Learning from heavy-ion collisions about the ions' geometry and the way QCD flows

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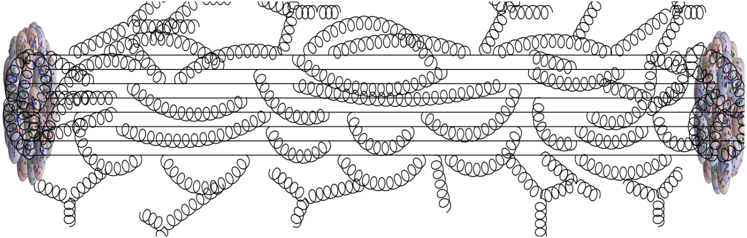
Weekly webinar, School of Particles and Accelerators
IPM, Tehran
23 November, 2022

From Lead-Lead to Proton-Proton at the LHC

Pb–Pb collision at the LHC, soft physics



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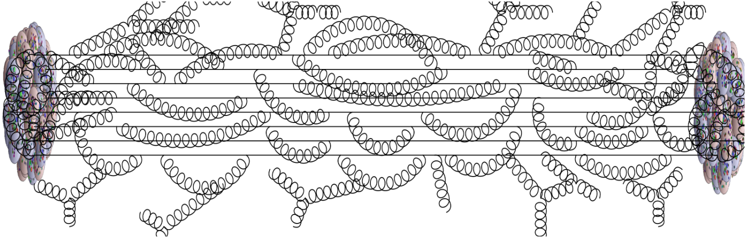

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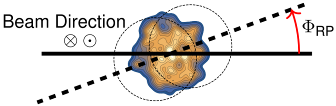
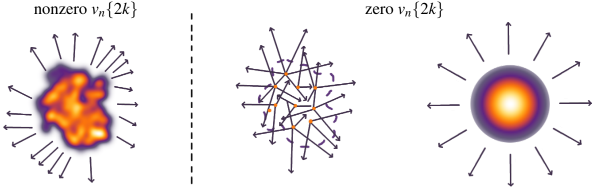
Color Glass Condensate **Causal Hydrodynamics** **Hadronic Gas Cascade**

Pb–Pb, U–U, Au–Au, Xe–Xe, Ru–Ru, Zr–Zr, Cu–Cu, O–O, Pb–p, **p–p**

Particle correlation, Collective evolution, Initial Geometry





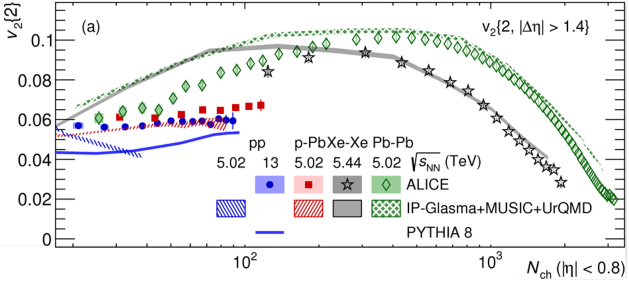
- Non-symmetric region
- Self-interacting medium

- Non-symmetric region
- No interaction

- Symmetric region
- Self-interacting medium

$$\langle e^{in\phi}\rangle \quad \rightarrow \quad \langle\langle e^{in(\phi_1-\phi_2)}\rangle\rangle$$

$$v_n\{2\} = \sqrt{\langle\langle e^{in(\phi_1-\phi_2)}\rangle\rangle}$$



Nucleon structure

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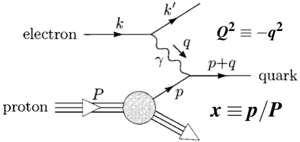

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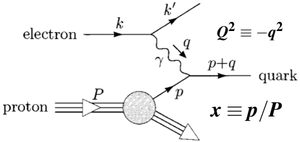

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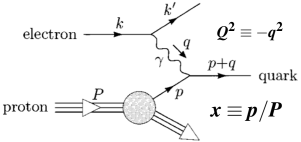

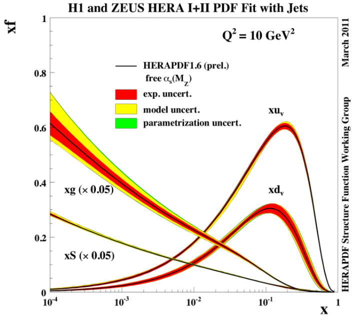

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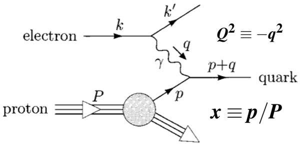

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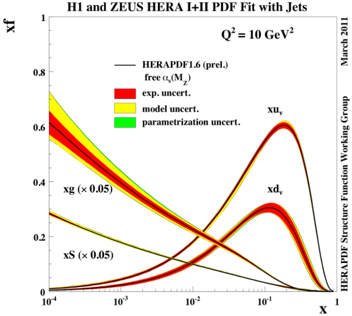

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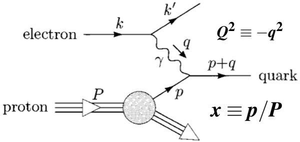
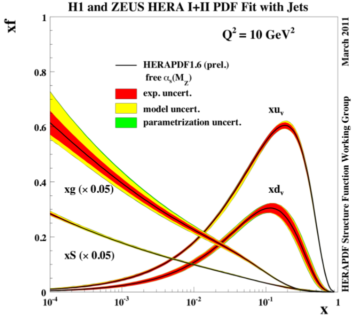
Perfect for high-x physics!

small-x sector is important in heavy-ion collision





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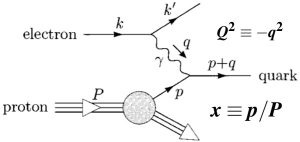

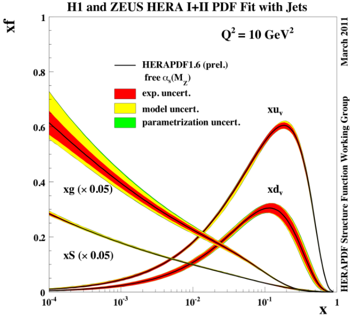

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At low-x, gluons are dominant.

Color Glass Condensate (CGC): a QCD effective theory

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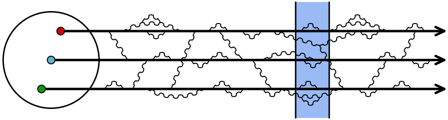

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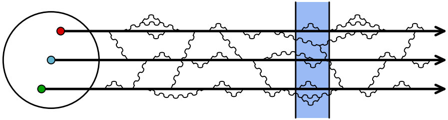

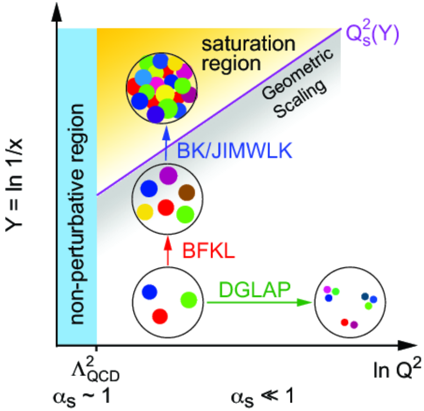

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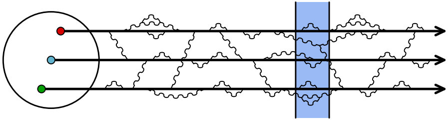
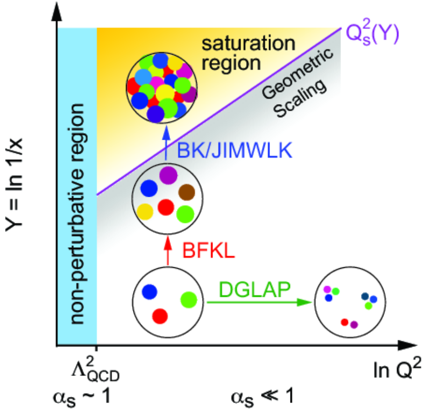
Low rapidity





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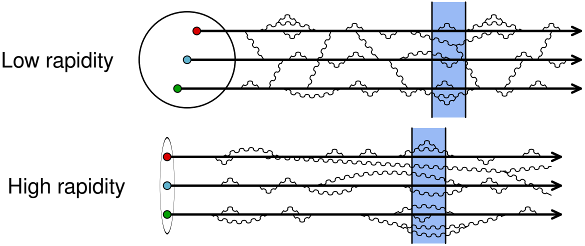

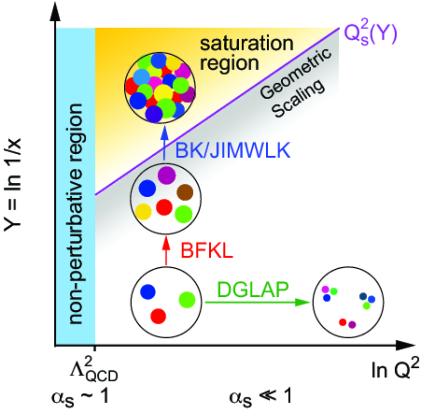

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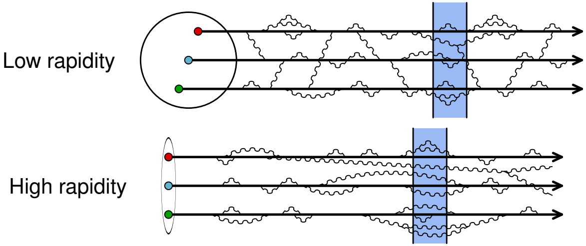

