A light shed on decays of heavy flavored mesons

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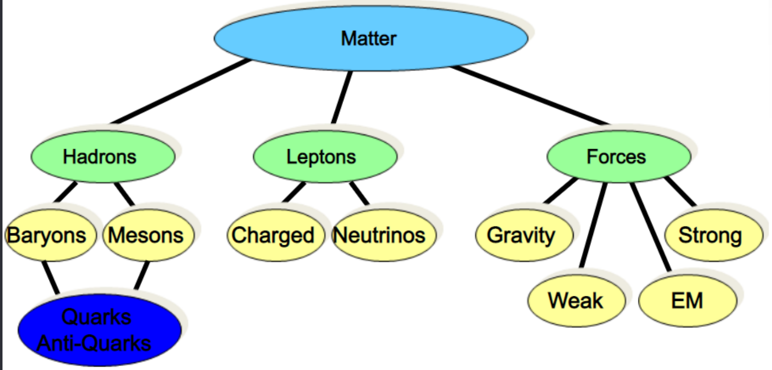
FUNDAMENTALS TO REVISIT

Matter & Forces



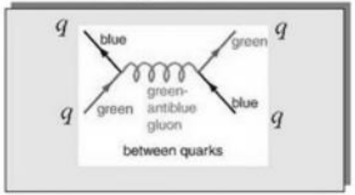
Quantum Numbers of quarks

QUARK	BARYON NO. (B)	SPIN	CHARGE (e)	ISOSPIN I	I_3	S	C	b	t	MASS (GeV)
u	1/3	1/2	2/3	1/2	1/2	0	0	0	0	0.34
d	1/3	1/2	-1/3	1/2	-1/2	0	0	0	0	0.34
s	1/3	1/2	-1/3	0	0	-1	0	0	0	0.51
c	1/3	1/2	2/3	0	0	0	1	0	0	1.6
b	1/3	1/2	-1/3	0	0	0	0	-1	0	5.0
t	1/3	1/2	2/3	0	0	0	0	0	1	174

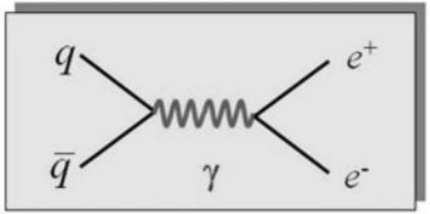
Light quarks (*u*, *d*, *s*)
Heavy quarks (*c*, *b*, *t*)

Quark Interactions

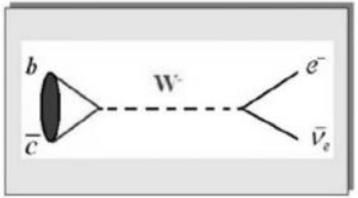
Strong Interaction



Electromagnetic Interaction

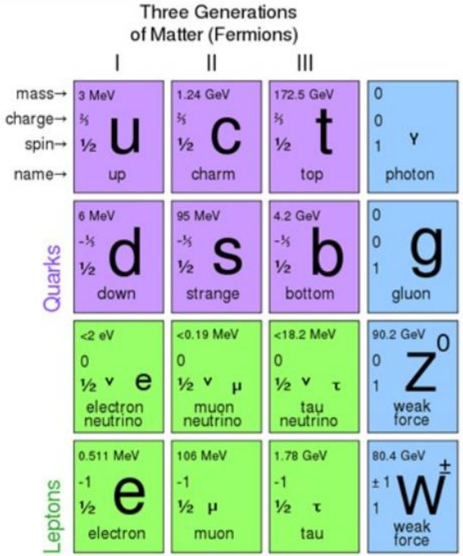


Weak Interaction



The Standard Model

- Quarks and leptons are the most fundamental particles of nature that we know about.
- Up & down quarks and electrons are the constituents of ordinary matter.
- The other quarks and leptons can be produced in cosmic ray showers or in high energy particle accelerators.
- Each particle has a corresponding antiparticle.



