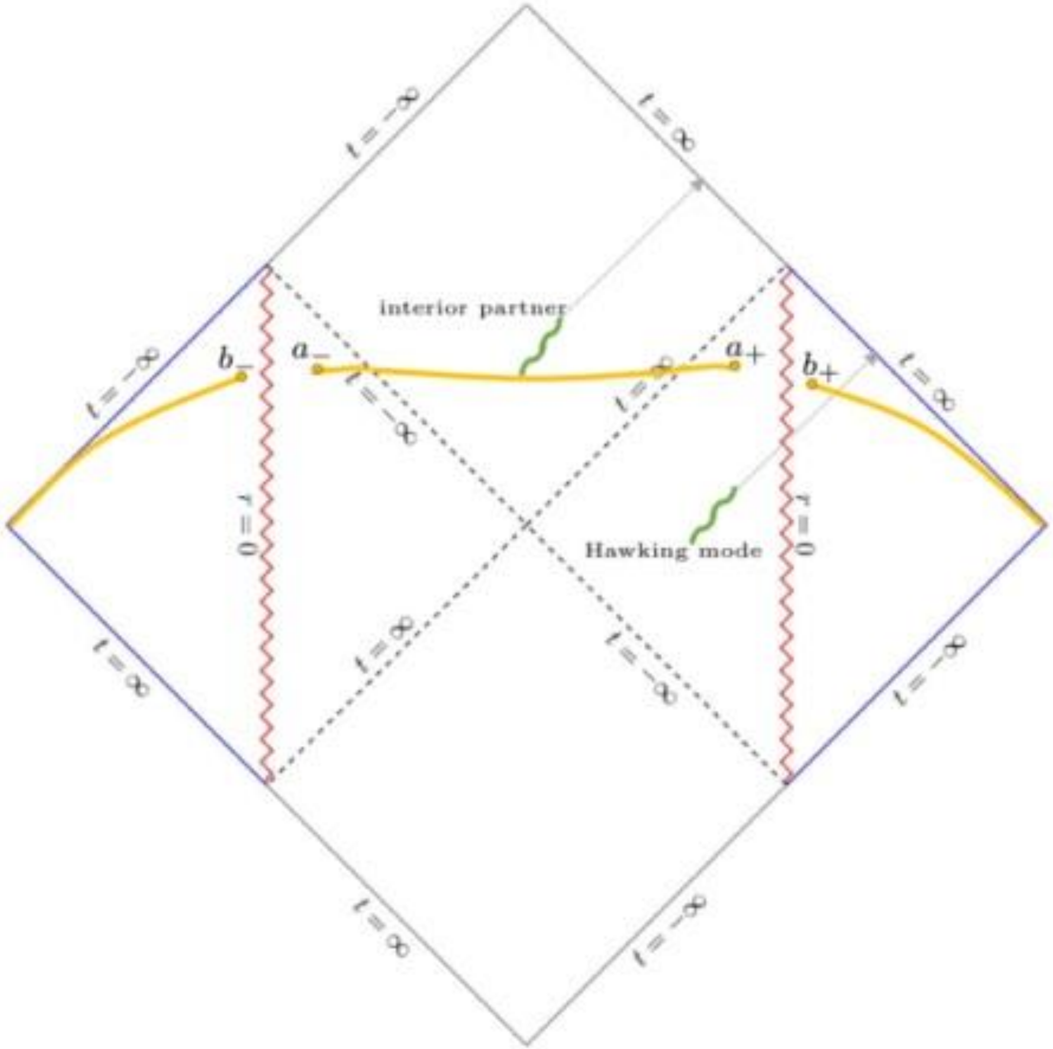
- $$f(r) = \frac{\hat{r}_+^2(r^2 - r_0^2)}{r^2} \quad , \quad N_\phi(r) = \frac{r_0\hat{r}_+}{r^2}$$

- They have event horizons, entropy proportional to the area of the event horizon and temperature proportional to the surface gravity.
- Hawking radiation started near its event horizon is reflected from the timelike singularity at the center.