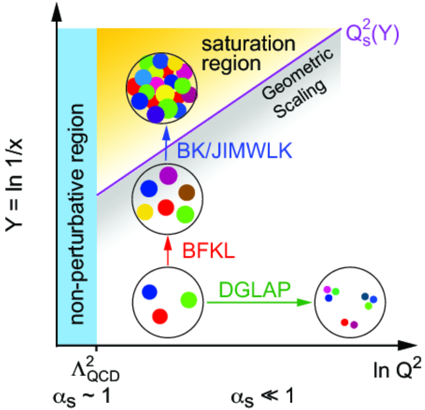

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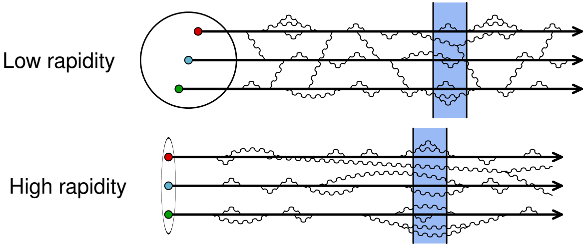

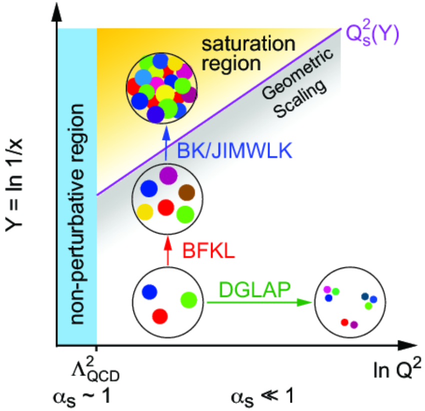

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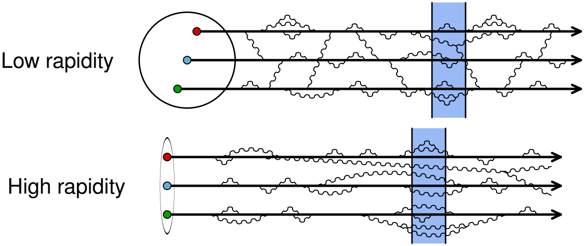

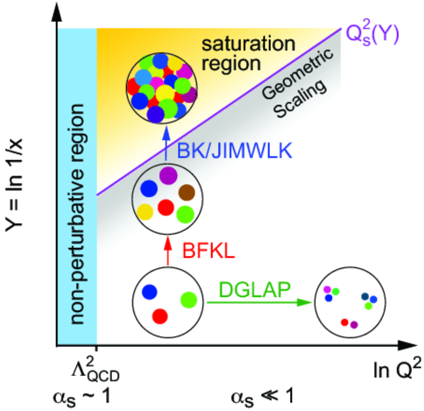

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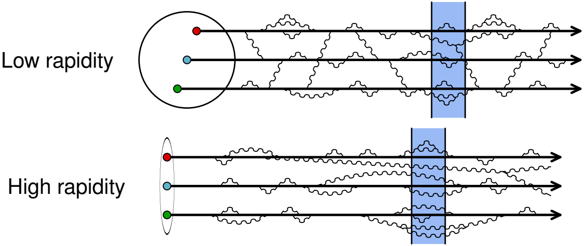

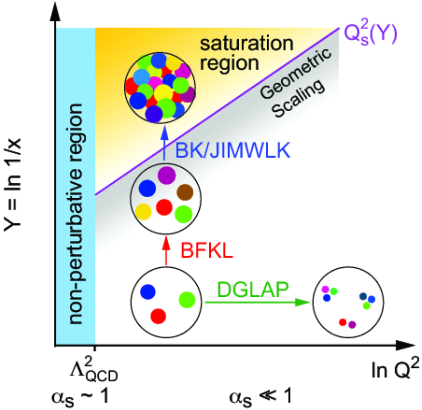

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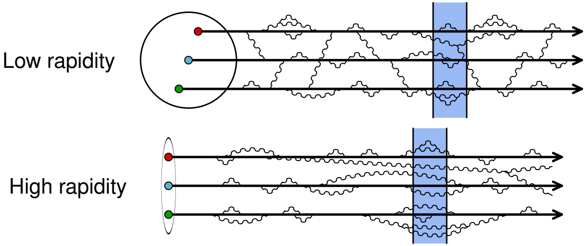

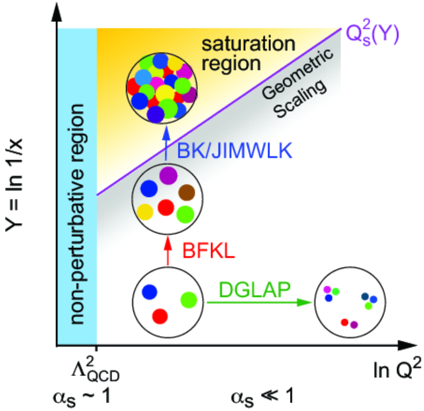

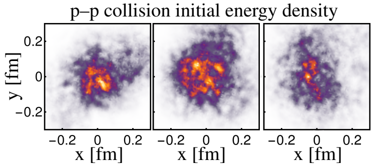

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MC-Glauber Vs IP-Glasma, (and T_RENTo)

[B. Schenke, Rept.Prog.Phys. 84 (2021) 8, 082301]

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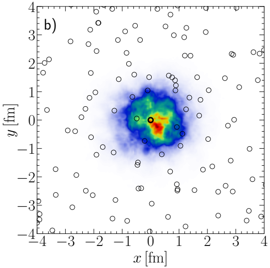
IP-Glasma

(round nucleons)
(gluons fluctuation inside nucleon)



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p-Pb collisions initial states

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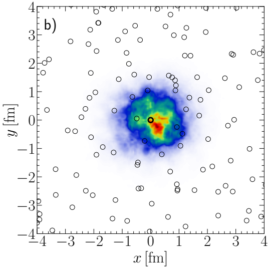

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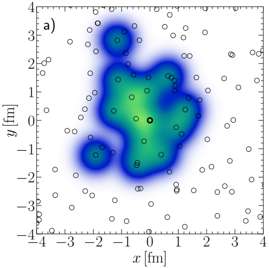

$T_p + T_{\text{wounded nucleon}}$

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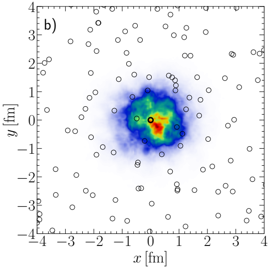

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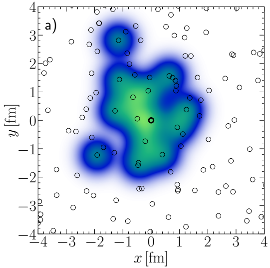

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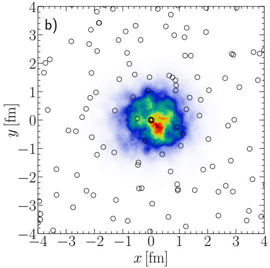

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- Thickness function $T_p = \int dz \rho(x,y,z)$. Here ρ is density of the nuclear matter.

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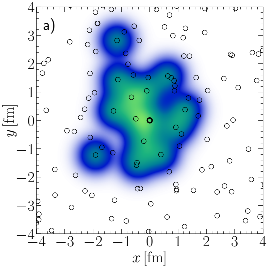

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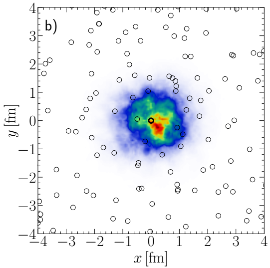
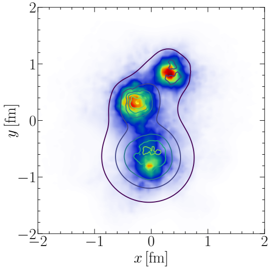


$T_p * T_{\text{wounded nucleon}}$

IP-Glasma

(nucleons substructure)
(quarks and gluon fluctuations inside p)



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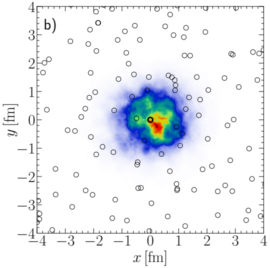

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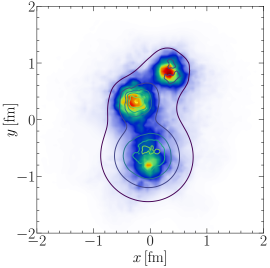

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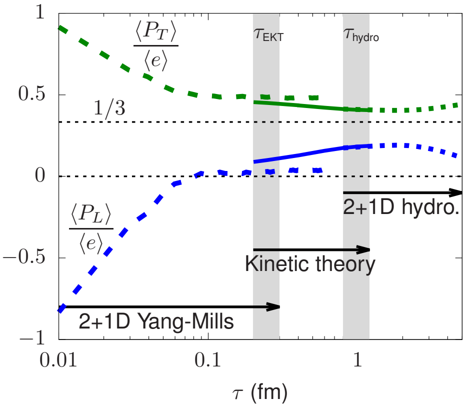
Pre-equilibrium, Hydrodynamics



- Color-Glass-Condensate ($\tau < \tau_{\mathrm{EKT}}$):

$$D_\mu F^{\mu\nu} = J^\nu$$

- Effective Kinetic Theory ($\tau_{\mathrm{EKT}} < \tau < \tau_{\mathrm{hydro}}$):

$$(\partial_t + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}}) f(\boldsymbol{x}, \boldsymbol{p}, t) = \mathscr{C}[f]$$