Low lying (s-wave) Hadrons

- (0^-) Pseudoscaler Mesons

$$\underbrace{\pi, K, \eta, \eta'}_{Light\ mesons}\ \underbrace{D, D_s, \eta_c \mid B, B_s, B_c, \eta_b}_{Heavy\ mesons}$$

- (1^-) Vector Mesons

$$\underbrace{\rho, K^*, \omega, \phi}_{Light\ mesons},\ \underbrace{D^*, D_s^*, J/\psi, B^*, B_s^*, B_c^*, \Upsilon}_{Heavy\ mesons}$$

- $(\frac{1}{2}^+)$ Baryons

$$(\underbrace{N, \Lambda, \Sigma, \Xi}_{Light\ Baryons} \mid \underbrace{\Lambda_c, \Xi_c, \Sigma_c, \Xi'_c, \Xi_{cc}, \Omega_{cc} \mid \Lambda_b, \Sigma_b, \Xi_b, \Omega_b, \Xi_{cb}, \Omega_{cb}, \Omega_{ccb}}_{Heavy\ baryons})$$

- $(\frac{3}{2}^+)$ Baryons

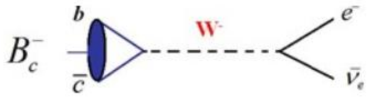
$$(\underbrace{\Delta, \Sigma^*, \Xi^*}_{Light\ Baryons} \mid \underbrace{\Sigma_c^*, \Xi_c^*, \Xi_{cc}^*, \Omega_{cc}^* \mid \Sigma_b^*, \Xi_b^*, \Omega_b^*, \Xi_{cb}^*, \Omega_{cb}^*, \Omega_{ccb}^*, \Xi_{bb}^*, \Omega_{bb}^*, \Omega_{cbb}^*, \Omega_{bbb}^*}_{Heavy\ Baryons})$$

Introduction to Standard Model

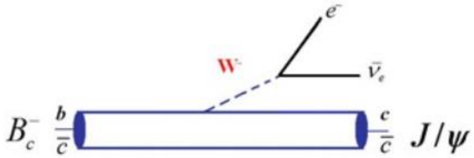
- Leptonic and semileptonic weak interactions of hadrons are explained accurately to a great precision by Standard Model. However, there exist serious problems in understanding the hadronic weak decays, as the theory deals with leptons and quarks, whereas the experiments are performed at hadronic level.
- Theoretical description of the exclusive weak hadronic decays based on Standard Model is not yet obtained as these experiences strong interaction interference.
- Weak currents in the Standard Model generate leptonic, semileptonic and hadronic decays of the heavy flavor hadrons.
- Since the quarks are confined inside the colorless hadrons, matching between theory and experiment requires an exact knowledge of the low energy strong interactions.
- The weak decays of heavy quark hadrons provide a unique opportunity to learn more about QCD particularly on the interface between the perturbative and nonperturbative regimes, to determine SM parameters and finally to search for the physics lying beyond the model.

Weak decays:

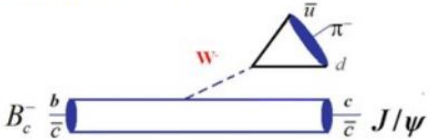
- Leptonic Decays: *e. g.* $B_c^- \to \tau^- \bar{\nu}_\tau$



- Semileptonic Decays: *e. g.* $B_c^- \to J/\psi e^- \bar{\nu}_e$



- Nonleptonic Decays: *e. g.* $B_c^- \to J/\psi \pi^-$



What is Lepton Flavour Universality (LFU)?

