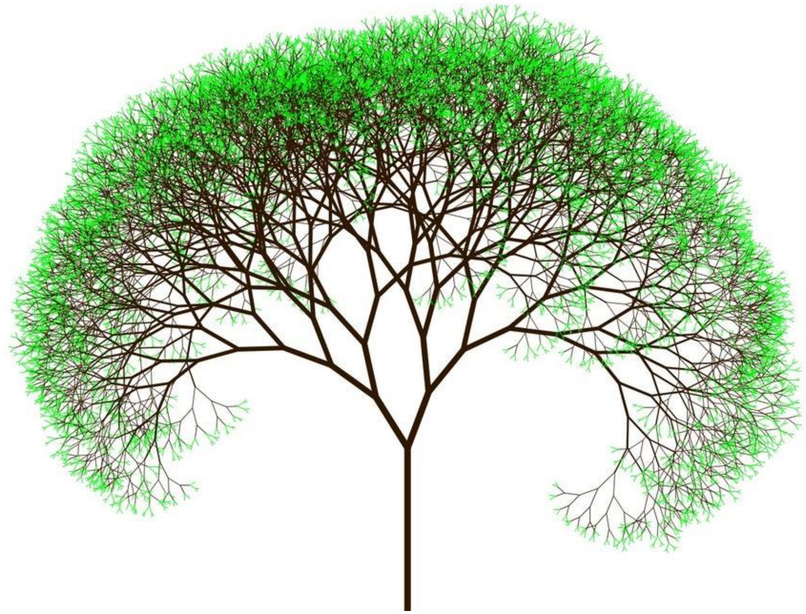
IPM

Institute for Research in Fundamental Sciences

Proton structure functions at low x: the Fractal approach



Samira Shoeibi

in collaboration with

Shahin Atashbar Tehrani, Fatemeh Taghavi-Shahri

Weekly Seminar, School of Particles and Accelerators, 26 October 2022

Outline

- **Proton structure functions and PDFs**
- **Fractals**
- **Two essential ingredients to obtain PDFs:**
  - **Parametrizations of PDFs in initial value of Q_0^2**
  - **Experimental observable**
- **Minimization processes and Discussion of the QCD fit results**
- **Result**

Proton structure functions and PDFs

❖ Quantum chromodynamics (**QCD**) describes the **properties and the internal structure of hadrons.**

❖ Hadrons are made up of **partons** (quarks and gluon) and at high energy scales (>200 MeV), these partons are probed almost free and the hadron's momentum are sharing among them. This information can be encoded in term of non-perturbative **Parton Distribution Functions** (PDFs).

❖ PDFs are critical component in **High Energy Physics (HEP) phenomenology ,for example**:

✓ used in the computation of physical observables, such as cross sections.

✓ are key component on the precision measurement of Standard Model parameters such as strong coupling constant.

❖ There are two main ways to **determining the PDFs**:

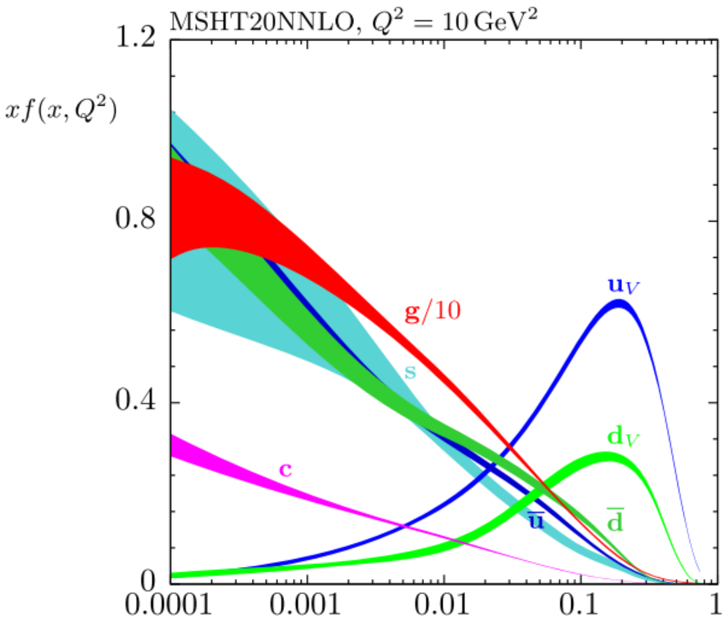
1) using **lattice QCD**.

2) using **large experimental data sets in global QCD analyses**

✓ particular functional form for the x dependence of the PDFs (at a scale Q0) like MSHT20 PDFs and CT14.

✓ Neural Networks (NNs) like NNPDF.

Proton structure functions and PDFs



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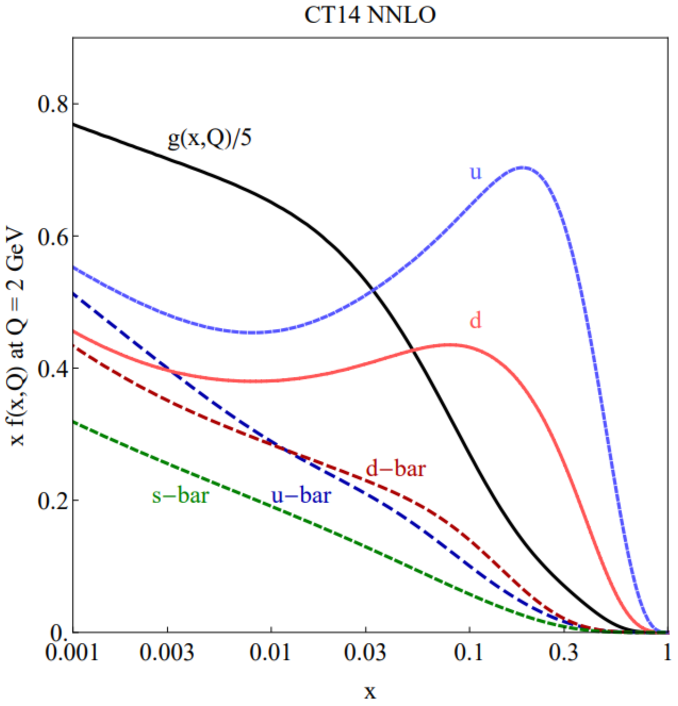
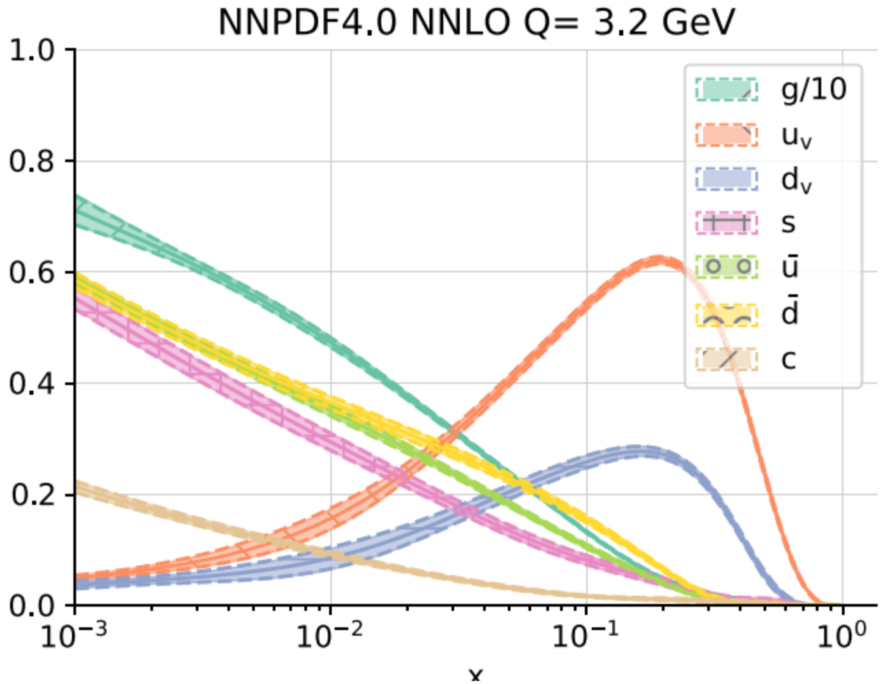


Phys. Rev. D 93, 033006 (2016)



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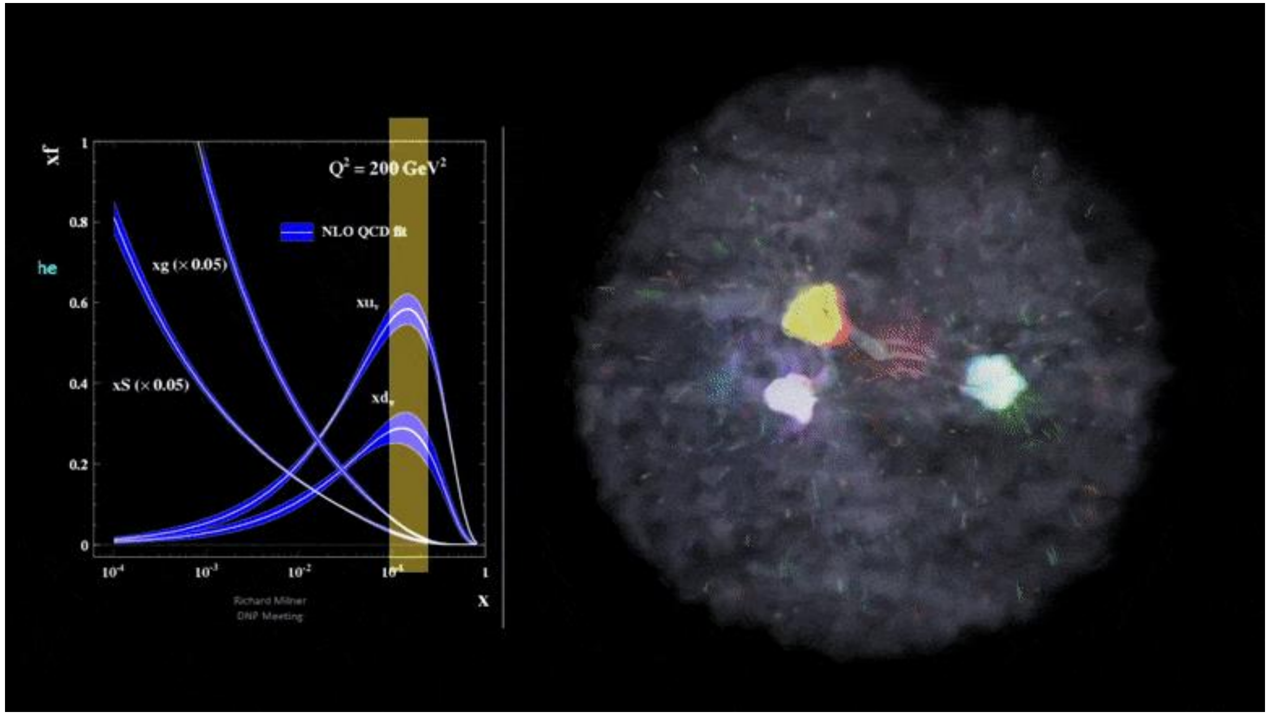
Proton structure functions and PDFs