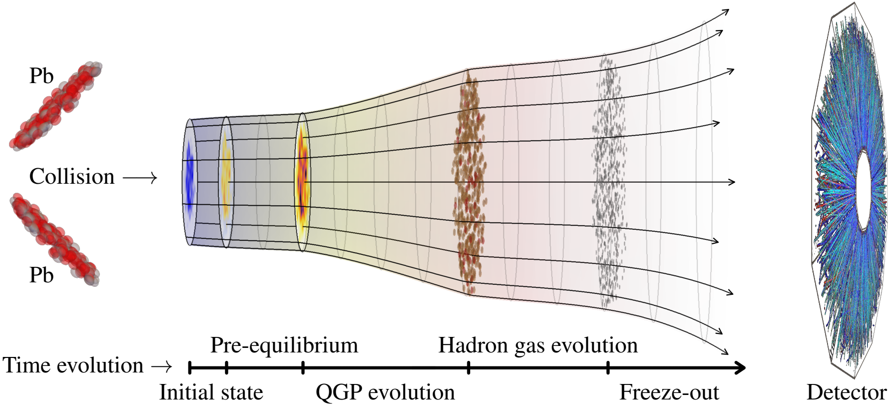
- Causal relativistic hydrodynamics ($\tau > \tau_{\mathrm{hydro}}$):

$$\partial_\mu T^{\mu\nu}(\boldsymbol{x}) = 0$$

$\varepsilon(\boldsymbol{x})$, $P(\boldsymbol{x})$, $u^\mu(\boldsymbol{x})$, EoS

$\eta/s(T)$, $\zeta/s(T)$, $\cdots$

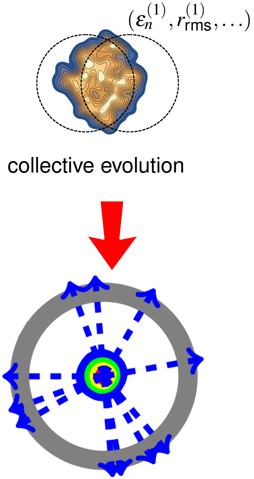
State-of-the-art heavy-ion collision models



Hybrid Model =	IP-Glasma T_RENTo MC-Glauber MC-KLN ⋮	⊗	$\mathbb{1}$ Free-streaming KøMPøST Gauge/Gravity ⋮	⊗	VISH2+1 MUSIC Trajectum VH2+1 ⋮	⊗	UrQMD SMASH B3D ⋮

[References in the backup slides]

Flow harmonics in a nutshell!

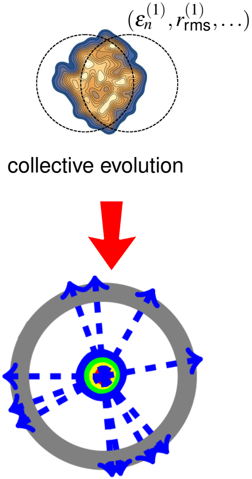


$(\varepsilon_n^{(1)}, r_{\mathrm{rms}}^{(1)}, \ldots)$

collective evolution

Flow harmonics in a nutshell!

$$\frac{d^2N}{p_T dp_T d\varphi} = N(p_T)\left[1+\sum_{n=1}^{\infty} 2\, v_n \cos\left[n(\varphi-\psi_n)\right]\right]$$



Flow harmonics, (v_n, ψ_n), depend on the initial state parameters, transport coefficients (η/s, $\zeta/s, \ldots$), …

Flow harmonics in a nutshell!

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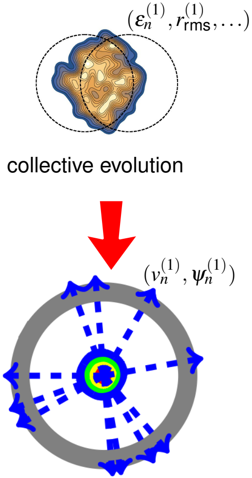

Flow harmonics in a nutshell!

$$\frac{d^2N}{p_T dp_T d\varphi} = N(p_T)\,[1+\sum_{n=1}^{\infty} 2\,v_n \cos[n(\varphi-\psi_n)]]$$

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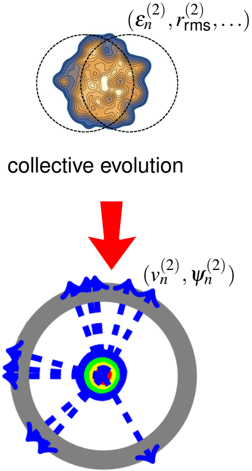

Flow harmonics in a nutshell!

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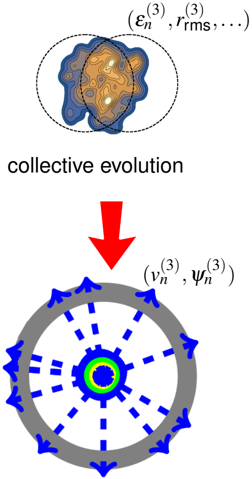
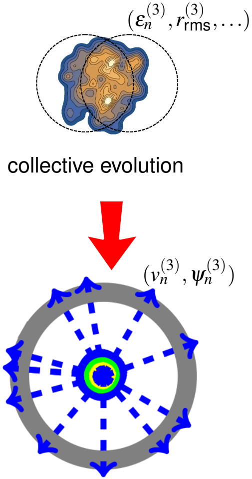


collective evolution

Flow harmonics in a nutshell!

$$\frac{d^2N}{p_T dp_T d\varphi} = N(p_T)\left[1+\sum_{n=1}^{\infty} 2\, v_n \cos\left[n(\varphi-\psi_n)\right]\right]$$

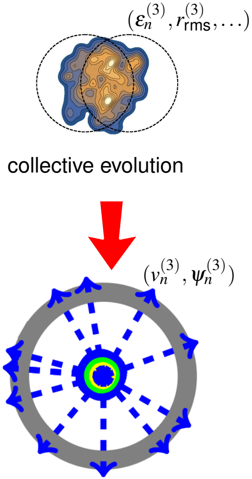


Flow harmonics, (v_n, ψ_n), depend on the initial state parameters, transport coefficients $(\eta/s,\ \zeta/s, \ldots)$, …

$$p([p_T], v_n, \psi_n, \ldots)$$

Flow harmonics in a nutshell!

$$\frac{d^2N}{p_T dp_T d\varphi} = N(p_T)\left[1 + \sum_{n=1}^{\infty} 2\, v_n \cos\left[n(\varphi - \psi_n)\right]\right]$$



Flow harmonics, (v_n, ψ_n), depend on the initial state parameters, transport coefficients (η/s, $\zeta/s, \ldots$), …

$$p([p_T], v_n, \psi_n, \ldots)$$

- Concentrate on a single harmonic flow amplitude $p(v_n)$,

$$v_n\{2\} \equiv \left(\langle v_n^2 \rangle\right)^{1/2}, \qquad v_n\{4\} \equiv \left(-\langle v_n^4 \rangle + 2\langle v_n^2 \rangle^2\right)^{1/4}, \qquad \cdots$$

[Borghini, Dinh, Ollitrault, PRC, 64, 054901 (2001)]

Theoretical models Vs experimental data

Initial state parameters

N($\sqrt{s_{\rm NN}}$)	Overall normalization
p	Entropy deposition parameter
w	Gaussian nucleon width

⋮

Pre-equilibrium parameters

$\tau_{\rm fs}$	Free-streaming time

⋮

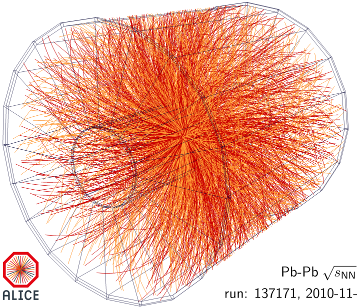
QGP evolution parameters

$\eta/s(T_c)$	Minimum $\eta/s(T)$
$(\eta/s)_{\rm slope}$	Slope of $\eta/s(T)$ above T_c
$(\eta/s)_{\rm curve}$	Curvature of $\eta/s(T)$ above T_c

⋮

Hadronic gas evolution parameters

⋮



Pb-Pb $\sqrt{s_{\rm NN}} = 2.76$ TeV
run: 137171, 2010-11-09 00:12:13

Experimental observables

dN/dy	Particle yields, $\pi^{\pm}$, $k^{\pm}$, ...
$\langle p_T \rangle$	Mean transverse momentum, $\pi^{\pm}$, $k^{\pm}$, ...
$v_n\{2\}$	Anisotropic flow two-particle correlation
$v_n\{4\}$	Anisotropic flow four-particle correlation

⋮

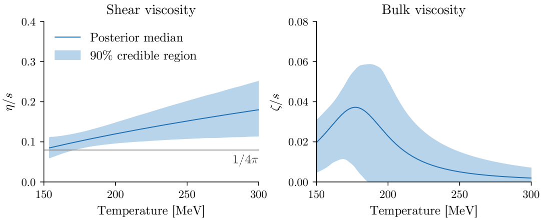
- What is the optimal value for the parameters to reproduce the experimental data, and how can we improve it?
- How much the models are applicable in small systems (Pb–Pb, Xe–Xe, . . . , O–O, . . . , Pb–p, p–p)?

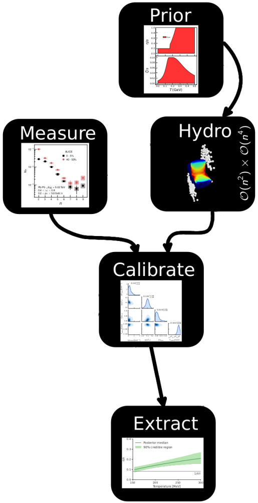
Model in the light of experimental data

- The initial the degree of belief to model parameters, *prior* distribution: $P(\text{theory})$
- Likelihood: $P(\text{Data}|\text{Theory})$
- Updated belief in the light of data. *posterior* distribution: $P(\text{Theory}|\text{Data})$.