In the SM:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs+Yukawa}}$$

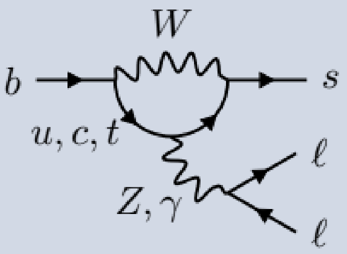
- LFU: e, μ, τ are all the same $(\gamma, W, Z) \rightarrow$ expect $\Gamma_e = \Gamma_\mu = \Gamma_\tau$
- LFUV: $m_e \neq m_\mu \neq m_\tau$
  $y_\tau \sim 10^{-2} \Rightarrow$ very small breaking, only in interactions with H

Beyond the SM:

- New Physics may distinguish between different lepton species

Hints of LFUV in $b \rightarrow s\ell\ell$ and $b \rightarrow c\ell\nu$

$b \to s\ell\ell$

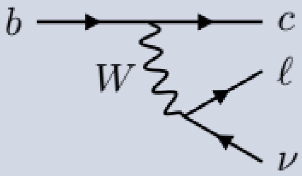


LFU ratios:

$$R_{X_s} = \frac{\mathcal{B}(B \to X_s \mu\mu)}{\mathcal{B}(B \to X_s ee)}$$

$$X_s = K, K^*, K_S, \phi$$

$b \to c\tau\nu$



LFU ratios:

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

$$\ell = \mu, e$$

- Excess of τ leptons

Combined explanations suggest TeV-scale new physics coupled mainly to the 3rd generation

- **Experimental data in B physics hints at non-standard LFU violations both in charged-current as well as neutral-current transitions:**
  - An overall 3.9σ violation from τ/ℓ universality ($\ell = \mu, e$) in the charged-current $b \to c$ decays [BaBar '13, Belle '15, LHCb '15, Fajfer, Kamenik and Nisandzic '12]

$$R^{\tau/\ell}_{D^{(*)}} = \frac{\mathcal{B}(\bar{B} \to D^{(*)}\tau\bar{\nu})_{\rm exp}/\mathcal{B}(\bar{B} \to D^{(*)}\tau\bar{\nu})_{\rm SM}}{\mathcal{B}(\bar{B} \to D^{(*)}\ell\bar{\nu})_{\rm exp}/\mathcal{B}(\bar{B} \to D^{(*)}\ell\bar{\nu})_{\rm SM}}$$

$$R^{\tau/\ell}_{D} = 1.37 \pm 0.17, \qquad R^{\tau/\ell}_{D^*} = 1.28 \pm 0.08$$

  - A 2.6σ deviation from μ/e universality in the neutral-current $b \to s$ transition

$$R^{\mu/e}_{K} = \frac{\mathcal{B}(B \to K\mu^+\mu^-)_{\rm exp}}{\mathcal{B}(B \to Ke^+e^-)_{\rm exp}} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

while $(R^{\mu/e}_K)_{SM} = 1$ up to few % corrections [Hiller et al,'07, Bordone, Isidori and Pattori, '16].

$$R_{J/\psi} = 0.71 \pm 0.17(\text{stat.}) \pm 0.18(\text{syst.})$$

With this context, we have also undertaken a detailed study of these decay processes in a model dependent framework i.e ***Relativistic Independent Quark Model***

WHY BC DECAYS ?

- *It is the **lowest bound state** having two heavy quarks(b and c) .*
- *The B_c meson lie intermediate in mass and size between charmonium($c\bar{c}$) and bottomonium($b\bar{b}$) family where the heavy quark interactions are understood well.*
- *As it has open flavours, B_c decays weakly but not via strong and radiative modes, and therefore it is long lived .*
- *The data available in this sector is scant. The masses of Bc excited states and even the ground state of Bc* have not yet been determined.*
- *The recent prediction of B_c (2S) state at ATLAS gives high statistics B_c events provide necessary motivation to investigate the decay process in radially excited states.*
- *It provides a fertile ground as well as challenging for both theoretical and experimental studies as it has neutrinos in their final states*

MODEL FRAMEWORK :

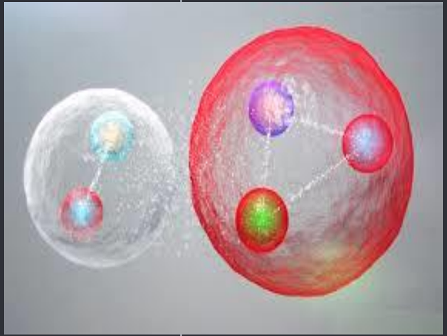
RELATIVISTIC INDEPENDENT QUARK MODEL

- *The description of physical phenomena such as the decay processes have not yet been possible from the first principle QCD application which involves strong interaction between quarks and antiquarks.*