- The Penrose diagram of FSC



- Similar to the study of information paradox in eternal AdS blackholes, we couple an auxiliary flat bath and impose a transparent boundary condition…



- To make ϕ−coordinate spacelike everywhere in the spacetime we make a coordinate transformation:

$$\psi = \phi - \frac{\hat{r}_+}{r_0} t$$

- So the metric will be: $$ds^2 = \frac{\hat{r}_+^2 (r^2 - r_0^2)}{r_0^2} dt^2 - \frac{r^2}{\hat{r}_+^2 (r^2 - r_0^2)} dr^2$$

- Then we go to the Kruskal coordinates: $$V(t,r) = e^{\kappa(t + r^*(r))} \quad U(t,r) = -e^{-\kappa(t - r^*(r))}$$

$$r^*_{FSC} = \frac{1}{2\kappa} \log(|r^2 - r_0^2|),$$

- So the metric will take the following form;

$$ds^2 = -\Omega^2 dV dU$$

$$\Omega_{FSC} = \frac{\hat{r}_+}{\sqrt{|r^2 - r_0^2|}} e^{\kappa r^*_{FSC}}, \qquad \Omega_{Bath} = \kappa e^{\kappa r^*_{Bath}},$$

- Examining the information paradox with and without islands!
- Assumptions:
- Divergence is absorbed in Newton's constant
- Backreaction of matter fields is negligible
- Entangling regions are far from each other hence some simplification

- 1. Without islands:
- The generalized entropy only consists of vN entropy of matter fields

$$S_{gen} = \frac{c}{6}\log[L(b_+, b_-)] = \frac{c}{6}\log[\frac{(V(b_+) - V(b_-))(U(b_-) - U(b_+))}{\Omega(b_+)\Omega(b_-)}],$$

- Entropy calculations indicate a linear growth of the entropy of radiation at late times

$$S_{gen} \sim \frac{c}{3}\log[\frac{2}{\kappa}\cosh(\kappa t_b)],$$

$$S_{gen} \sim \frac{c}{3}\kappa t_b + \ldots$$

- This is the famous information paradox!

- 2. With the islands:

- $S_{Rad} = Min\{Ext[\frac{Area(\partial I)}{4G_N} + S_{vN}(R \cup I)]\},$ $\quad S_{vN} = \frac{c}{6}Log\frac{L(a_+,a_-)L(b_+,b_-)L(a_+,b_+)L(a_-,b_-)}{L(a_+,b_-)L(a_-,b_+)}$

$$S_{Rad} = \frac{\pi a}{G_N} + \frac{c}{3}\log[L(a_+,b_+)],$$

By extremizing this generalized entropy w.r.t. the island one finds the location of the island:

$$a = r_0(1 - \frac{2c^2 e^{-2\kappa b} G_N^2}{9\pi^2}),$$

- By substituting the island in the generalized entanglement entropy we'll see that it's not time dependent anymore:

$$S_{gen} = 2S_{thermal} + \frac{c}{6}\log(\frac{\beta^3 e^{\frac{4\pi b}{\beta}}}{8\pi^3 r_0}) - \frac{2c^2 r_0 e^{\frac{4\pi b}{\beta}} G_N}{3\pi} + O(G_N^2),$$

- So the monotonic growth of the entropy of the radiation stops!

* Page time: the time that entropy stops growing transition from trivial island (no island) to non-trivial island

$$t_{Page} = \frac{3\beta}{\pi c} S_{th} + \frac{\beta}{4\pi}\log(\frac{\beta e^{\frac{4\pi b}{\beta}}}{2\pi r_0}) - \frac{c\beta r_0 e^{-\frac{4\pi b}{\beta}} G_N}{\pi^2} + O(G_N^2),$$

*Scrambling time:

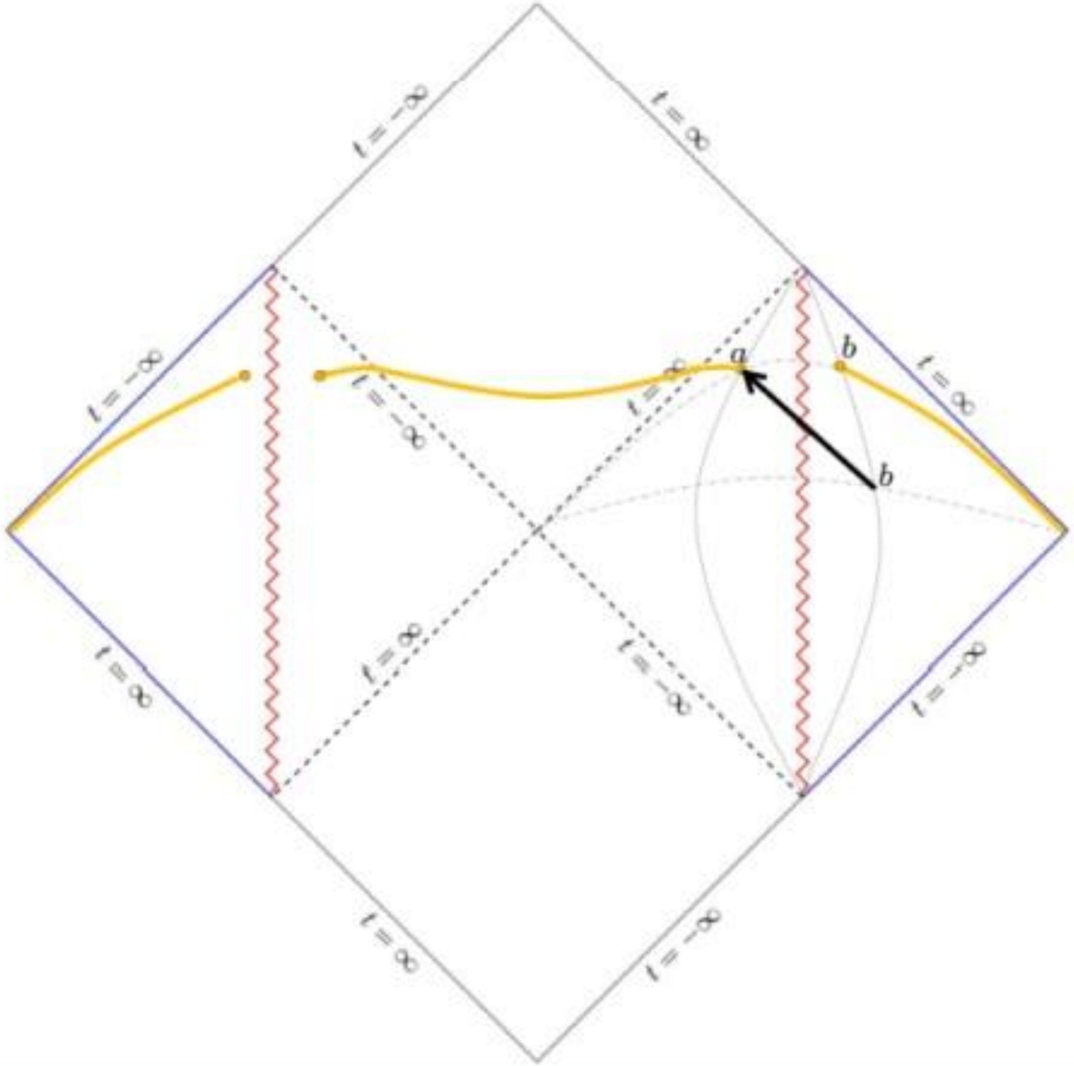
According to the conjecture → the density matrix of the radiation is given by that of union of regions R and I

Information thrown into the island can be recovered from radiation

$$v(t_0, b) - v(t_a, a) = \left(t_0 + r^*(b)\right) - \left(t_a + r^*(a)\right),$$

$$t_{scr} \equiv t_a - t_0$$

$$t_{scr} = \frac{\beta}{2\pi} \log S_{th} + 2b - \frac{\beta}{2\pi} \log(\frac{c r_0^2}{3}),$$



$t = -\infty$
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- Kerr- de Sitter metric in three dimensions:

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 \left(d\phi + N^\phi dt\right)^2$$

$$N^2 = \mu - \frac{r^2}{l^2} + \frac{16G^2J^2}{r^2} \qquad N^\phi = \frac{4GJ}{r^2}$$