Bayes's theorem: $P(\textit{Theory}|\textit{Data}) \propto P(\textit{Data}|\textit{Theory})P(\textit{Theory})$

Ref. [1]:




Prior

Measure

Hydro

$\mathcal{O}(n^2) \times \mathcal{O}(n^4)$

Calibrate

Extract

- Theoretical developments: collectivity **[2]**, jet-quenching **[3]**, nucleon substructure **[4]**

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[1] Bernhard, PhD Thesis, arXiv: 1804.06469; Bernhard, Moreland, Bass, Nature Phys. 15 (2019) 11, 1113-1117
[2] Auvinen, et al., PRC 102 (2020) 044911, Nijs et al., PRL 126 (2021) 202301, JETSCAPE, PRC 103 (2021) 054904
[3] JETSCAPE, PRC 104 (2021), 024905
[4] Mäntysaari, Schenke, Shen, Zhao, arXiv: 2202.01998

Many new measurements

Observable	
Single-harmonic observables [1, 2]	$v_2\{2\}, \ldots, v_7\{2\}$
Symmetric cumulants [3]	NSC$(2,3)$, NSC$(2,4)$, NSC$(3,4)$
Higher-order symmetric cumulants [4]	NSC$(2,3,4)$, NSC$(2,3,5)$
Symmetry plane correlations [2,5]	$\rho_{4,22}, \rho_{5,23}, \rho_{6,222}, \rho_{6,33}$
Non-linear mode couplings [2,6]	$\chi_{4,22}, \chi_{5,23}, \chi_{6,222}, \chi_{6,33}$
Symmetry plane correlation (GE) [7]	$\langle\cos(4\psi_2 - 4\psi_4)\rangle_{\text{GE}}, \ldots$
Asymmetric cumulants [8]	AC$_{2,1}(m,n), \ldots$

$$\text{NSC}(m,n) = \frac{\langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle}{\langle v_m^2\rangle\langle v_n^2\rangle}, \quad \chi_{4,22} = \frac{\langle v_2^2 v_4 \cos(4\psi_2 - 4\psi_4)\rangle}{\langle v_2^4\rangle}, \quad \ldots$$

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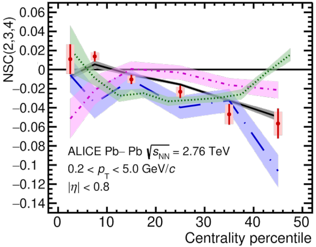
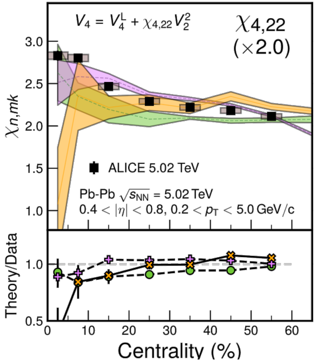
[1] Borghini, et al., PRC 64 (2001) 054901, ALICE Collaboration, PRL, 107 (2011) 032301, ALICE Collaboration, PRL, 116 (2016) 13, 132302.
[2] ALICE Collaboration, JHEP 05 (2020) 085, ALICE Collaboration, 773 (2017) 68-80.
[3] Bilandzic, et al., PRC 89 (6) (2014) 064904, ALICE Collaboration, PRC 97 (2018) 2, 024906, PRL, 117 (2016) 182301.
[4] Mordasini, et al., PRC 102 (2) (2020) 024907, ALICE Collaboration, PRL, 127 (2021) 9, 092302.
[5] Bhalerao, et al., PLB, 742 (2015) 94-98, Yan, et al. PLB, 744 (2015) 82-87.
[6] Qiu, et al., PLB, 717 (2012) 261-265, ATLAS Collaboration, PRC, 90 (2014) 2, 024905.
[7] A. Bilandzic, M. Lesch, SFT, PRC 102, 024910 (2020)
[8] A. Bilandzic, M. Lesch, C. Mordasini, SFT, PRC 105, 024912 (2022)

Ref. [4]



Ref. [2]

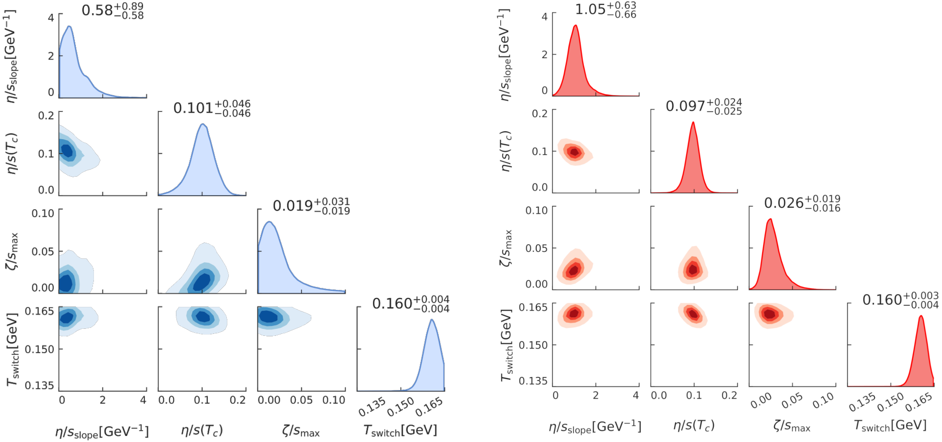


MAP parameters

Parameter	Description	Range	MAP
N(2.76 TeV)	Overall normalization (2.76 TeV)	[11.152, 18.960]	14.373
N(5.02 TeV)	Overall normalization (5.02 TeV)	[16.542, 25]	21.044
p	Entropy deposition parameter	[0.0042 , 0.0098]	0.0056
σ_k	Std. dev. of nucleon multiplicity fluctuations	[0.5518, 1.2852]	1.0468
d^3_{min}	Minimum volume per nucleon	$[0.889^3, 1.524^3]$	1.2367^3
τ_{fs}	Free-streaming time	[0.03, 1.5]	0.71
T_c	Temperature of const. $\eta/s(T)$, $T < T_c$	[0.135, 0.165]	0.141
$\eta/s(T_c)$	Minimum $\eta/s(T)$	[0, 0.2]	0.093
$(\eta/s)_{\mathrm{slope}}$	Slope of $\eta/s(T)$ above T_c	[0, 4]	0.8024
$(\eta/s)_{\mathrm{curve}}$	Curvature of $\eta/s(T)$ above T_c	$[-1.3, 1]$	0.1568
$(\zeta/s)_{\mathrm{peak}}$	Temperature of $\zeta/s(T)$ maximum	[0.15, 0.2]	0.1889
$(\zeta/s)_{\mathrm{max}}$	Maximum $\zeta/s(T)$	[0, 0.1]	0.01844
$(\zeta/s)_{\mathrm{width}}$	Width of $\zeta/s(T)$ peak	[0, 0.1]	0.04252
T_{switch}	Switching / particlization temperature	[0.135, 0.165]	0.1595

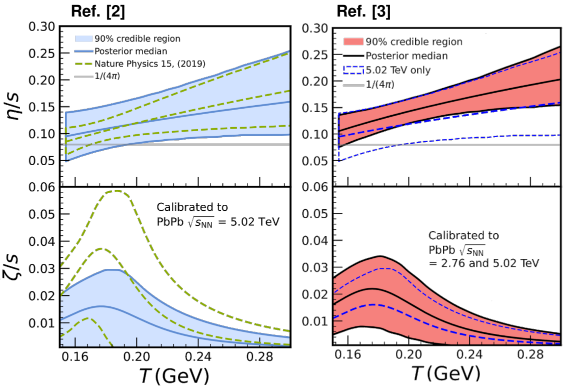
$$(\eta/s)(T) = (\eta/s)(T_c) + (\eta/s)_{\mathrm{slope}}(T - T_c)\left(\frac{T}{T_c}\right)^{(\eta/s)_{\mathrm{curve}}}, \qquad (\zeta/s)(T) = \frac{(\zeta/s)_{\mathrm{max}}}{1 + \left(\frac{T-(\zeta/s)_{T_{\mathrm{peak}}}}{(\zeta/s)_{\mathrm{width}}}\right)^2}.$$

Posterior distribution



Transport properties of QGP

$\mathrm{T_RENTo} \otimes$ Free-streaming $\otimes$ VISH2+1 $\otimes$ UrQMD



Significant improvement in uncertainties, especially in bulk viscosity.

	Ref. [1]	**Ref. [2] New!**	**Ref. [3] New!**
2.76 TeV	PID multi. N_{ch} PID $\langle p_T \rangle$ $\delta p_T / \langle p_T \rangle$ E_T $v_2, \ldots, v_4$		N_{ch} PID $\langle p_T \rangle$ $v_2, \ldots, v_4$ NSC$(3,2)$, NSC$(4,3)$ NSC$(2,3,4)$, NSC$(2,3,5)$ $\rho_{4,22}$ to $\rho_{6,mk}$ $\chi_{4,22}$ to $\chi_{6,mk}$
5.02 TeV	N_{ch} $v_2, \ldots, v_4$	PID multi. N_{ch} PID $\langle p_T \rangle$ $v_2, \ldots, v_4$ $v_5, \ldots, v_7$ NSC$(3,2)$ to NSC$(4,3)$ $\chi_{4,22}$ to $\chi_{6,mk}$	PID multi. N_{ch} PID $\langle p_T \rangle$ $v_2, \ldots, v_4$ $v_5, \ldots, v_7$ NSC$(3,2)$ to NSC$(4,3)$ $\chi_{4,22}$ to $\chi_{6,mk}$ $\rho_{4,22}$ to $\rho_{6,mk}$