- *Theoretical attempts have been made to describe hadronic phenoma using various phenomenological models like Bag Model, Cloud Bag Model, LFQM, Bethe-Salpeter approach, QCD Sum Rule, and Non-relativistic and Relativistic Model etc. Ours is a Relativistic Independent Quark (RIQ) Model.*

- *Last 35 years we have established this phenomenological model describing wide ranging hadronic phenomena in light as well as heavy flavor sector.*

- *In this model a meson is pictured as a colour-singlet assembly of constituents (quark & anti-quark) moving relativistically inside the meson bound state with an average flavour independent potential in the form :*



$$U(r) = \frac{1}{2}(1 + \Upsilon_0)(ar^2 + V_0)$$

r = the relative distance between quark and antiquark inside meson;
a & V_0 = the potential parameters

$$£= \overline{\psi_q}[\frac{1}{2}i\Upsilon^{\mu}\partial_{\mu} - U(r) - m_q]\,\psi_q(r)$$

The Dirac Equation:

$$(\alpha.p + \beta m_q + U(r))\psi_q(r) = E\,\psi_q(r)$$

Where, $\alpha = \frac{\Upsilon_i}{\Upsilon_0}$ *and* $\beta = \Upsilon_0$

Solving the Dirac equations we get positive and negative energy solutions known as quark and antiquark orbitals respectively in the general form:

$$\psi^{+}_{nlj}(\vec{r}) = \begin{pmatrix} \dfrac{i g_{nlj}(r)}{r} \\ (\vec{\sigma}.\hat{r})\dfrac{f_{nlj}(r)}{r} \end{pmatrix} \varphi_{ljm_j}(\hat{r})$$

$$\psi^{-}_{nlj}(\vec{r}) = \begin{pmatrix} i\vec{\sigma}.\dfrac{\hat{r} f_{nj}(r)}{r} \\ g_{nlj}(r)/r \end{pmatrix} (-1)^{j+m_j-l} \varphi_{ljm_j}(\hat{r})$$

Where, $g_{nlj}(r) = N_q \left(\frac{r}{r_{nl}}\right)^{l+1} e^{-r^2/2r_{nl}^2} L_{n-1}^{l+1/2}\left(\frac{r^2}{r_{nl}^2}\right)$

and $f_{nlj}(r) = -N_q \frac{1}{r_{nl}\lambda_{nl}} \left(\frac{r}{r_{nl}}\right)^{l+2} e^{-r^2/2r_{nl}^2} \left[L_{n-2}^{l+3/2}\left(\frac{r^2}{r_{nl}^2}\right) + L_{n-1}^{l+3/2}\left(\frac{r^2}{r_{nl}^2}\right) \right]$

$$\psi_q^+(\vec{r}) = \frac{1}{\sqrt{4\pi}}\begin{pmatrix} \frac{ig_q(r)}{r} \\ (\overrightarrow{\sigma}.\hat{r})\frac{f_q(r)}{r} \end{pmatrix}\chi_\lambda \qquad \psi_q^-(\vec{r}) = \frac{1}{\sqrt{4\pi}}\begin{pmatrix} i(\overrightarrow{\sigma}.\hat{r})\frac{f_q(r)}{r} \\ g_q(r)/r \end{pmatrix}\widetilde{\chi}_\lambda$$

$$g_q(r) = N_q\left(\frac{r}{r_{0q}}\right)^2 e^{\left(-r^2/2r_{0q}^2\right)} \qquad f_q(r) = -\frac{N_q}{\lambda_q r_{0q}}\left(\frac{r}{r_{0q}}\right)^2 e^{\left(-r^2/2r_{0q}^2\right)}$$

$$N_q^2 = \frac{8\lambda_q}{r_{0q}\sqrt{\pi}}\frac{1}{(3E_q' + m_q')}$$

- *The condition required to be satisfied in obtaining these quark orbitals is binding energy condition i.e*

$$\sqrt{\lambda_{nl}/a}\;(E'_{nl} - m') = (4n+2l-1) \quad \text{where } E'_q = E - V_0/2 \text{ and } m'_q = m - V_0/2$$