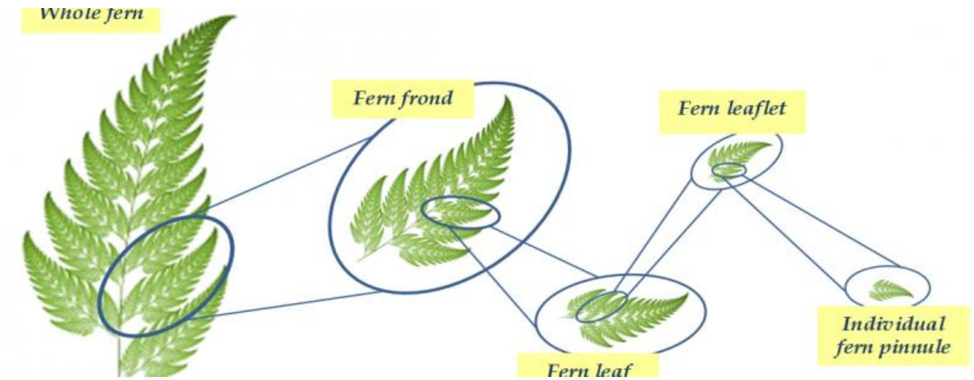
https://scitechdaily.com/visualizing-the-proton-physicists-innovative-animation-depicts-the-subatomic-world-in-a-new-way/

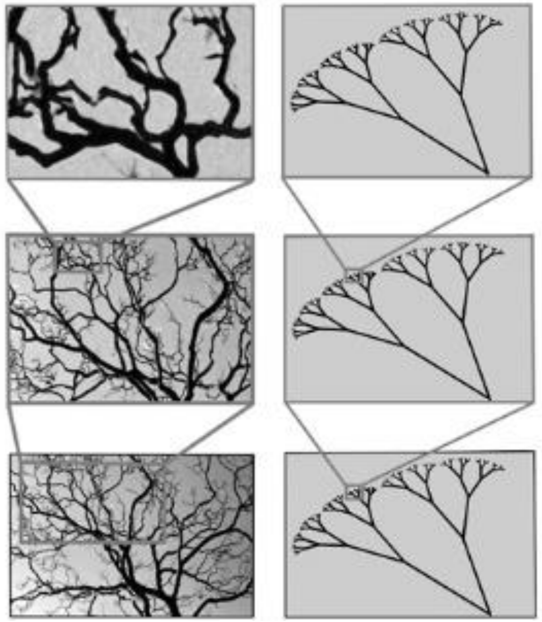
Fractals

Fractal is a curve or geometrical figure, **each part of which has the same statistical character as the whole** and they have some features such as:

- **Self-similar**
- **Fractal Dimension**

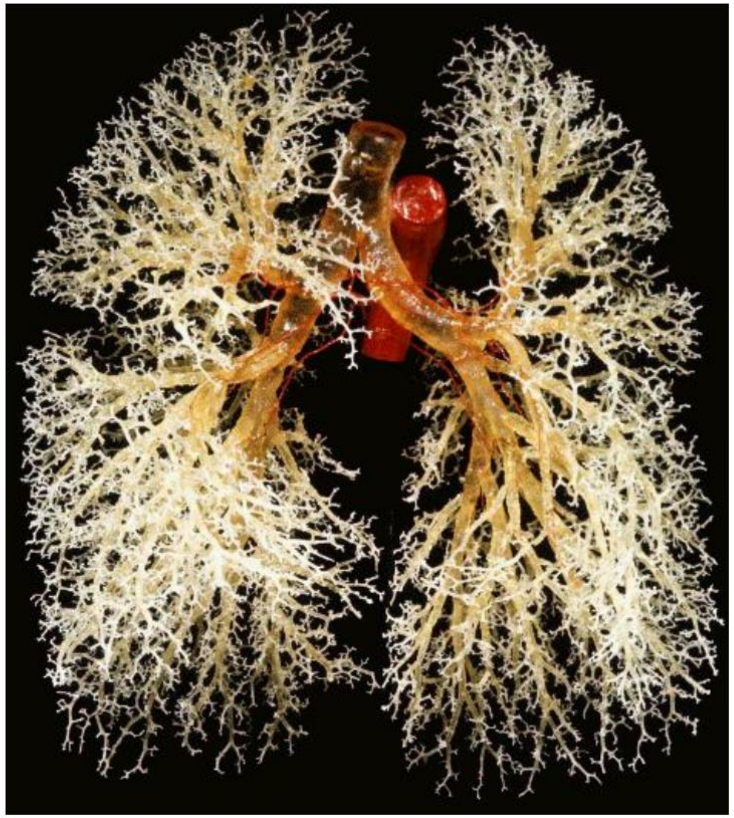
Fractals in Nature:





Fractals

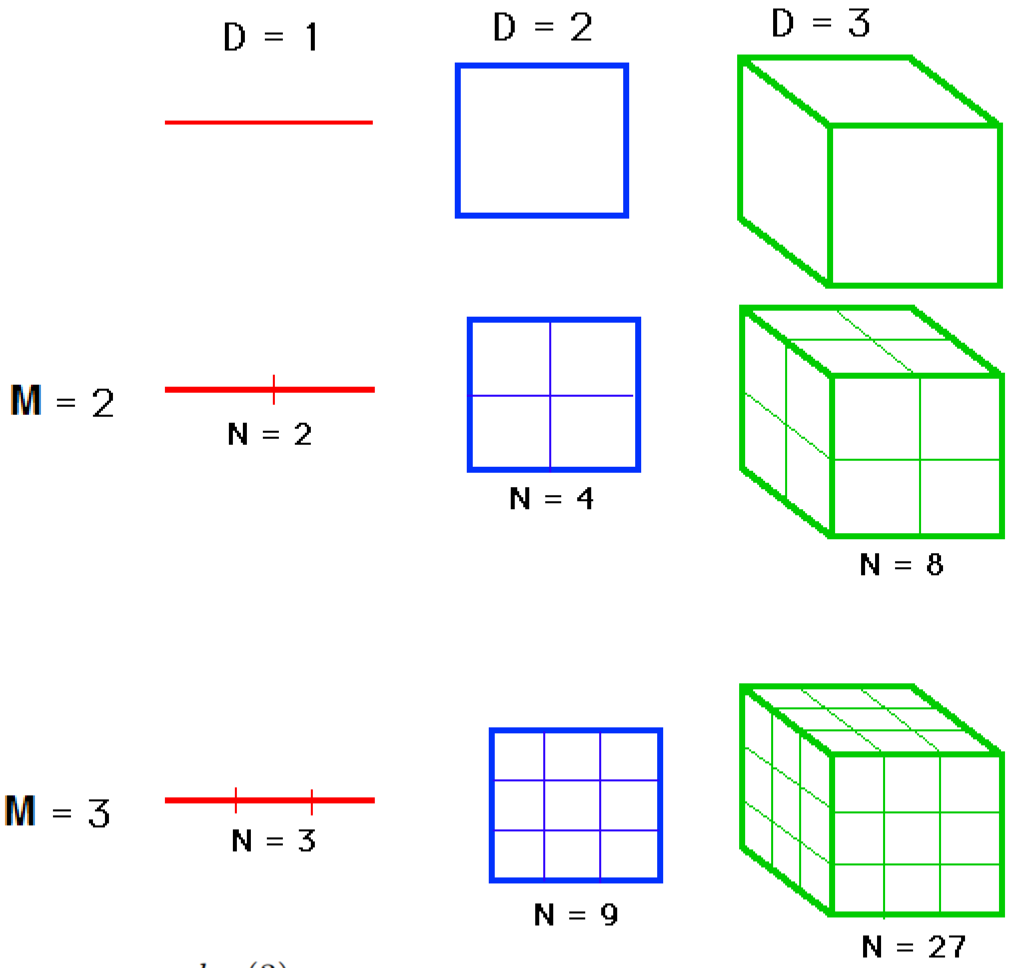
Fractals in Animal Bodies:



Fractal Lightning and Electricity:

Fractals

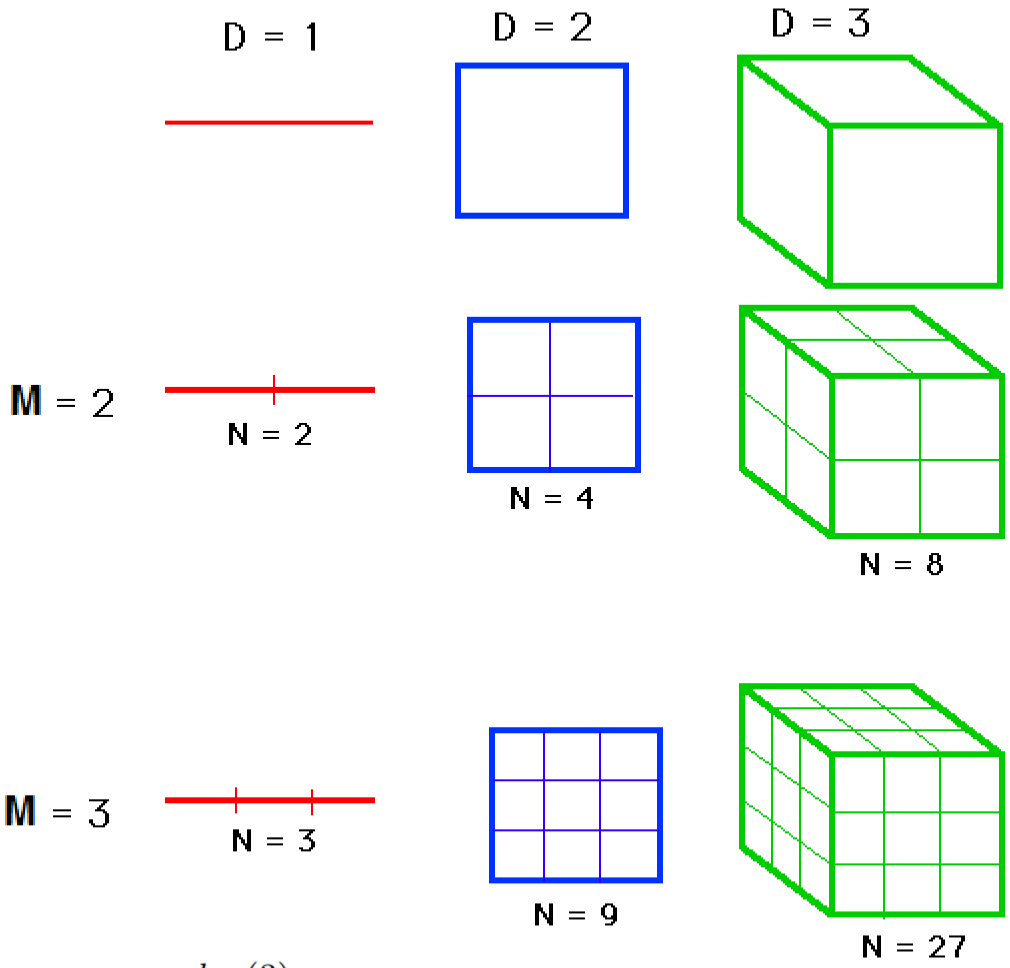
Fractal Dimension:



$$D=\frac{log(3)}{log(3)}=1 \qquad D=\frac{log(3^2)}{log(3)}=2 \qquad D=\frac{log(3^3)}{log(3)}=3$$

Fractals

Fractal Dimension:

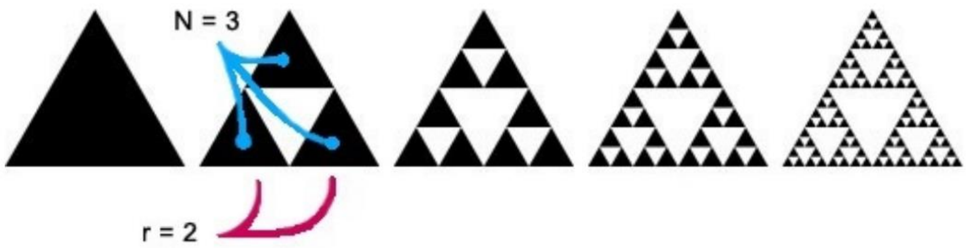


$$D = \frac{log(3)}{log(3)} = 1 \qquad D = \frac{log(3^2)}{log(3)} = 2 \qquad D = \frac{log(3^3)}{log(3)} = 3$$

$$D = \frac{log(number\ of\ self\ similar\ objects)}{log(magnification\ factor)} = \frac{log(N)}{log(r)}$$

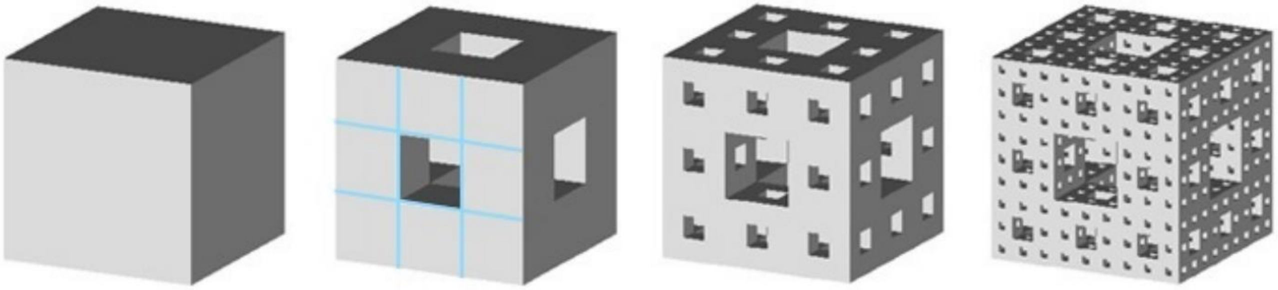
Fractals

Sierpinski Gasket:



$$D = \frac{\log(N)}{\log(r)} = \frac{\log(3)}{\log(2)} = 1.585$$

Menger Sponge:



$$D = \frac{\log(N)}{\log(r)} = \frac{\log(20)}{\log(3)} = 2.726$$

The fractal dimension describes how complicated or how large a self-similar object is. A plane is "larger" than a line. The Sierpinski gasket is not a line but also far from being a plane.

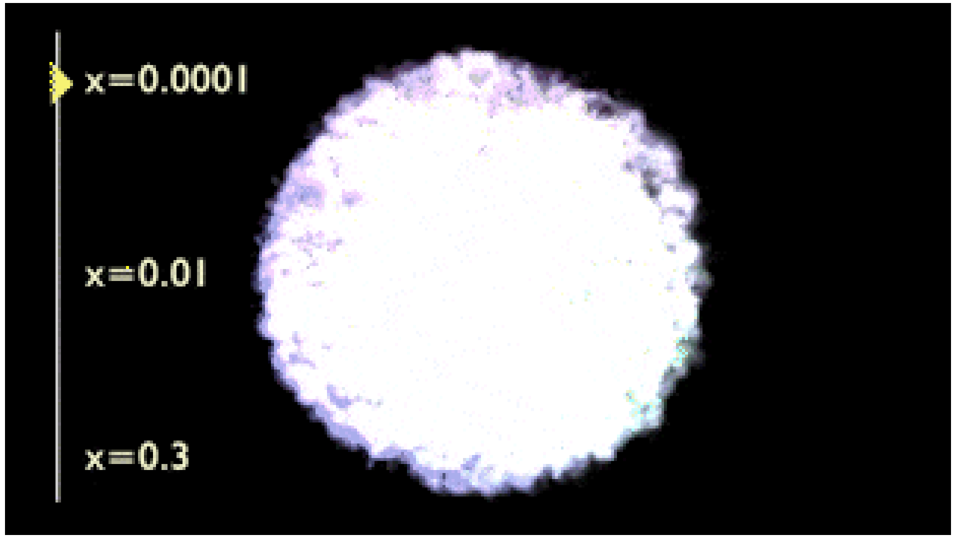
Fractals

If $M \equiv z$ and the number of the self-similar objects $= f(z)$, then

$$D = \frac{\partial \log f(z)}{\partial \log z} \Longrightarrow \log f(z) = D.\log z + D_0 \Longrightarrow f(z) \propto z^D$$

In general, fractals may have two independent magnification factors, z and y. In this case the density f(z, y) is written in the following way:

$$\log f(z, y) = D_{zy}.\log z.\log y + D_z.\log z + D_y.\log y + D_0$$



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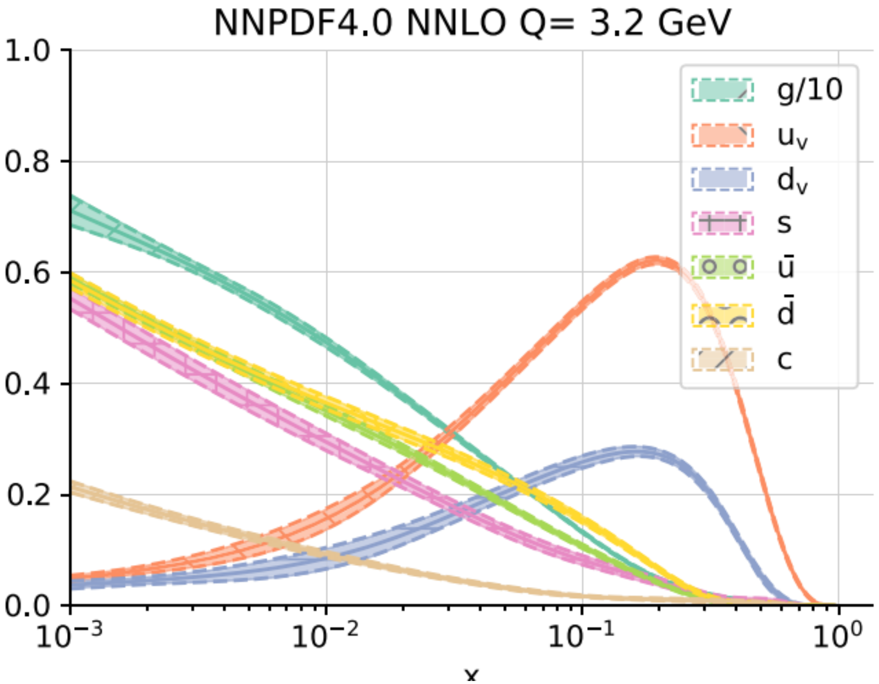

- In small x (x < 0.01), more gluon–gluon interactions can be observed which increase the gluon and the sea quark densities. Therefore we can consider the proton as a soup of sea quarks and gluon and we can say in analogy to fractals, they follow self-similarity.

1) Parametrizations of PDFs in initial value of Q_0^2

Eur. Phys. J. C 24, 529–533 (2002)
Digital Object Identifier (DOI) 10.1007/s10052-002-0973-3

THE EUROPEAN PHYSICAL JOURNAL C

Self-similar properties of the proton structure at low x

T. Laštovička[1,2,a]

[1] DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany
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Received: 28 March 2002 /
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Abstract. Self-similar properties of proton structure in the kinematic region of low values of the Bjorken variable x are introduced and studied numerically. A description of the proton structure function $F_2(x, Q^2)$ reflecting self-similarity is proposed with a few parameters which are fitted to recent HERA data. The specific parametrisation provides an excellent description of the data which cover the region of four momentum transfer squared $0.045 \leq Q^2 \leq 120\,\mathrm{GeV}^2$, and of the Bjorken variable x $6.2 \cdot 10^{-7} \leq x \leq 0.01$.