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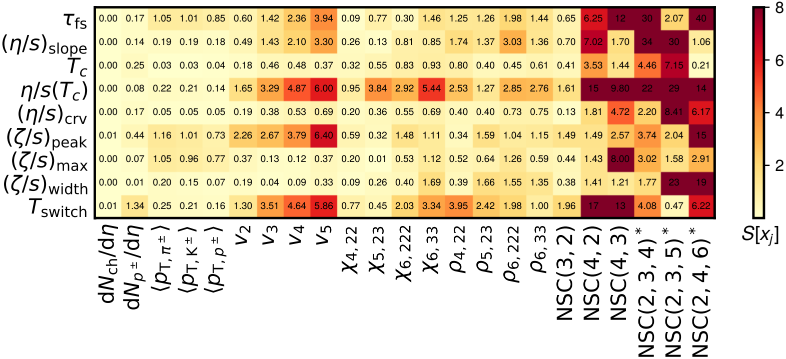
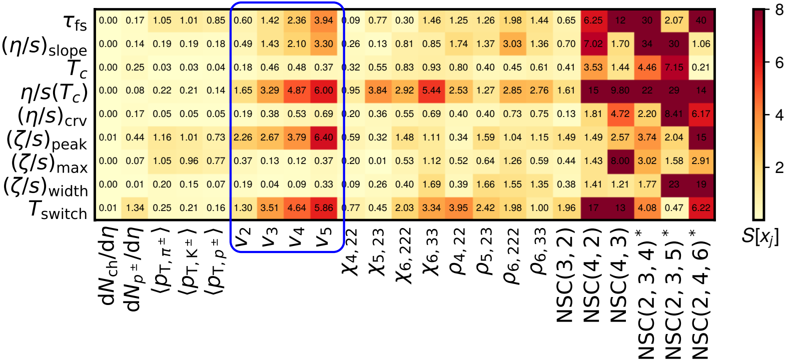
[1] Bernhard, et al., Nature Phys. 15 (2019) 11, 1113-1117.
[2] J.E. Parkkila, A. Onnerstad, D.J. Kim, Phys.Rev.C 104 (2021) 5, 054904.
[3] J.E. Parkkila, A. Onnerstad, SFT, C. Mordasini, A. Bilandzic, D.J. Kim, arXiv: 2111.08145.

Sensitivity index $S_{\hat{O}}[x_j] = \frac{1}{\delta}\frac{|\hat{O}(\vec{x}') - \hat{O}(\vec{x})|}{\hat{O}(\vec{x})}$ of the input parameters



	$dN_{ch}/d\eta$	$dN_{p^\pm}/d\eta$	$\langle p_{T,\pi^\pm}\rangle$	$\langle p_{T,K^\pm}\rangle$	$\langle p_{T,p^\pm}\rangle$	v_2	v_3	v_4	v_5	$\chi_{4,22}$	$\chi_{5,23}$	$\chi_{6,222}$	$\chi_{6,33}$	$\rho_{4,22}$	$\rho_{5,23}$	$\rho_{6,222}$	$\rho_{6,33}$	NSC(3, 2)	NSC(4, 2)	NSC(4, 3)	NSC(2, 3, 4)*	NSC(2, 3, 5)*	NSC(2, 4, 6)*
τ_{fs}	0.00	0.17	1.05	1.01	0.85	0.60	1.42	2.36	3.94	0.09	0.77	0.30	1.46	1.25	1.26	1.98	1.44	0.65	6.25	12	30	2.07	40
$(\eta/s)_{slope}$	0.00	0.14	0.19	0.19	0.18	0.49	1.43	2.10	3.30	0.26	0.13	0.81	0.85	1.74	1.37	3.03	1.36	0.70	7.02	1.70	34	30	1.06
T_c	0.00	0.25	0.03	0.03	0.04	0.18	0.46	0.48	0.37	0.32	0.55	0.83	0.93	0.80	0.40	0.45	0.61	0.41	3.53	1.44	4.46	7.15	0.21
$\eta/s(T_c)$	0.00	0.08	0.22	0.21	0.14	1.65	3.29	4.87	6.00	0.95	3.84	2.92	5.44	2.53	1.27	2.85	2.76	1.61	15	9.80	22	29	14
$(\eta/s)_{crv}$	0.00	0.17	0.05	0.05	0.05	0.19	0.38	0.53	0.69	0.20	0.36	0.55	0.69	0.40	0.40	0.73	0.75	0.13	1.81	4.72	2.20	8.41	6.17
$(\zeta/s)_{peak}$	0.01	0.44	1.16	1.01	0.73	2.26	2.67	3.79	6.40	0.59	0.32	1.48	1.11	0.34	1.59	1.04	1.15	1.49	1.49	2.57	3.74	2.04	15
$(\zeta/s)_{max}$	0.00	0.07	1.05	0.96	0.77	0.37	0.13	0.12	0.37	0.20	0.01	0.53	1.12	0.52	0.64	1.26	0.59	0.44	1.43	8.00	3.02	1.58	2.91
$(\zeta/s)_{width}$	0.00	0.01	0.20	0.15	0.07	0.19	0.04	0.09	0.33	0.09	0.26	0.40	1.69	0.39	1.66	1.55	1.35	0.38	1.41	1.21	1.77	23	19
T_{switch}	0.01	1.34	0.25	0.21	0.16	1.30	3.51	4.64	5.86	0.77	0.45	2.03	3.34	3.95	2.42	1.98	1.00	1.96	17	13	4.08	0.47	6.22

Sensitivity index $S_{\hat{O}}[x_j] = \frac{1}{\delta}\frac{|\hat{O}(\vec{x}') - \hat{O}(\vec{x})|}{\hat{O}(\vec{x})}$ of the input parameters



	$dN_{ch}/d\eta$	$dN_{p^\pm}/d\eta$	$\langle p_{T,\pi^\pm}\rangle$	$\langle p_{T,K^\pm}\rangle$	$\langle p_{T,p^\pm}\rangle$	v_2	v_3	v_4	v_5	$\chi_{4,22}$	$\chi_{5,23}$	$\chi_{6,222}$	$\chi_{6,33}$	$\rho_{4,22}$	$\rho_{5,23}$	$\rho_{6,222}$	$\rho_{6,33}$	NSC(3, 2)	NSC(4, 2)	NSC(4, 3)	NSC(2, 3, 4)*	NSC(2, 3, 5)*	NSC(2, 4, 6)*
τ_{fs}	0.00	0.17	1.05	1.01	0.85	0.60	1.42	2.36	3.94	0.09	0.77	0.30	1.46	1.25	1.26	1.98	1.44	0.65	6.25	12	30	2.07	40
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T_c	0.00	0.25	0.03	0.03	0.04	0.18	0.46	0.48	0.37	0.32	0.55	0.83	0.93	0.80	0.40	0.45	0.61	0.41	3.53	1.44	4.46	7.15	0.21
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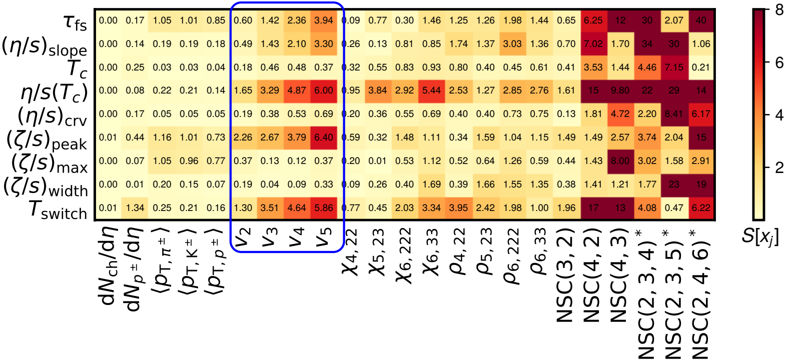

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$\eta/s(T_c)$	0.00	0.08	0.22	0.21	0.14	1.65	3.29	4.87	6.00	0.95	3.84	2.92	5.44	2.53	1.27	2.85	2.76	1.61	15	9.80	22	29	14
$(\eta/s)_{crv}$	0.00	0.17	0.05	0.05	0.05	0.19	0.38	0.53	0.69	0.20	0.36	0.55	0.69	0.40	0.40	0.73	0.75	0.13	1.81	4.72	2.20	8.41	6.17
$(\zeta/s)_{peak}$	0.01	0.44	1.16	1.01	0.73	2.26	2.67	3.79	6.40	0.59	0.32	1.48	1.11	0.34	1.59	1.04	1.15	1.49	1.49	2.57	3.74	2.04	15
$(\zeta/s)_{max}$	0.00	0.07	1.05	0.96	0.77	0.37	0.13	0.12	0.37	0.20	0.01	0.53	1.12	0.52	0.64	1.26	0.59	0.44	1.43	8.00	3.02	1.58	2.91
$(\zeta/s)_{width}$	0.00	0.01	0.20	0.15	0.07	0.19	0.04	0.09	0.33	0.09	0.26	0.40	1.69	0.39	1.66	1.55	1.35	0.38	1.41	1.21	1.77	23	19
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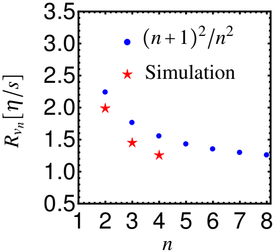

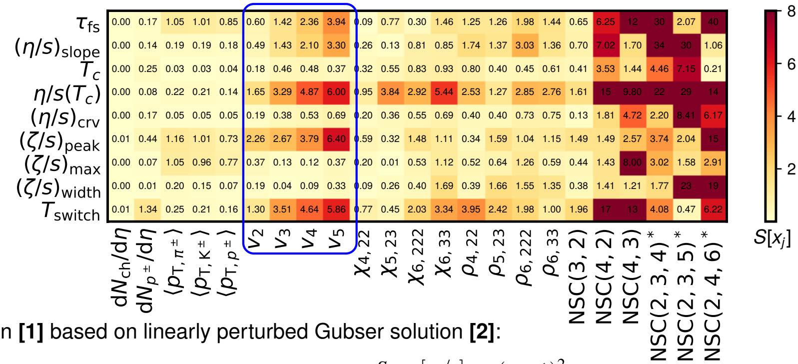

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τ_{fs}	0.00	0.17	1.05	1.01	0.85	0.60	1.42	2.36	3.94	0.09	0.77	0.30	1.46	1.25	1.26	1.98	1.44	0.65	6.25	12	30	2.07	40
$(\eta/s)_{\mathrm{slope}}$	0.00	0.14	0.19	0.19	0.18	0.49	1.43	2.10	3.30	0.26	0.13	0.81	0.85	1.74	1.37	3.03	1.36	0.70	7.02	1.70	34	30	1.06
T_c	0.00	0.25	0.03	0.03	0.04	0.18	0.46	0.48	0.37	0.32	0.55	0.83	0.93	0.80	0.40	0.45	0.61	0.41	3.53	1.44	4.46	7.15	0.21
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what about $R_n[\zeta/s]$?