- *Which for the ground state (n=1 and l=0) is reduced to the form:* $\sqrt{\lambda_{10}/a}\;(E'_{10} - m') = 3$

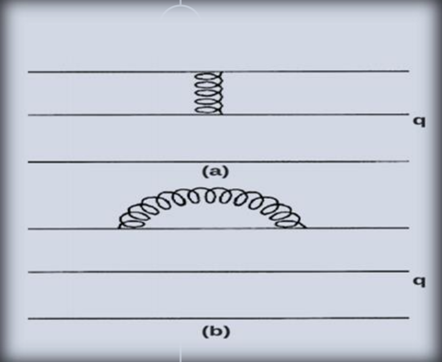
- *The core energy of the meson in zeroth order is sum of the binding energy for the quark and antiquark i.e*

$$E_M^0 = \sum_q E_q$$

GLUONIC CORRECTION

- *Inside the mesons quark and antiquark are bound. The binding is due to exchange of gluons. Taking one gluon exchange the interaction lagrangian density is in the form of*

$$\pounds_I^g = \sum_\infty J_i^{\mu\,a}\, A_\mu^a(x)$$



(One gluon exchange contribution to the energy conservation)

Where $A_\mu^a(x)$ *is the vector-gluon fields and* $J_i^{\mu\,a}$ *is the colour current*

Since at small distance the quarks should be almost free, it is reasonable to calculate the shift in the energy of meson core using first oder perturbation theory.

Energy shift is in the form:

$$(\Delta E)_g = (\Delta E)_g^{\epsilon} + (\Delta E)_g^{M}$$

$$(\Delta E_M)_g^{\mathcal{E}} = \alpha_s \sum_{i,j} \left\langle \sum_a \lambda_i^a \lambda_j^a \right\rangle \frac{1}{\sqrt{\pi}\, R_{ij}} \left(1 - \frac{\alpha_i + \alpha_j}{R_{ij}^2} + \frac{3\alpha_i \alpha_j}{R_{ij}^4}\right)$$

$$(\Delta E_M)_g^{\mathcal{M}} = \alpha_s \sum_{i<j} \left\langle \sum_a \lambda_i^a \lambda_j^a \sigma_i \sigma_j \right\rangle \frac{256}{9\sqrt{\pi}} \frac{1}{(3E_i' + m_i')(3E_j' + m_j')} \frac{1}{R_{ij}^3}.$$

So total energy of meson in its ground state is

$$E_M = E_M^0 + (\Delta E)_g^{\epsilon} + (\Delta E)_g^{M}$$

CENTER OF MASS CORRECTION

- *The independent motion of quarks inside the hadron core does not lead to a state of definite total momentum*
- *The energy associated with the spurious centre of mass motion must provide a further correction to the hadron energy obtained*
- *This prescription was given by:*

1. Wong C W Phys.Rev. D 24, 1416 (1981)
2. Duck I Phys.Lett. 77, 223, 1978
3. Bertelski J et al. Phys.Rev.D. 29, 1035 (1984)

The static meson core state is decomposed into plane-wave momentum eigen state:

$$|M(x)> = \int \frac{d^3P}{W_M(P)} \exp(iP.X)\, \varphi_M(P)\, |M(P)>$$

The inverse relation is:

$$|M(P)> = \frac{1}{(2\pi)^3} \frac{W_M(P)}{\varphi_M(P)} \int d^3X exp(-iP.X) |M(X)>$$