- The conserved charges associated with time translation and rotational symmetries are:

$$M := Q_{\partial_t} = -\frac{\mu}{8G} \qquad Q_{\partial_\phi} = J$$

- Rewriting the metric with “M”;

$$ds^2 = -\frac{(r^2 + r_+^2)(r_-^2 - r^2)}{l^2 r^2} dt^2 + \frac{l^2 r^2}{(r^2 + r_+^2)(r_-^2 - r^2)} dr^2 + r^2 \left(d\phi + \frac{r_+ r_-}{l r^2} dt\right)^2$$

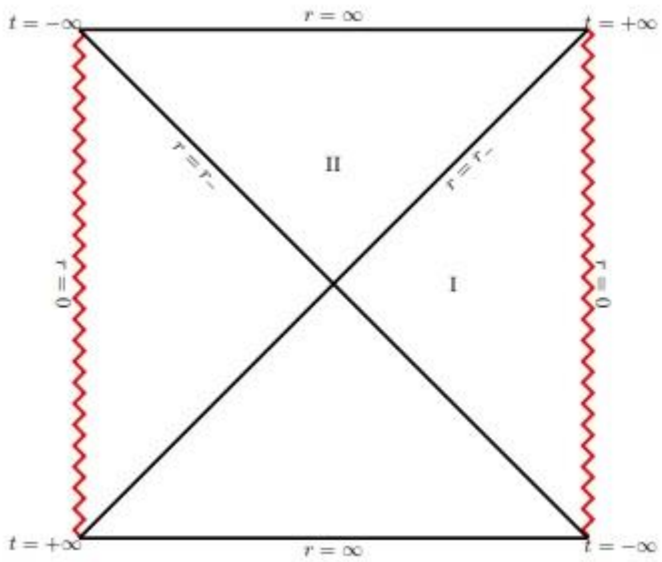
- Location of the cosmological horizon: $r_-^2 = 4GMl^2(\sqrt{1+\left(\frac{J}{Ml}\right)^2} - 1),$

A parameter ⇒ $r_+^2 = 4GMl^2(\sqrt{1+\left(\frac{J}{Ml}\right)^2} + 1),$

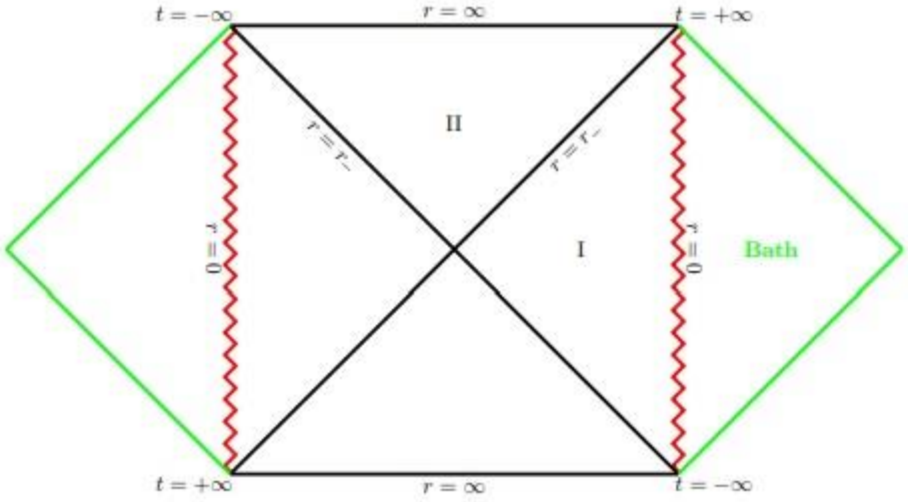
- This metric and its surface gravity recover the FSC metric and surface gravity at the flat limit $r_- \to r_0, r_+ \to \hat{r}_+ l$ $l \to \infty$
- Again in order to have a angular coordinate that is spacelike everywhere we should make a coordinate transformation: $\psi = \phi + \frac{r_+}{lr_-} t$
- So the metric reads:

$$ds^2 = \frac{(r_+^2 + r_-^2)(r^2 - r_-^2)}{r_-^2 l^2} dt^2 - \frac{r^2 l^2}{(r_+^2 + r^2)(r^2 - r_-^2)} dr^2$$