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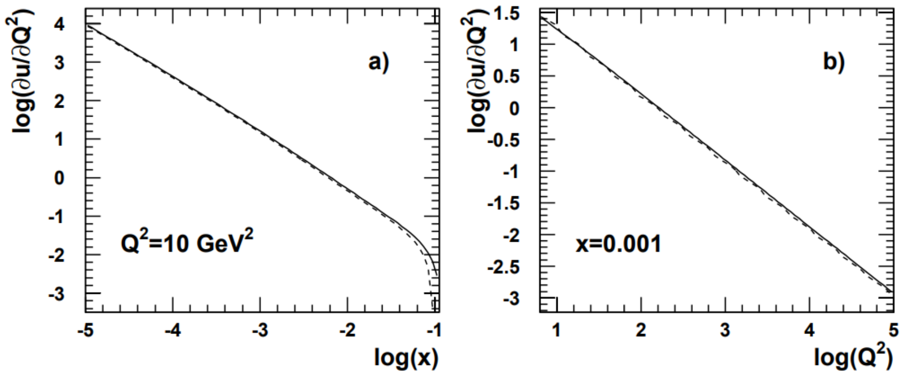

Fig. 2a,b. Logarithm of the unintegrated u-quark density $\partial u(x, Q^2)/\partial Q^2$ as a function of the Bjorken variable x **a** and Q^2 **b**. The full and dashed lines correspond to GRV parametrisations in LO and NLO [10], respectively

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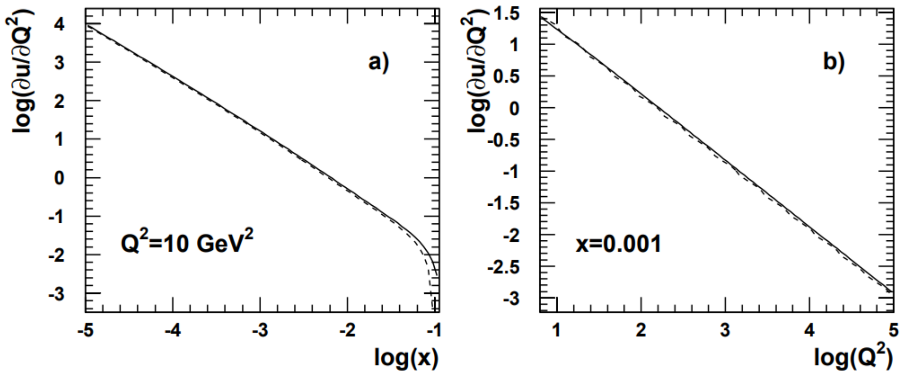

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$$log(f_{q/p}(x, k_t^2)) = D_1^q log(\frac{1}{x}) log(1 + \frac{k_t^2}{q_0^2}) + D_2^q log(\frac{1}{x}) + D_3^q log(1 + \frac{k_t^2}{q_0^2}) + D_0^q$$

Quark Parton Model → $F_2 = x \sum_i e_i^2 (q_i + \bar{q}_i)$

1) Parametrizations of PDFs in initial value of Q_0^2

Towards precision determination of uPDFs

Magnus Hansson[1] and Hannes Jung[2]

1- Lund University

2- DESY, FRG

The unintegrated Parton Density Function of the gluon is obtained from a fit to dijet production in DIS as measured at HERA. Reasonable descriptions of the measurements are obtained, and a first attempt to constrain the intrinsic transverse momentum distribution at small $k_\perp$ is presented [1].

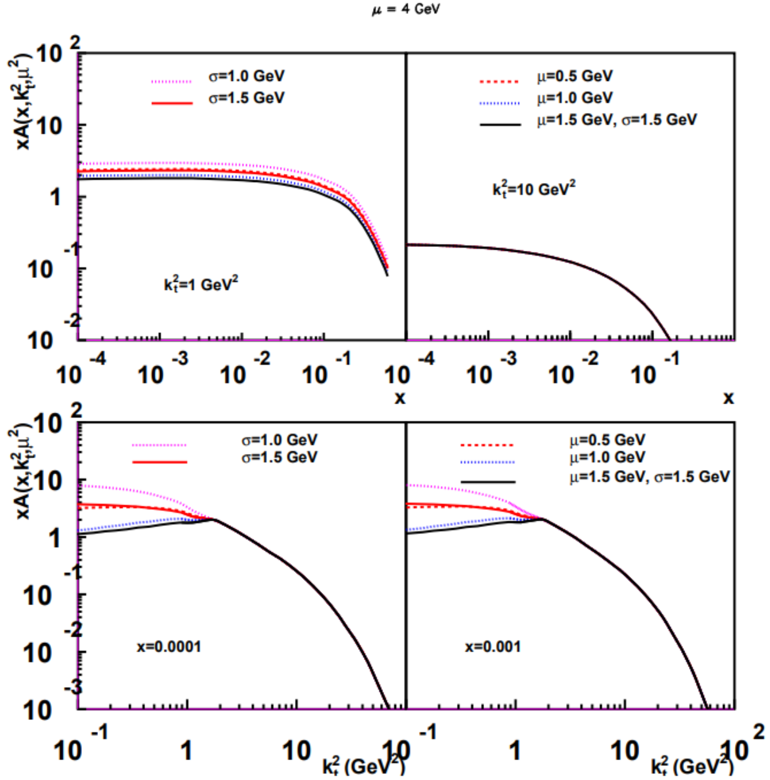


Figure 1: The unintegrated gluon distribution at a scale $\bar{q} = 4$ GeV for different values of μ and σ of the intrinsic $k_\perp$ distribution as a function of x for fixed $k_\perp$(top) and as a function of $k_\perp$ (bottom) for fixed x

1) Parametrizations of PDFs in initial value of Q_0^2

$$log(f_{q/p}(x,k_t^2)) = D_1^q log(\frac{1}{x}) log(1+\frac{k_t^2}{q_0^2}) + D_2^q log(\frac{1}{x}) + D_3^q log(1+\frac{k_t^2}{q_0^2}) + D_0^q - log(M^2)$$

$$xf_{g/p}(x,k_t^2) = (\frac{1}{x})^{B^g}(1-x)^{C^g}(1-D^g x)\ .\ e^{-\frac{(\mu-k_t)^2}{\sigma^2}} \frac{1}{M^2}$$

$$xf_{i/p}(x,Q^2) = \int_0^{Q^2} xf_{i/p}(x,k_t^2)\ dk_t^2$$

Initial PDFs

$$xf_{q/p}(x,Q_0^2) = \frac{e^{D_0^q} q_0^2 x^{-D_2^q+1}}{M^2(1+D_3^q - D_1^q log(x))}(x^{-D_1^q log(1+\frac{Q_0^2}{q_0^2})}(1+\frac{Q_0^2}{q_0^2})^{D_3^q+1} - 1)$$

$$xf_{g/p}(x,Q_0^2) = A'(\frac{1}{x})^{B^g}(1-x)^{C^g}(1-D^g x)$$

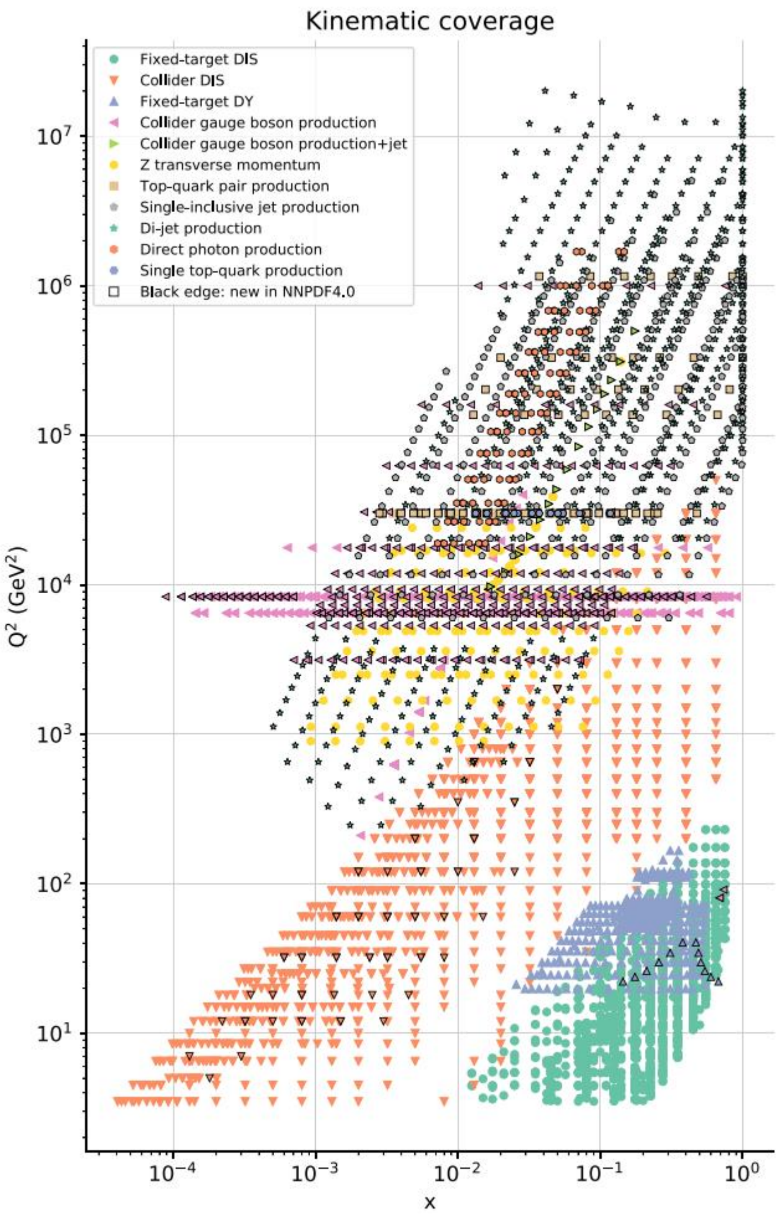
here A' is:

$$A' = \frac{A}{M^2}(exp^{-\frac{(\mu-1)^2}{\sigma^2}} + exp^{-\frac{\mu^2}{\sigma^2}})\sqrt{\pi}\mu(Erf(\frac{1-\mu}{\sigma}) + Erf(\frac{\mu}{\sigma}))$$