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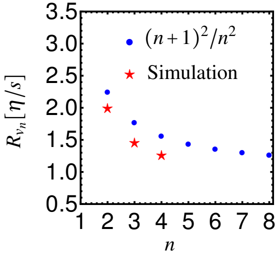

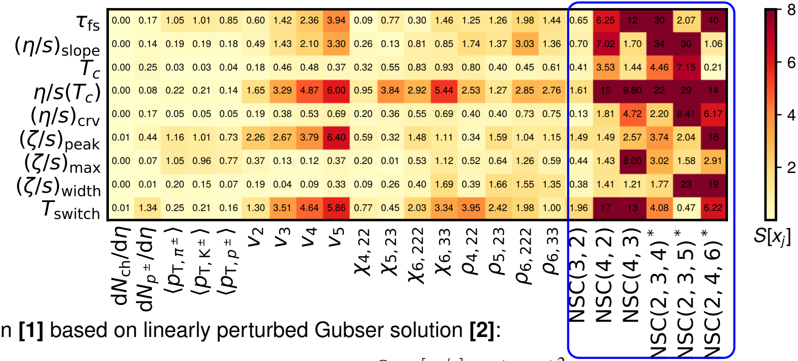

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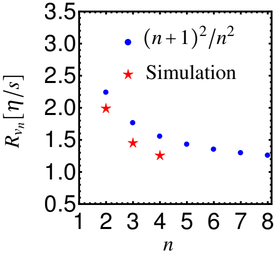

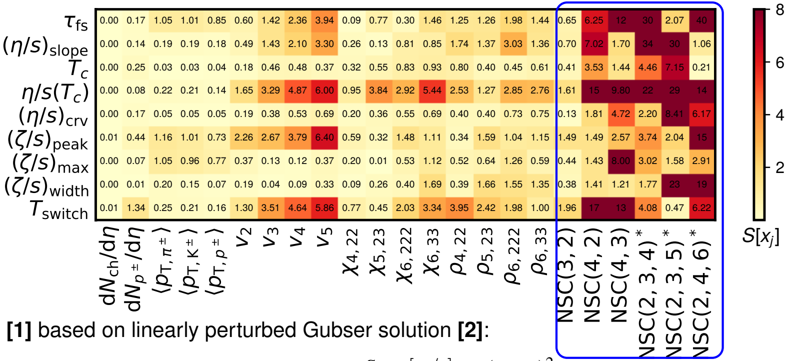

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what about $R_n[\zeta/s]$?

- Sensitivity of normalized symmetric cumulants:

$$\text{NSC}(m,n) = \frac{\langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle}{\langle v_m^2\rangle\langle v_n^2\rangle}, \quad \to \quad S_{\text{NSC}(m,n)}[\eta/s] \sim \mathbf{0} + \textbf{non-linear!?}$$

---

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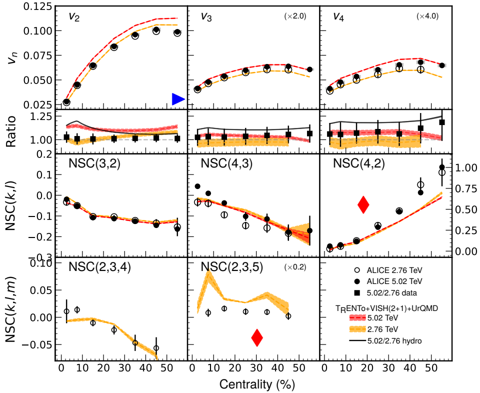
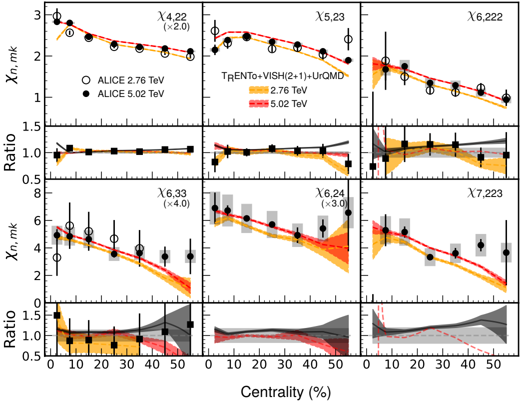
[2] Gubser, Yarom, Nucl.Phys.B 846 (2011) 469-511

Maximum A Posteriori parametrization

Overall agreement, with only few discrepancies





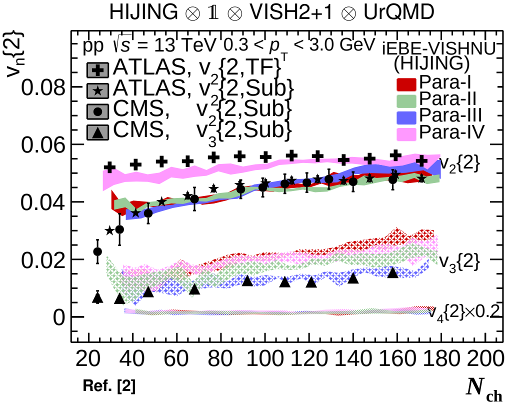
- ► The energy dependence of v_2.
- ♦ Deviation from simulation and data in NSC$(2,4)$ and NSC$(2,3,5)$.
  More plots in **Refs. [1,2]**.

---

[1] J.E. Parkkila, A. Onnerstad, D.J. Kim, Phys.Rev.C 104 (2021) 5, 054904.

[2] J.E. Parkkila, A. Onnerstad, SFT, C. Mordasini, A. Bilandzic, D.J. Kim, arXiv: 2111.08145.

Pushing the model's limit to its extreme!



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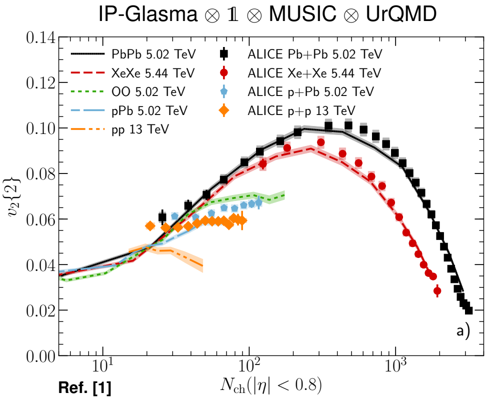

Ref. [1]

- Pre-equilibrium dynamics is missing in these models.
- The model predictions are worsen at lower multiplicities.
- Removing the non-flow effects are challenging, especially at low multiplicities.

---

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[2] Zhao, Zhou, Xu, Deng, Song, PLB 780 (2018) 495-500

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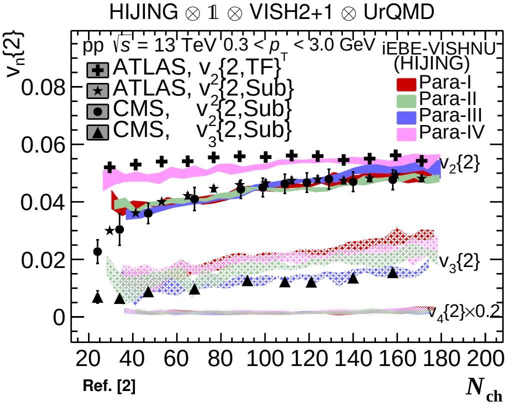

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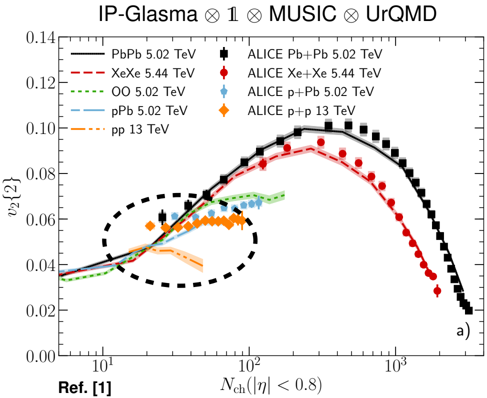

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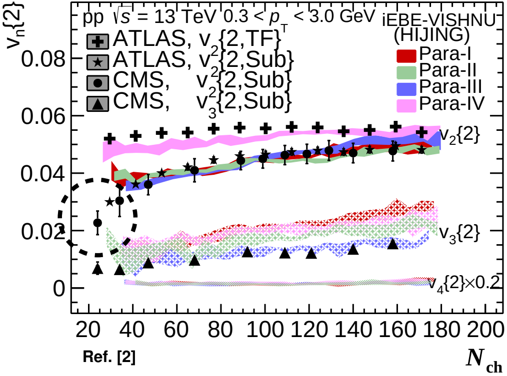
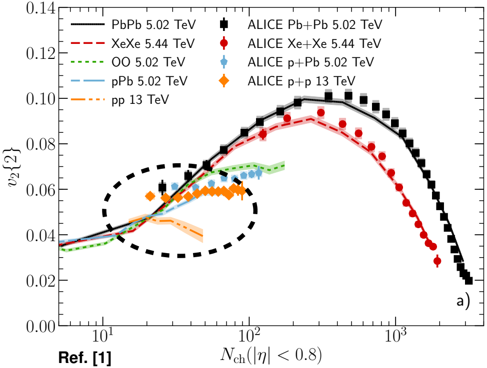
Pushing the model's limit to its extreme!