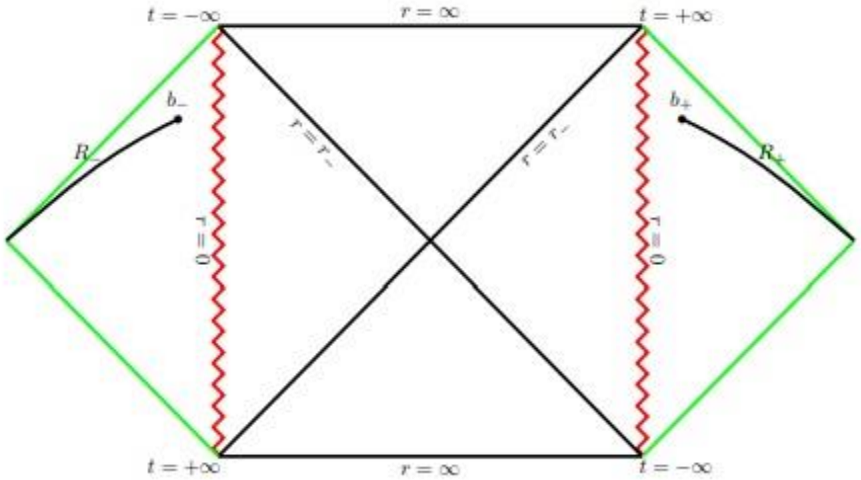
- The Penrose diagram of KdS:



- Like the previous case we put it in thermal equilibrium with the flat auxiliary bath



- Just like the FSC case we will get an ever increasing entropy for radiation without islands



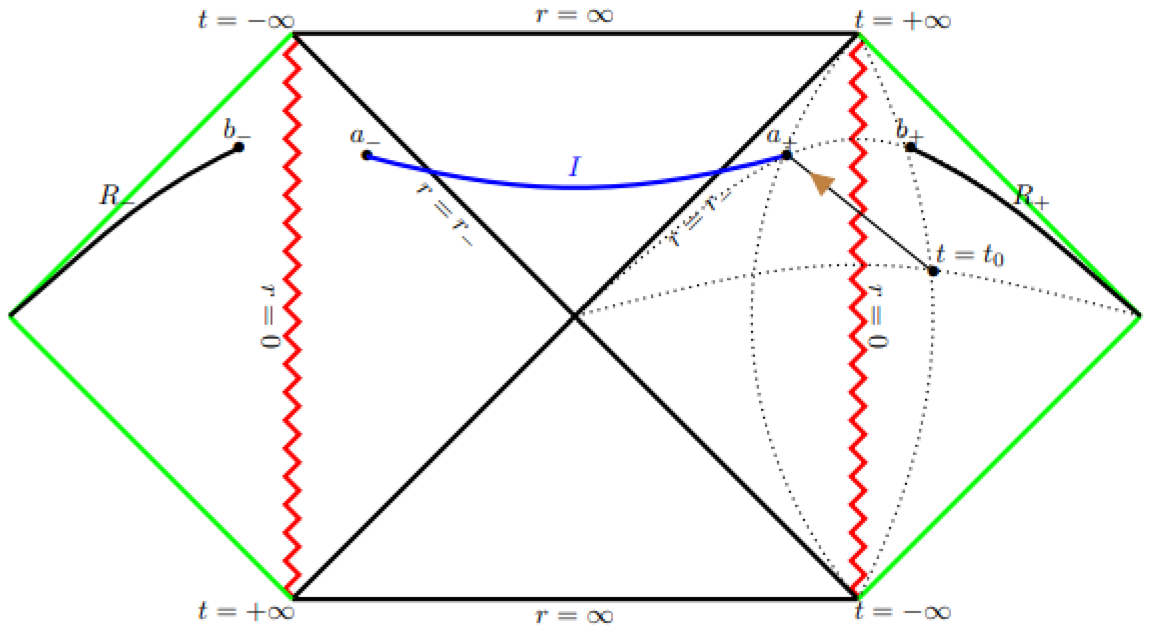
$$S_{gen} = \frac{c}{6}\log[L(b_+, b_-)] = \frac{c}{6}\log[\frac{(V(b_+) - V(b_-))(U(b_-) - U(b_+))}{\Omega(b_+)\Omega(b_-)}],$$

$$S_{gen} \sim \frac{c}{3}\kappa t_b + \ldots$$

Entropy in the presence of the islands

By extremizing the generalized entropy w.r.t. the island one finds te location of island to be:

$$a_{KdS} = r_- - \frac{2 r_- r_+^2 c^2 e^{-2\kappa b} G_N^2}{9(r_-^2 + r_+^2)\pi^2},$$