where $Erf(z)$ is the "error function" which is defined by

$$Erf(z) \equiv \frac{2}{\pi}\int_0^z exp^{-t^2} dt$$

2) Experimental observable



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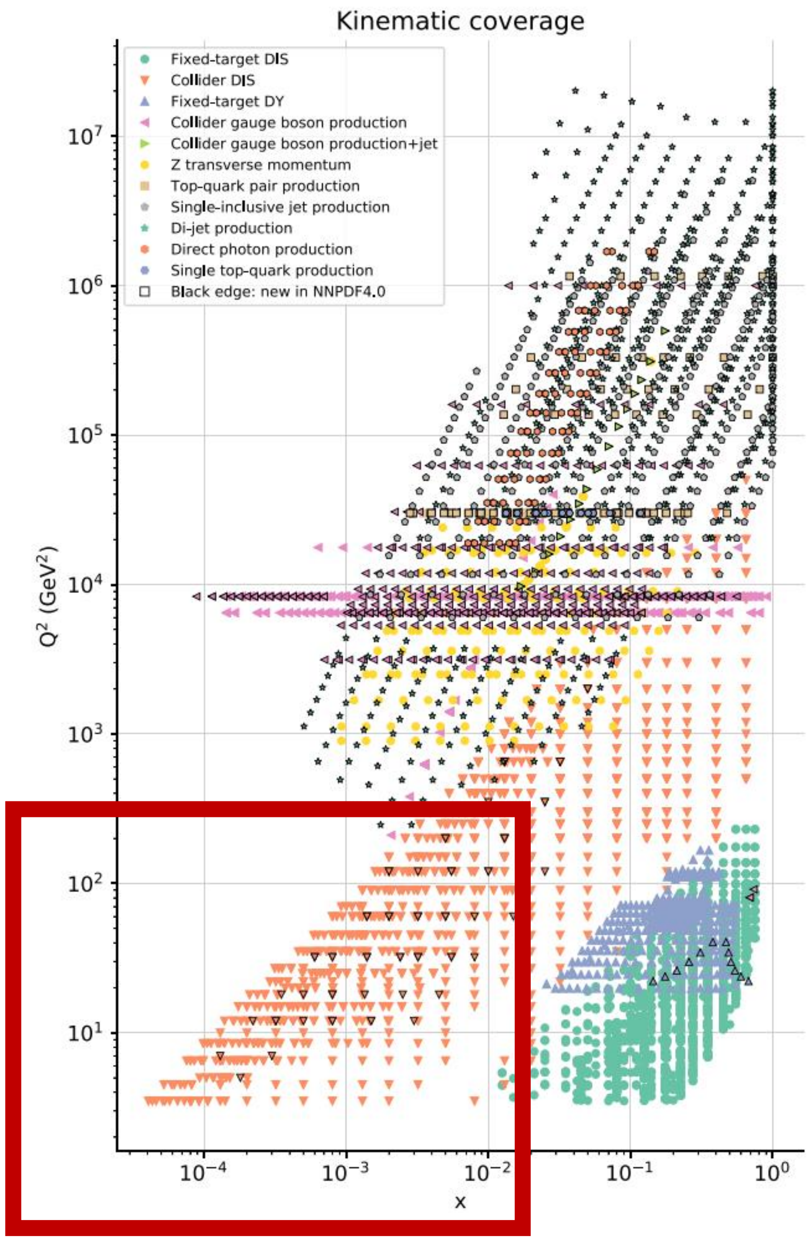

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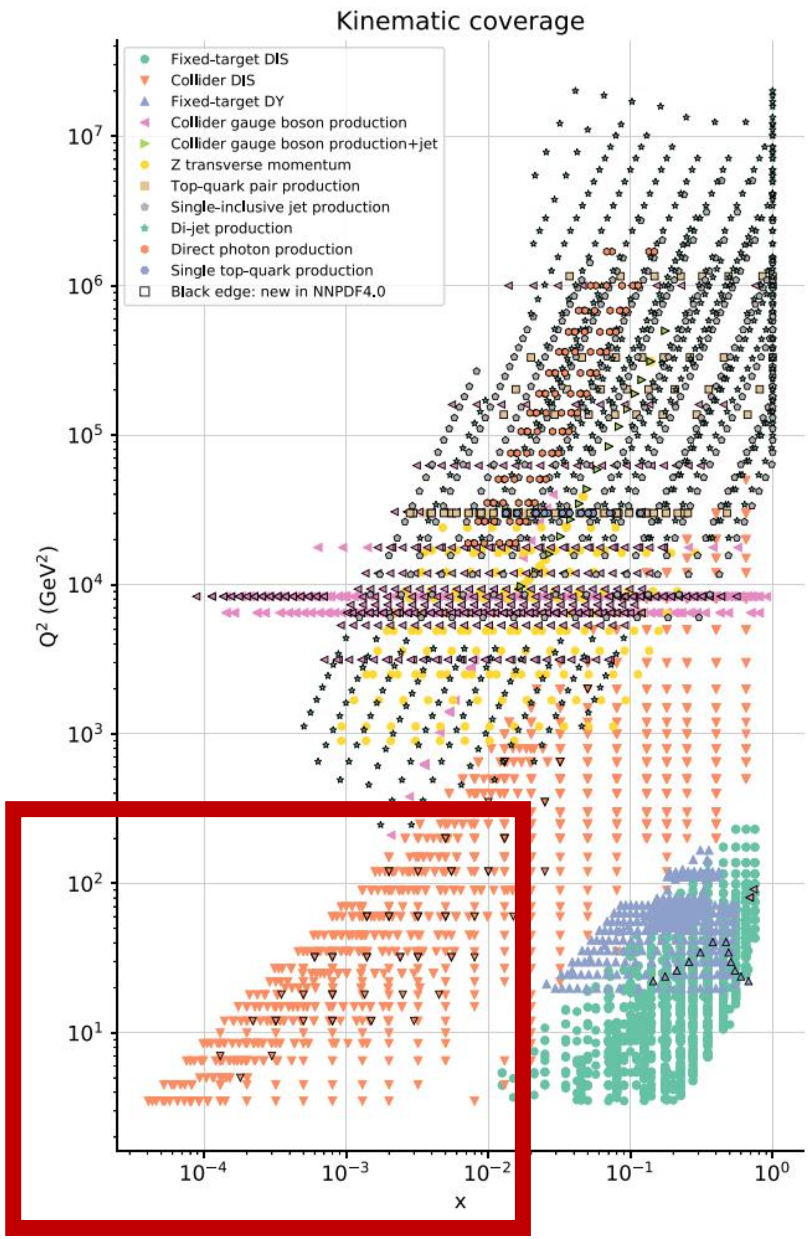

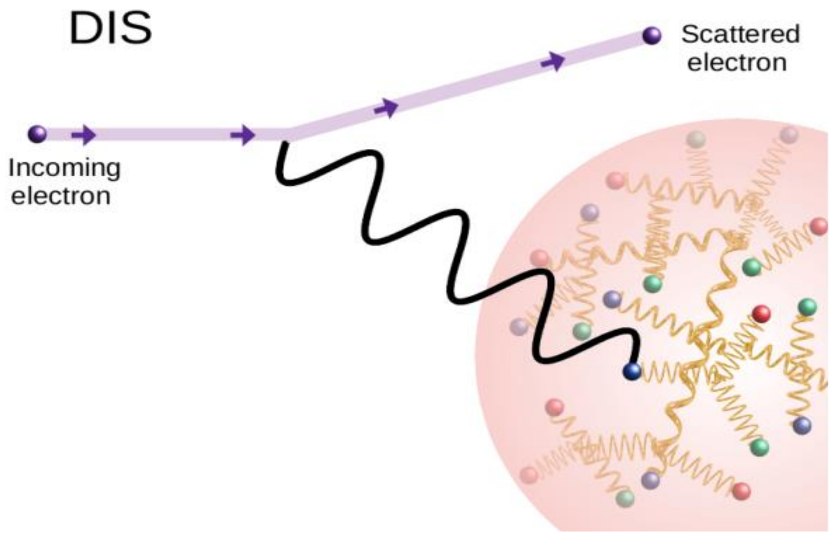

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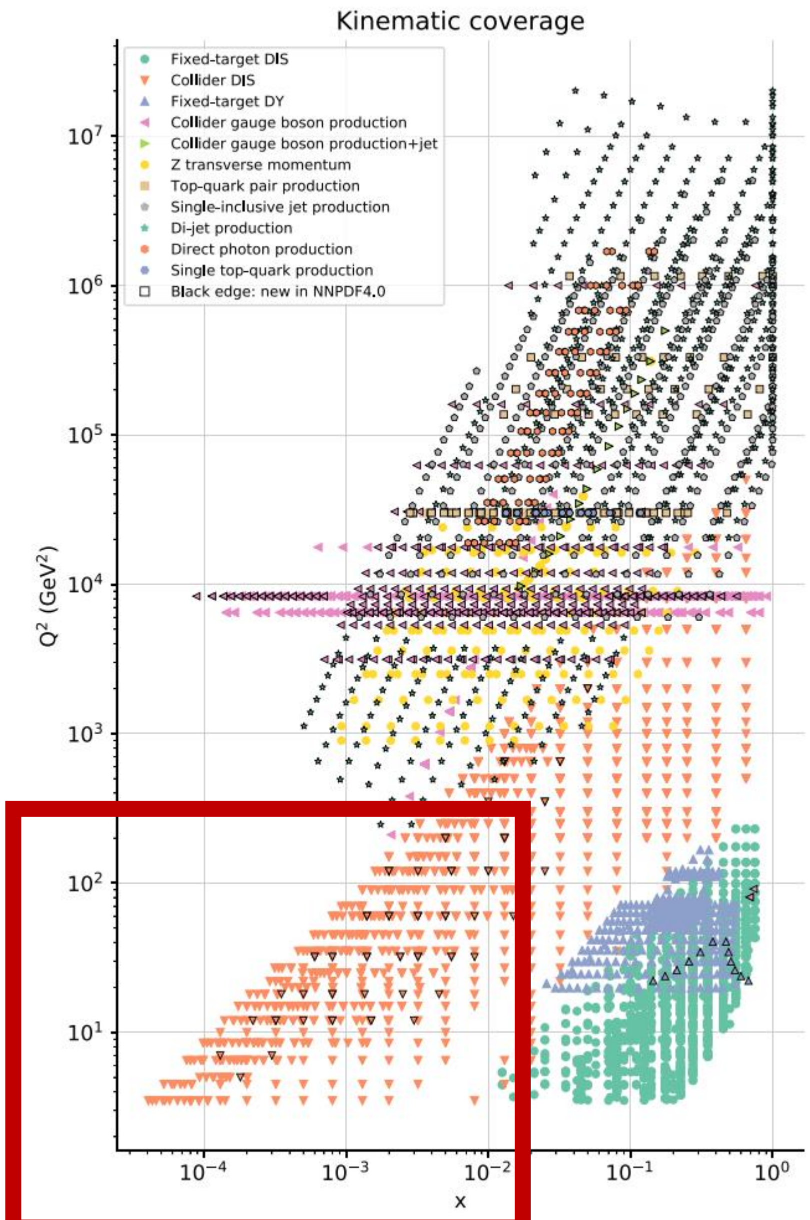