HIJING ⊗ 𝟙 ⊗ VISH2+1 ⊗ UrQMD



Ref. [2]

IP-Glasma ⊗ 𝟙 ⊗ MUSIC ⊗ UrQMD



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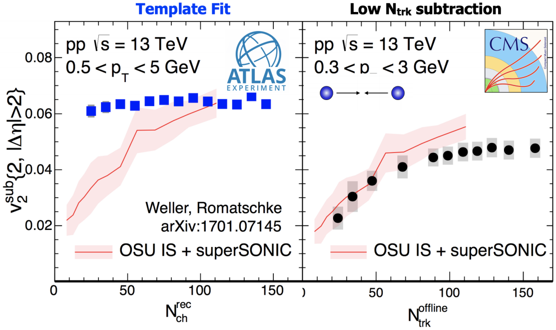
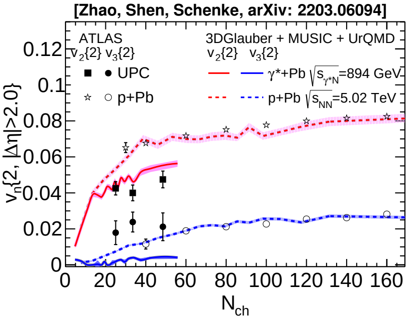
What is happening in small systems / low multiplicities?





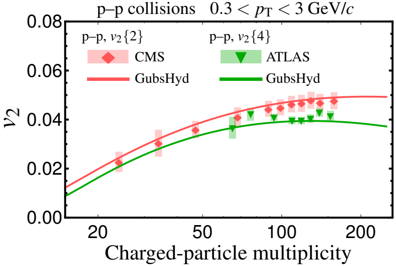
- In superSOINUC: as a pre-equilibrium stage $T^{\mu\nu}_{\text{AdS/CFT}}$ is matched to $T^{\mu\nu}_{\text{hydro}}$ at τ_{hydro}.
- In 3DGlauber⊗MUSIC⊗UrQMD: A 3D dynamical initial state is considered.

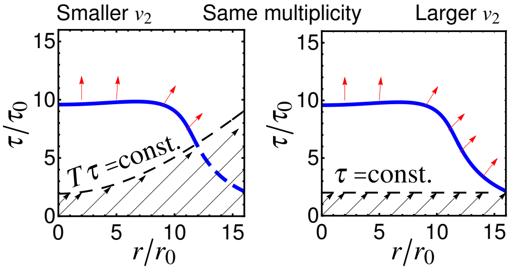
- IP-Glasma $\otimes$ $\mathbb{1}$ $\otimes$ MUSIC $\otimes$ UrQMD
- HIJING $\otimes$ $\mathbb{1}$ $\otimes$ VISH2+1 $\otimes$ UrQMD
- T$_R$ENTo$\otimes$ Free-streaming $\otimes$ VISH2+1 $\otimes$ UrQMD

- MC-Glauber (OSU) $\otimes$ Gage/Gravity $\otimes$ VH2+1 $\otimes$ B3D
- 3DGlauber⊗Dynamical IS⊗MUSIC⊗UrQMD

Small system and GubsHyd

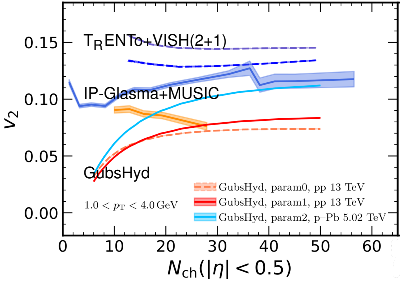
[SFT, PRC 102 (2020) 024910]





In GubsHyd:

- The hydrodynamic evolution starts at $T\tau =$const. Evidences:
  - Attractors in Gubser flow using Kin. Theory, $w(T\tau) = w_0$ **[1]**
  - Non-hydrodynamic modes decay time: $e^{-z_0 T\tau}$ **[2]**
  - “Inhomogeneous longitudinal cooling”!? **[3]**
- Cold corona region contributes to the multiplicity.
- Compared to $\tau =$const., less evolution time → smaller v_n.



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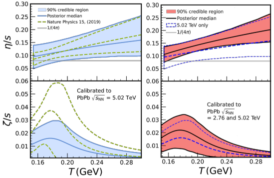
[1] Behtash, Cruz-Camacho, Martinez, PRD 97 (2018) 044041
[2] Heller, Janik, Witaszczyk, PRL 108 (2013) 211602
[3] Ambrus, Schlichting, Werthmann, PRD 105 (2022) 014031

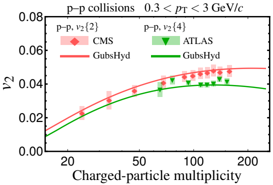
Summary

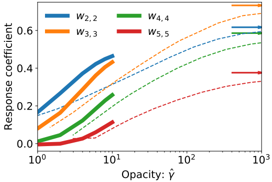
- Importance of observables to understand the models. We need to choose cleverly!
- Improvement of the transport coefficient uncertainties.
- Including small system information into a Bayesian analysis needs a careful consideration.
- The pre-equilibrium dynamics could have a substantial influence in small system collisions.

Outlook

- Observables sensitive to initial state: isobar ratio
- Collective models with dynamical pre-equilibrium
- Framework beyond hydrodynamics in an event-by-event basis







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Thank You!

Backup Slides

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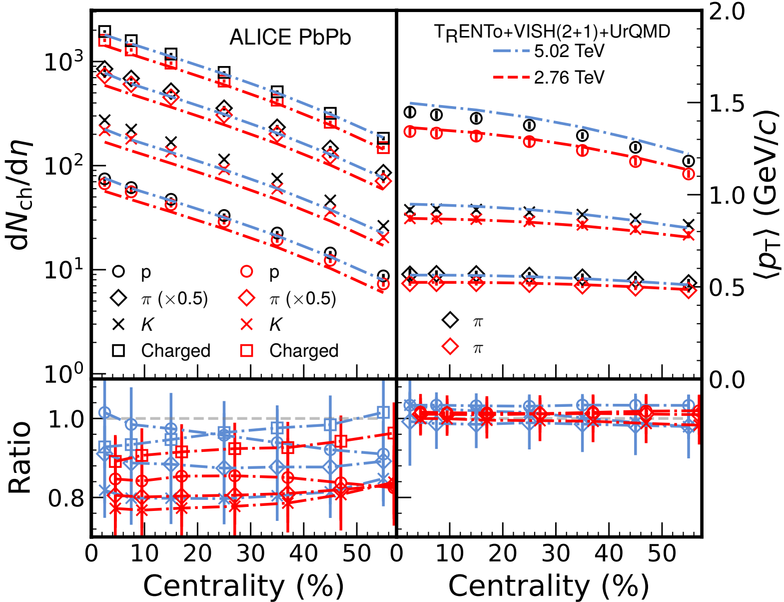
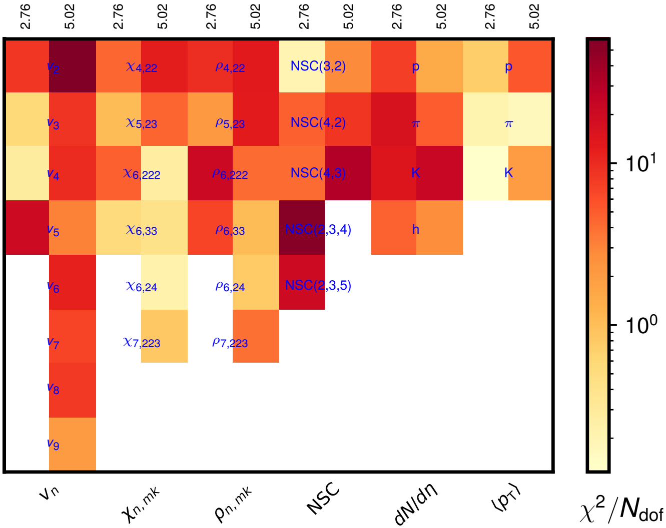
MAP parametrization



Chi-square


2.76
5.02
2.76
5.02
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5.02
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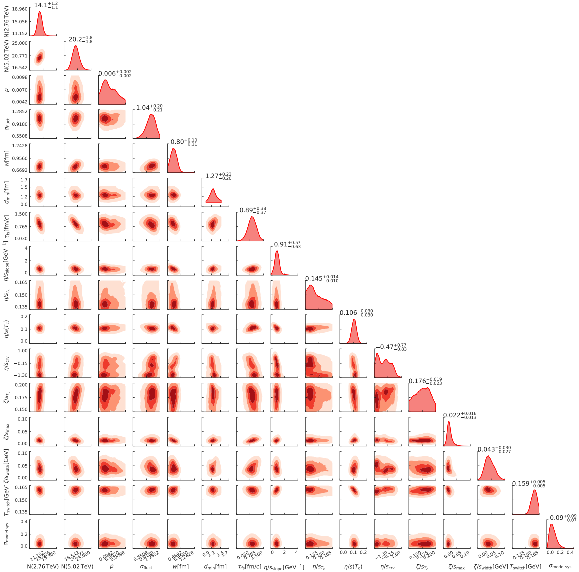
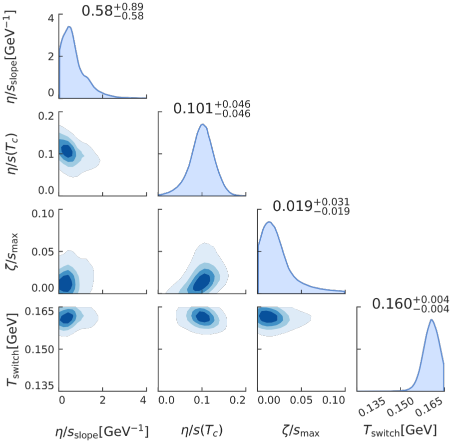
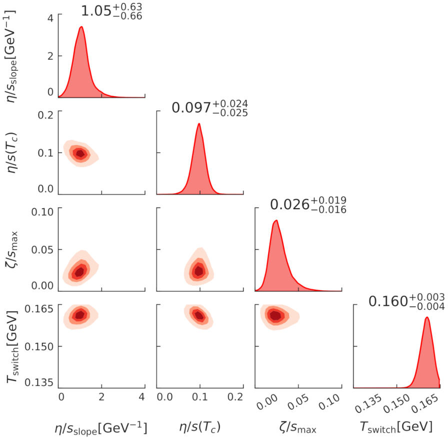
2.76
5.02
v_2
$\chi_{4,22}$
$\rho_{4,22}$
NSC(3,2)
p
p
v_3
$\chi_{5,23}$
$\rho_{5,23}$
NSC(4,2)
π
π
v_4
$\chi_{6,222}$
$\rho_{6,222}$
NSC(4,3)
K
K
v_5
$\chi_{6,33}$
$\rho_{6,33}$
NSC(2,3,4)
h
v_6
$\chi_{6,24}$
$\rho_{6,24}$
NSC(2,3,5)
v_7
$\chi_{7,223}$
$\rho_{7,223}$
v_8
v_9
v_n
$\chi_{n,mk}$
$\rho_{n,mk}$
NSC
$dN/d\eta$
$\langle p_T \rangle$
10^1
10^0
χ^2/N_{dof}