*Location of the KdS Island recovers the FSC one at flat limit

The Page time for the KdS spacetime:

$$t_{Page(KdS)} = \frac{3\beta}{\pi c} S_{th} + \frac{\beta}{4\pi} \log\left[\frac{(r_-^2 + r_+^2)^2 e^{\frac{4\pi b}{\beta}} \beta^4}{16\pi^4 r_-^2 r_+^2 l^2}\right] - \frac{\beta r_- r_+^2 c^2 e^{-\frac{4\pi b}{\beta}} G_N}{\pi^2 (r_-^2 + r_+^2)}$$

*Page time of KdS recovers the FSC one at the flat limit

- The scrambling time for the KdS:

$$t_{scr(KdS)} = \frac{\beta}{2\pi}\log S_{th} + 2b - \frac{\beta}{2\beta}\log(\frac{cr_-^2 r_+^2}{3(r_-^2 + r_+^2)}),$$

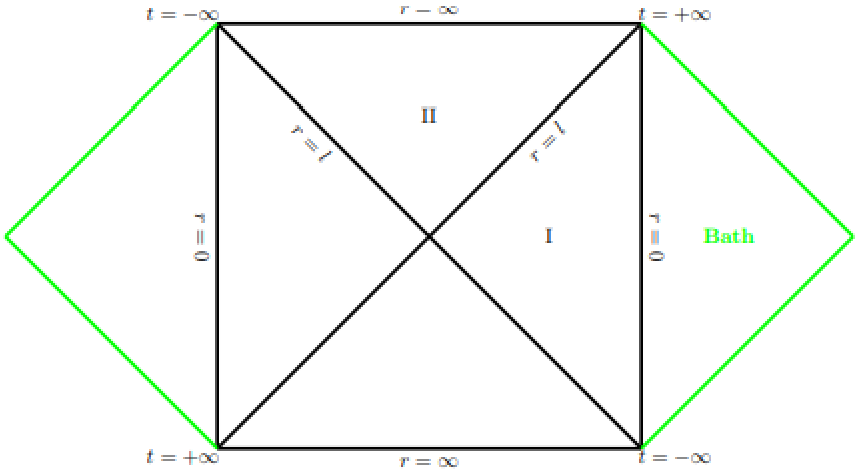
*The scramblimg time of KdS recovers the FSC one in the flat limit.

- What about de Sitter spacetime?

$$ds^2 = -(1 - \frac{r^2}{l^2})dt^2 + \left(1 - \frac{r^2}{l^2}\right)^{-1} dr^2$$

$$r_{ds}^*(r) = \frac{1}{2\kappa}\log[\frac{l - r}{l + r}],$$

$$\Omega_{ds}(r) = \frac{\kappa}{\sqrt{1 - \frac{r^2}{l^2}}} e^{\kappa r_{Kds}^*} \qquad \Omega_{Bath}(r) = \kappa e^{\kappa r_{Bath}^*}$$



- Location of the island: $$a_{dS} = l - \frac{c^2 e^{2\kappa b} G_N^2}{18 l \pi^2}$$
- Page time: $$t_{Page(dS)} = \frac{3\beta}{c\pi} S_{th} + \frac{c}{6} \log[\frac{e^{\frac{4\pi b}{\beta}} \beta^4}{4\pi^4}] - \frac{\beta c e^{-\frac{4\pi b}{\beta}} G_N}{4 l \pi^2}$$
- Scrambling time: $$t_{scr(dS)} = \frac{\beta}{2\pi} \log S_{th} + 2b + \frac{\beta}{2\pi} \log \frac{12}{c}$$
- The final result confirms that de Sitter space is a fast scrambler!

Thank you for your attention