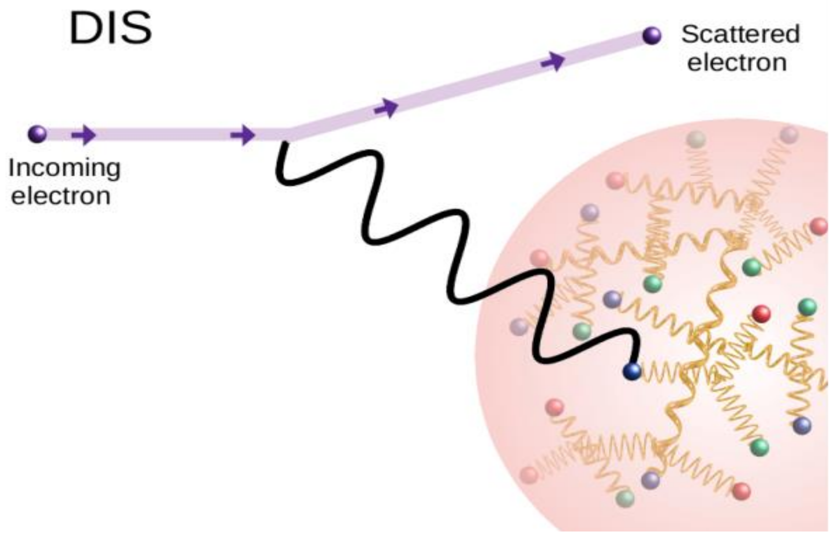

Q^2 GeV^2	x_{Bj}	$\sigma^+_{r,NC}$	δ_{stat} %	δ_{uncor} %	δ_{cor} %	δ_{rel} %	$\delta_{\gamma p}$ %	δ_{had} %	δ_1 %	δ_2 %	δ_3 %	δ_4 %	δ_{tot} %
15	0.200×10^{-3}	1.258	3.21	3.61	1.00	0.54	−0.11	−0.14	−0.01	−0.01	−0.48	−0.15	4.99
15	0.246×10^{-3}	1.360	1.17	1.52	1.43	0.76	−1.11	−0.46	−0.13	0.01	−0.39	−0.10	2.82
15	0.320×10^{-3}	1.290	0.94	1.36	1.00	0.68	−0.63	−0.22	−0.05	0.00	−0.19	−0.10	2.17
18	0.268×10^{-3}	1.318	3.26	3.67	0.99	0.54	−0.11	−0.17	−0.01	−0.01	−0.46	−0.15	5.06
18	0.328×10^{-3}	1.362	1.25	1.67	1.09	0.62	−0.67	−0.47	0.06	0.00	−0.18	−0.08	2.58
18	0.500×10^{-3}	1.266	0.93	1.01	0.95	0.71	−0.44	−0.29	−0.08	0.00	−0.14	−0.07	1.89
22	0.500×10^{-3}	1.335	2.57	1.22	1.18	1.36	−0.85	−0.43	−0.08	0.02	−0.07	0.04	3.50
27	0.335×10^{-3}	1.389	4.11	3.92	0.98	0.54	−0.12	−0.15	−0.01	−0.01	−0.45	−0.14	5.81
27	0.410×10^{-3}	1.405	1.16	2.10	1.15	0.63	−0.70	−0.40	−0.04	−0.01	−0.08	−0.11	2.85
27	0.500×10^{-3}	1.407	1.15	1.46	0.92	0.77	−0.59	−0.31	−0.02	0.00	−0.21	−0.07	2.32
27	0.800×10^{-3}	1.286	2.46	0.83	0.87	0.77	−0.48	−0.28	0.02	−0.02	0.02	0.04	2.89
35	0.574×10^{-3}	1.507	1.36	2.01	0.99	0.59	−0.47	−0.35	−0.03	−0.03	−0.19	−0.14	2.76
35	0.800×10^{-3}	1.399	0.95	0.81	0.91	0.95	−0.60	−0.25	−0.01	−0.01	−0.15	−0.17	1.94
45	0.800×10^{-3}	1.486	1.81	1.45	1.44	0.80	−1.20	−0.53	−0.06	−0.07	0.08	−0.09	3.14
45	0.130×10^{-2}	1.306	0.95	0.62	0.86	0.73	−0.03	0.05	0.00	−0.02	−0.07	−0.21	1.61

Minimization processes and Discussion of the QCD fit results

H1 and ZEUS Collaborations experimental data
(Eur.Phys.J.C 75 (2015) 12, 580)

Parametrized form of PDFs

Minimization processes and Discussion of the QCD fit results

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Parametrized form of
PDFs

QCDNUM

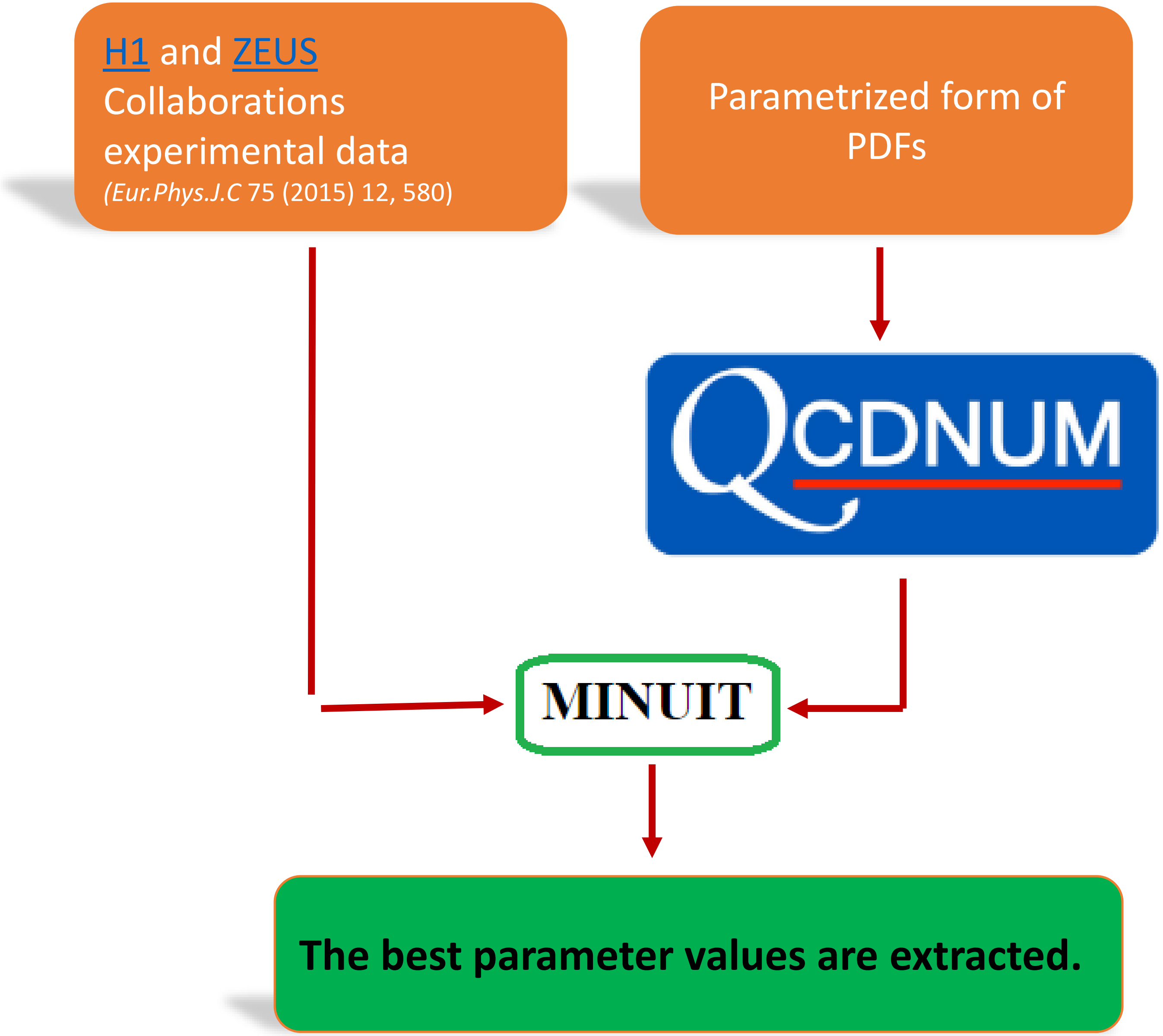
$$\frac{dq_i(x,Q^2)}{dlnQ^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}[\sum_j q_j(\xi,Q^2)P_{qq}^{(0)}(\frac{x}{\xi}) + g(\xi,Q^2)P_{qg}^{(0)}(\frac{x}{\xi})]$$

$$\frac{dg(x,Q^2)}{dlnQ^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}[\sum_j q_j(\xi,Q^2)P_{gq}^{(0)}(\frac{x}{\xi}) + g(\xi,Q^2)P_{gg}^{(0)}(\frac{x}{\xi})]$$

$$F_2(x,Q^2) = x\sum_{i=q,\bar{q},g}\int_x^1 \frac{d\xi}{\xi}C_2^i(\frac{x}{\xi},Q^2,\mu_f^2,\mu_r^2)f_i(\xi,\mu_f^2)$$