Posterior distribution



Posterior distribution

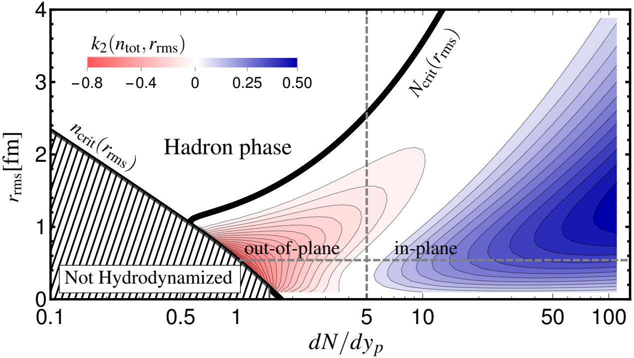




GubsHyd:

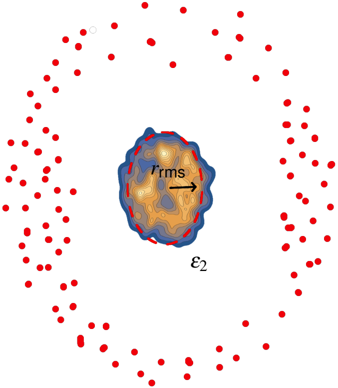
[SFT, Phys.Rev.C 104 (2021) 5, 054906]

A semi-analytical toy model based on the analytical Gubser hydrodynamic solution



Linear response approximation $v_2 \simeq k_2 \varepsilon_2$

$$v_2 \simeq k_2(n_{\rm tot}, r_{\rm rms})\varepsilon_2$$

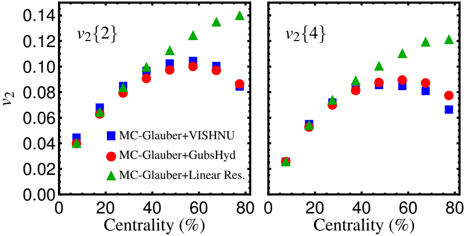


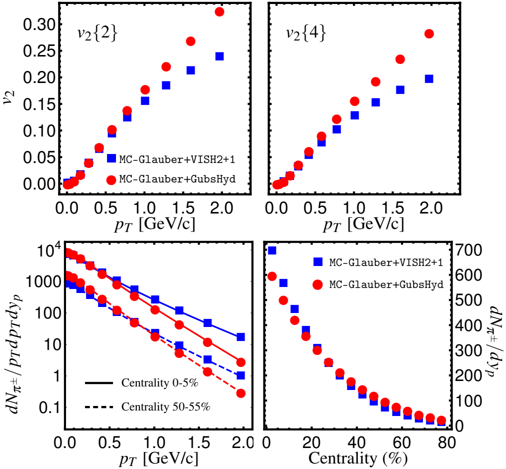
$n_{\rm tot} \equiv dN/dy_p$

VALIDATION: MC-Glauber+VISH(2+1) Vs MC-Glauber+GubsHyd

Pb–Pb collision, $\sqrt{s_{NN}} = 2.76$ *TeV*

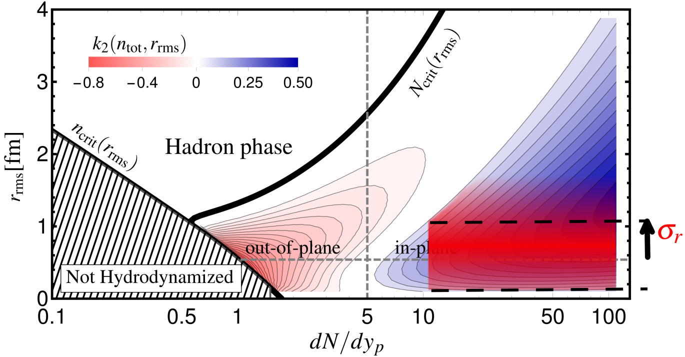
- **Same initial state** set for both and **GubsHyd**





GubsHyd for small system collisions

We model the initial state fluctuation



We assume ε_2 and r_{rms} fluctuate as follows:

$$p_{\mathrm{init}}(\varepsilon_2, r_{\mathrm{rms}}) = \left[\frac{r_{\mathrm{rms}}}{\sigma_r{}^2} e^{-r_{\mathrm{rms}}^2/2\sigma_r{}^2}\right] \times \left[\frac{\varepsilon_2}{\sigma_\varepsilon{}^2} e^{-\varepsilon_2^2/2\sigma_\varepsilon{}^2}\left[1 + \frac{\Gamma_2^\varepsilon}{2} L_2(\varepsilon_2^2/2\sigma_\varepsilon{}^2) + \cdots\right]\right]$$

Then the two particle correlation is given by

$p_{\mathrm{final}}(v_2)$:

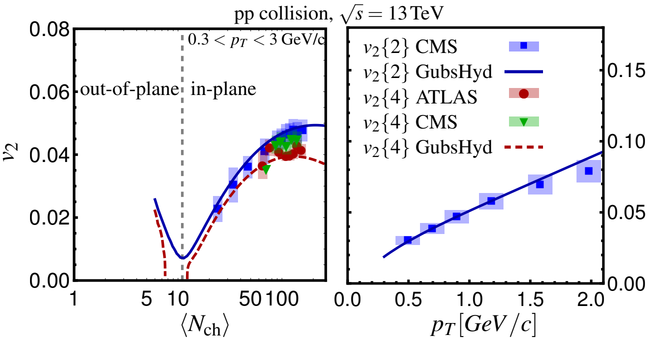
$$v_2\{2\} = \chi\, \sigma_\varepsilon \sqrt{2\langle k_2^2(n_{\mathrm{ntot}}, r_{\mathrm{rms}})\rangle_r}$$

$$v_2\{4\} = \chi\, \sigma_\varepsilon \left[8\langle k_2^2\rangle_r^2 - 4(2 + \Gamma_2^\varepsilon)\langle k_2^4\rangle_r\right]^{1/4}$$

Two- and four-particle correlation of proton–proton collision

[SFT, Phys.Rev.C 104 (2021) 5, 054906]

- By fitting the model to data, we obtain the free parameters:



$$p_{final}(v_2) \quad \longrightarrow \quad p_{init}(\varepsilon_2, r_{rms})$$

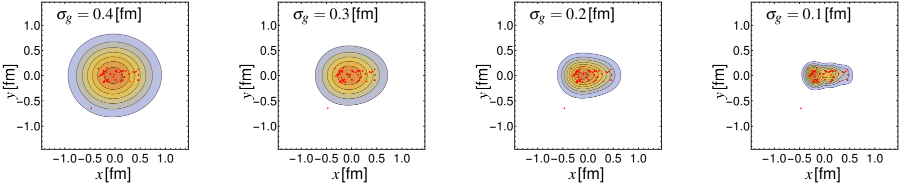
The initial state fluctuation should has the following properties:

$$\sigma_r \approx 0.4 \text{ [fm]}$$

$$\chi\,\sigma_\varepsilon \approx 0.097$$

$$\Gamma_2^\varepsilon \equiv -\left(\frac{\varepsilon_2\{4\}}{\varepsilon_2\{2\}}\right)^4 \approx -0.75$$

Do AMPT (only for initial state) and T$_R$ENTo fulfill these conditions?



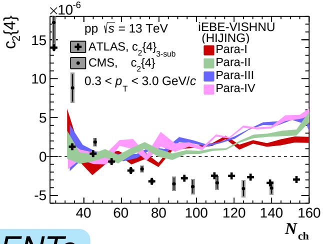
- To cancel out the effect of $\chi\,\sigma_\varepsilon$, we define $\Gamma_2^v = -\left(\frac{v_2\{4\}}{v_2\{2\}}\right)^4 \equiv \frac{c_2\{4\}}{c_2^2\{2\}}$

σ_g [fm]	σ_r^{AMPT} [fm]	$\Gamma_2^{\varepsilon(\mathrm{AMPT})}$	$\Gamma_2^{v(\mathrm{AMPT+Gubs})}$
0.5	0.48	0.53	0.80
0.4	0.41	0.18	0.53
0.3	0.35	-0.17	0.26
0.2	0.30	-0.48	0.01
0.1	0.26	-0.73	-0.20

$$\sigma_r \approx 0.4\ [\mathrm{fm}]$$

$$\Gamma_2^\varepsilon \approx -0.75$$

TENSION!



A similar behavior is observed for T$_R$ENTo.