$$\sigma_{r,NC}^{\pm} = F_2(x,Q^2) - \frac{y}{Y_+}F_{\rm L}(x,Q^2)\ .$$

Minimization processes and Discussion of the QCD fit results



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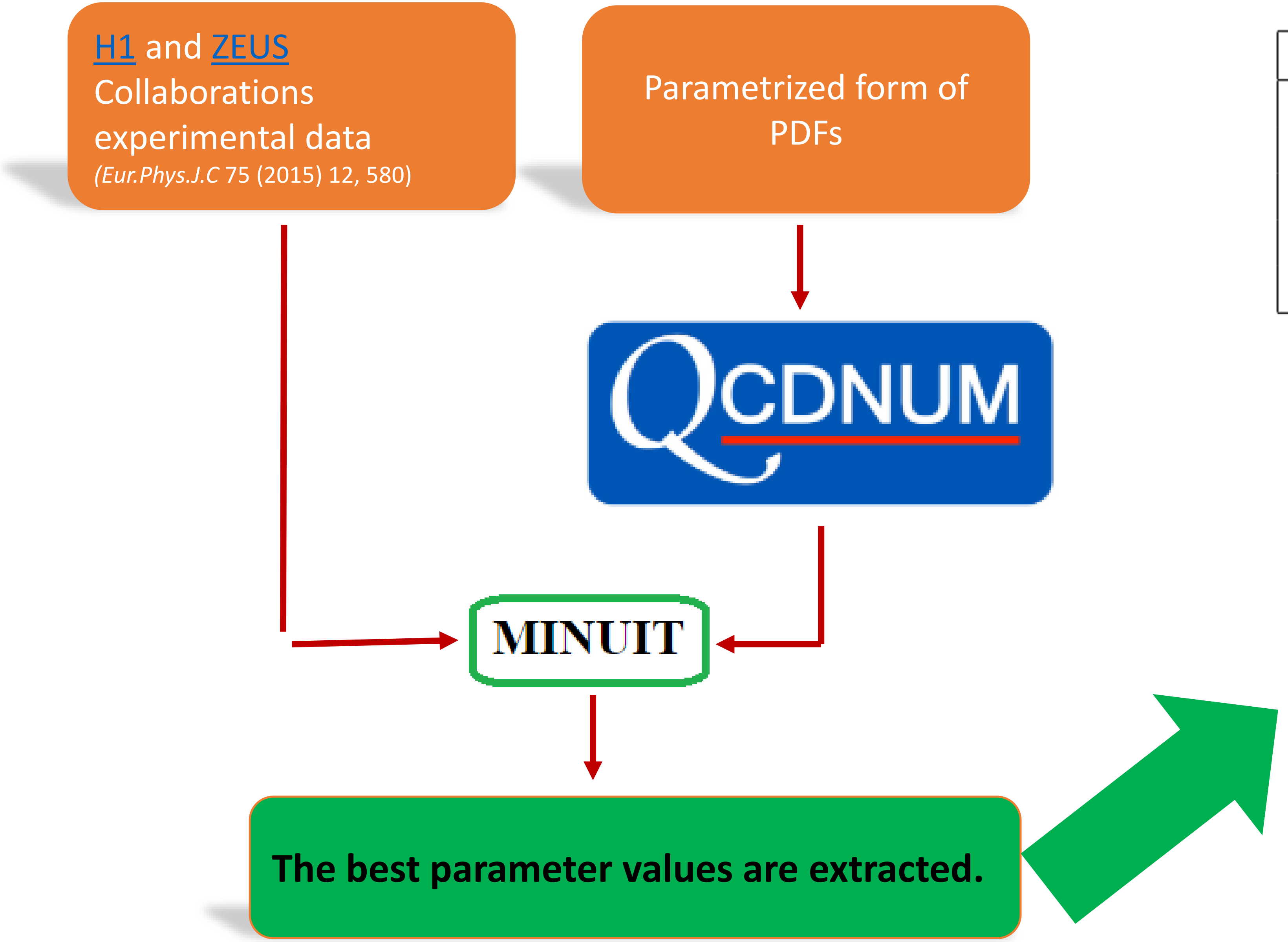

Parameters	$xf_{q/p}(x, Q_0^2)$
D_0^q	4.81 ± 0.01
q_0	0.0187 ± 0.0001
D_1^q	-0.0051 ± 0.0009
D_2^q	1.138 ± 0.002
D_3^q	-1.285 ± 0.005

Parameters	$xf_{g/p}(x, Q_0^2)$
A	0.191 ± 0.005
B^g	-3.01 ± 0.03
C^g	4^*
D^g	-3071.1 ± 80.6
σ	7.15 ± 6.18
μ	1.5^*

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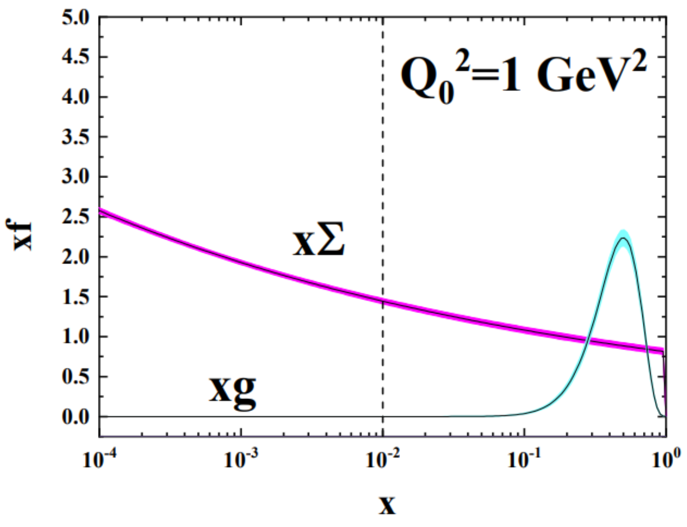


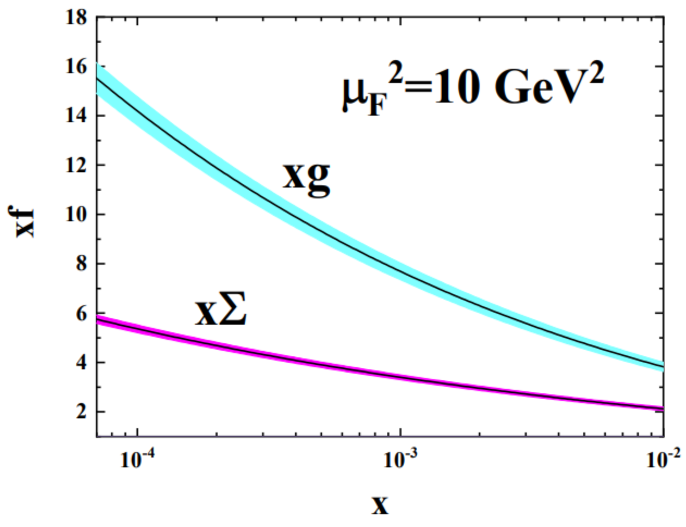
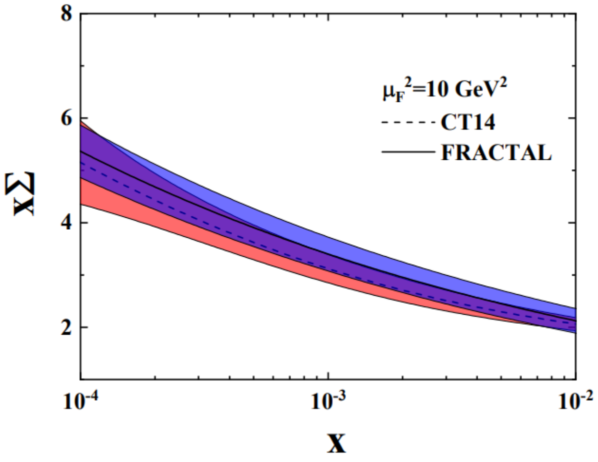


Fig. 3. (left) The parton distribution functions at $Q_0^2 = 1GeV^2$. (Right) The same but for $Q^2 = 10GeV^2$ and in the low x region bellow $x < 0.01$. Note that the fractal PDFs are valid only for $x < 0.01$.



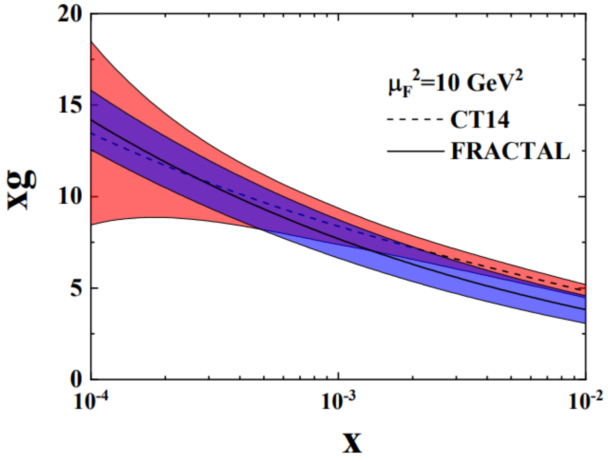


Fig. 5. The parton distribution functions with uncertainties band at low x in comparison with those from CT14 at $Q^2 = 10GeV^2$.

Minimization processes and Discussion of the QCD fit results

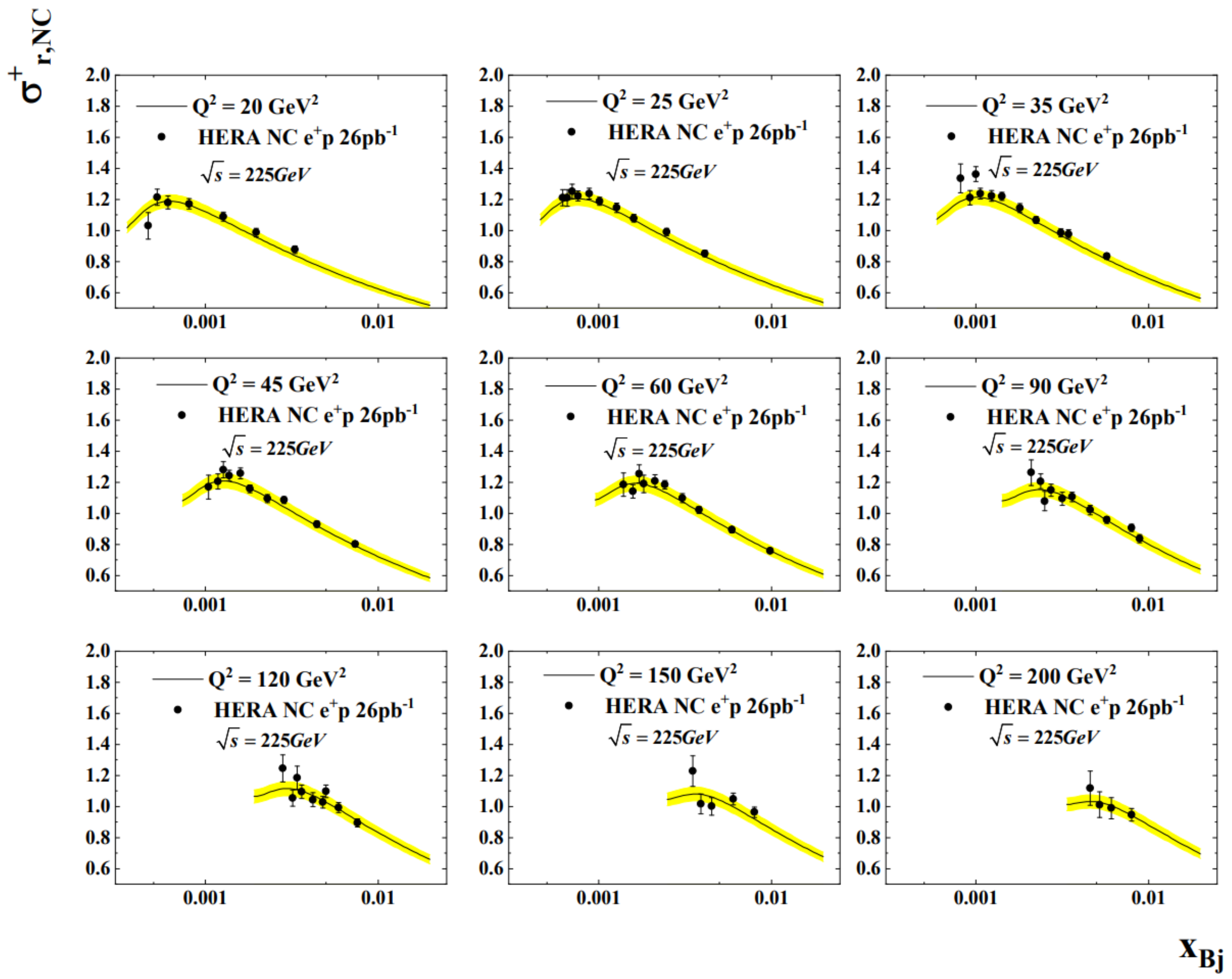


Fig. 6. The prediction for the reduced cross- section in the low x region bellow $x < 0.01$ for $Q^2 = 20GeV^2$ to $Q^2 = 200GeV^2$ and $\sqrt{s} = 225GeV$. Data points are from NC interactions in HERA positron- proton DIS processes.

Minimization processes and Discussion of the QCD fit results

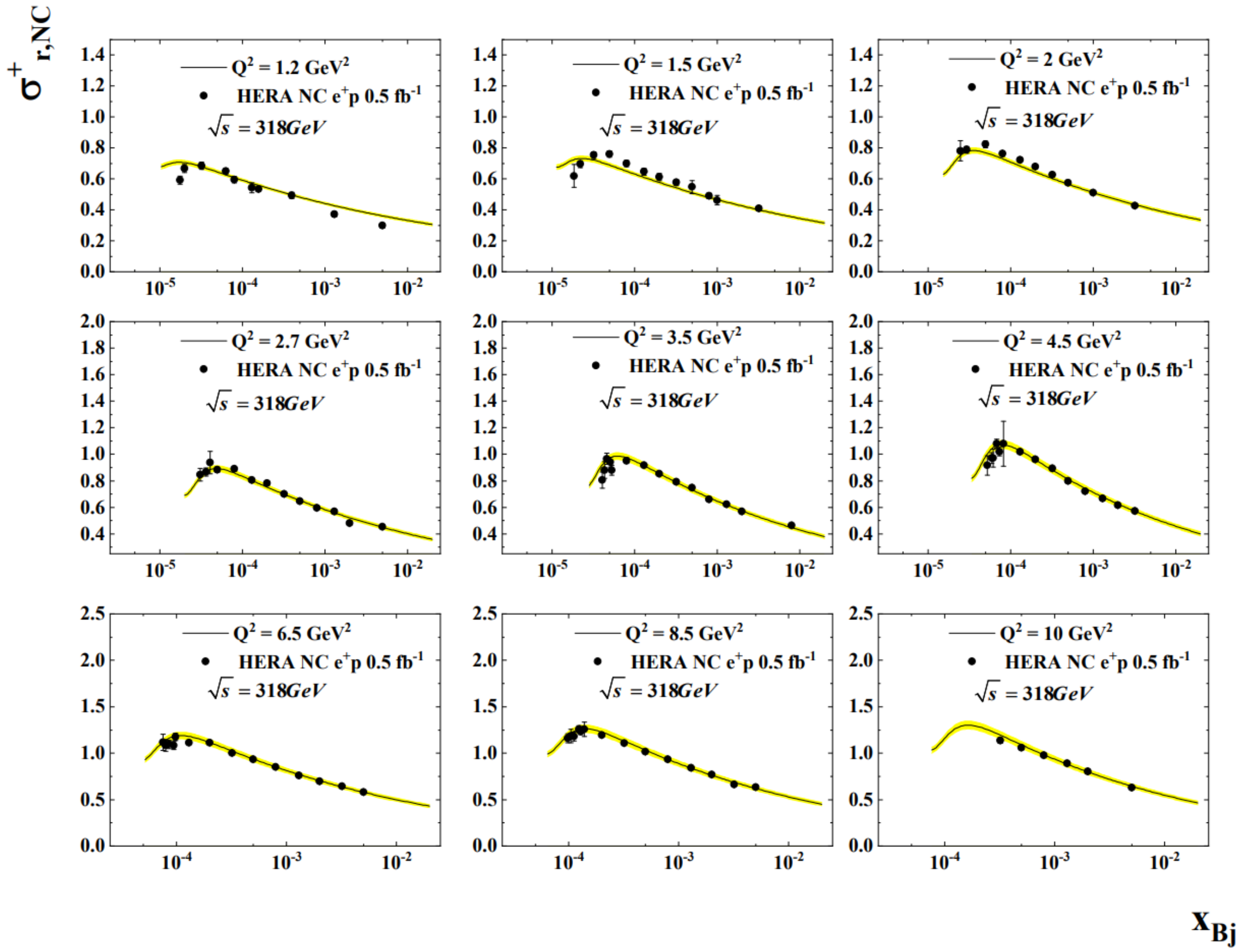


Fig. 7. The prediction for the reduced cross- section in the low x region bellow $x < 0.01$ for $Q^2 = 1.2GeV^2$ to $Q^2 = 10GeV^2$ and $\sqrt{s} = 318GeV$. Data points are from NC interactions in HERA positron- proton DIS processes.

Minimization processes and Discussion of the QCD fit results

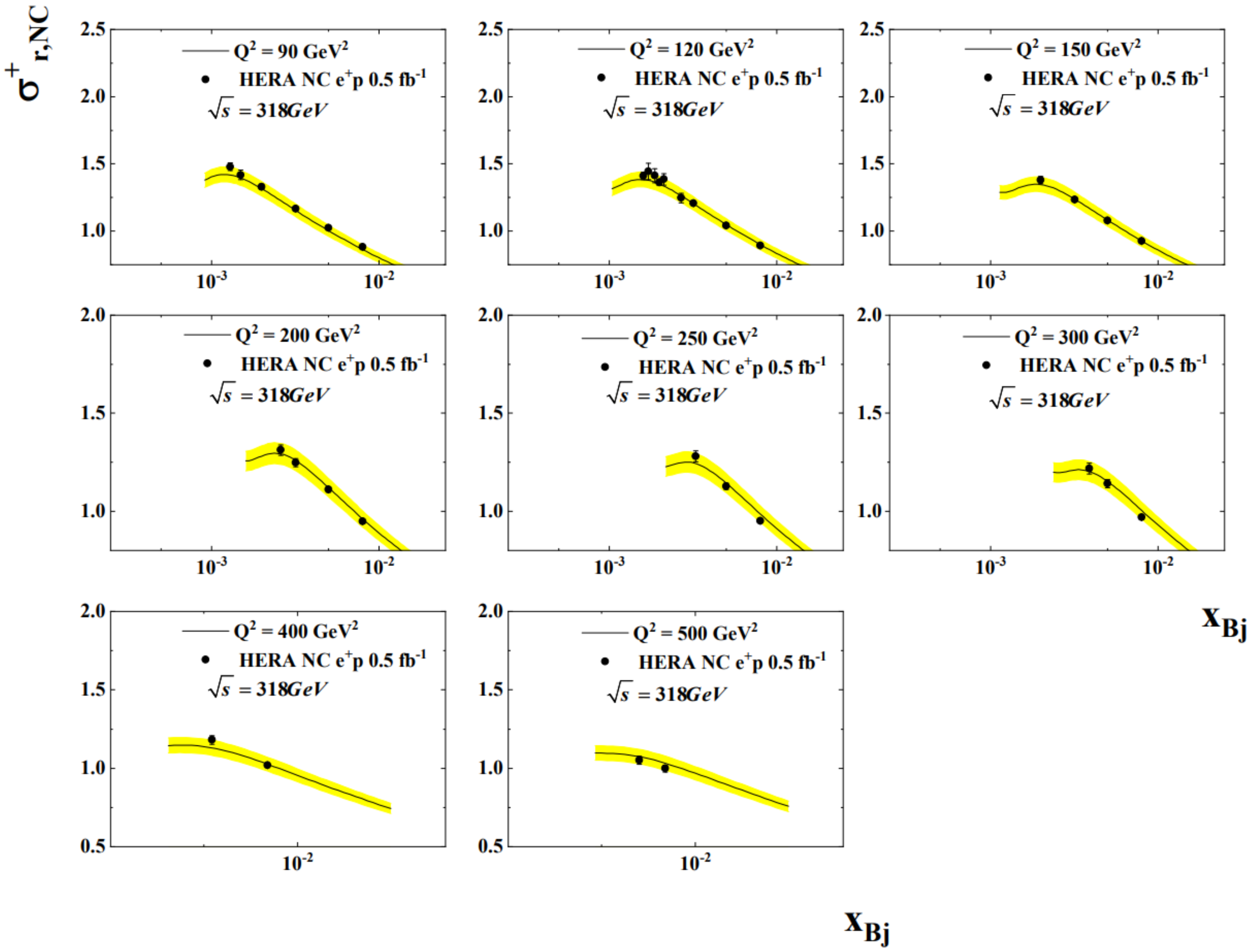


Fig. 9. The prediction for the reduced cross- section in the low x region bellow $x < 0.01$ for $Q^2 = 90GeV^2$ to $Q^2 = 500GeV^2$ and $\sqrt{s} = 318GeV$. Data points are from NC interactions in HERA positron- proton DIS processes.

Result

- **Fractal PDFs can describe low x physics well.**


Thank you!

Back up

$$\chi^2_{\text{global}}(\{\xi_i\}) = \sum_{m=1}^{m^{\exp}} w_m \, \chi^2_m(\{\xi_i\}) \, ,$$

$$\chi^2_m(\{\xi_i\}) = \left(\frac{1-\mathcal{N}_m}{\Delta\mathcal{N}_m}\right)^2 + \sum_{j=1}^{N_m^{\text{data}}} \left(\frac{(\mathcal{N}_m \, \mathcal{O}_j^{\text{data}} - \mathcal{T}_j^{\text{theory}}(\{\xi_i\})}{\mathcal{N}_m \, \Delta_j^{\text{data}}}\right)^2 .$$

$$\frac{\chi^2}{d.o.f} = 1.148$$