- *The normalisation is as follows:*

  $<M(P)|M(P')> = (2\pi)^3 2E_p \delta(P - P')$

  $$\varphi_M^2 = \frac{W_n(P)}{(2\pi)^3} \widetilde{I_M}(p)$$

- *This permits ready estimates of the centre of mass-momentum P*

  $$<P^2> = \int d^3p \widetilde{I_m}(P) P^2$$

  $$= \sum_q < p^2 >_q$$

  Where $< p^2 > = \frac{(11E'_q + m'_Q)(E'^2_q - m'^2_q)}{6(3E'_q + m'_q)}$

- *Here* $<p^2>_q$ *is the average value of the square of the individual quark-momentum*

- *Mass of Meson* : $m_M = [\{E_M^0 + (\Delta E)_g^{\epsilon} + (\Delta E)_g^{*M}\}^2 - < P^2 >]^{1/2}$

MESON	MESON MASS (GEV)	
	PREDICTED	EXPERIMENTAL
$D^{\pm *}$	2.0149	2.0101
$D^{\pm}$	1.8538	1.8694
$D_s^{\pm *}$	2.0731	2.1103
$D_s^{\pm}$	1.9149	1.9690
$B^{\pm *}$	5.3292	5.3246
$B^{\pm}$	5.2643	5.2786
B_s^{0*}	5.3720	5.4256
B_s^{0}	5.3055	5.3786
$B_c^{\pm *}$	6.3142	-
$B_c^{\pm}$	6.2707	6.2749

$$(a, V_0) \equiv (0.017166\ GeV^3, -0.1375\ GeV)$$

Quark q	m_q (GeV)
u	0.07875
d	0.07875
s	0.31575
c	1.49276
b	4.77659

- *Barik and Dash: Phys. Rev.D. 33, 1925 (1986)*
  - *Pramana-J. Phys: 29(6), 543-557 (1987)*

$$G_b(\vec{p}_b) = \frac{i\pi \mathcal{N}_b}{2\alpha_b \omega_b} \sqrt{\frac{(E_{p_b} + m_b)}{E_{p_b}}} (E_{p_b} + E_b) \exp\left(-\frac{\vec{p}^2}{4\alpha_b}\right)$$

$$\tilde{G}_c(\vec{p}_c) = -\frac{i\pi \mathcal{N}_c}{2\alpha_c \omega_c} \sqrt{\frac{(E_{p_c} + m_c)}{E_{p_c}}} (E_{p_c} + E_c) \exp\left(-\frac{\vec{p}^2}{4\alpha_c}\right)$$

MOMENTUM PROBABLITY AMPLITUDES

$$G_b(\vec{p}_b) = \frac{i\pi \mathcal{N}_b}{2\alpha_b} \sqrt{\frac{(E_{p_b} + m_b)}{E_{p_b}}} \frac{(E_{p_b} + E_b)}{(E_b + m_b)} \left(\frac{\vec{p}_b^2}{2\alpha_b} - \frac{3}{2}\right) \exp\left(-\frac{\vec{p}_b^2}{4\alpha_b}\right)$$

$$\tilde{G}_c(\vec{p}_c) = \frac{i\pi \mathcal{N}_c}{2\alpha_c} \sqrt{\frac{(E_{p_c} + m_c)}{E_{p_c}}} \frac{(E_{p_c} + E_c)}{(E_c + m_c)} \left(\frac{\vec{p}_c^2}{2\alpha_c} - \frac{3}{2}\right) \exp\left(-\frac{\vec{p}_c^2}{4\alpha_c}\right).$$

$$G_b(\vec{p}_b) = \frac{i\pi\mathcal{N}_b}{2\alpha_b}\sqrt{\frac{(E_{p_b}+m_b)}{E_{p_b}}\frac{(E_{p_b}+E_b)}{(E_b+m_b)}}\left(\frac{\vec{p}_b^{\,4}}{8\alpha_b{}^2}-\frac{5\vec{p}_b^{\,2}}{4\alpha_b}+\frac{15}{8}\right)\exp\left(-\frac{\vec{p}_b^{\,2}}{4\alpha_b}\right)$$

$$\tilde{G}_c(\vec{p}_c) = \frac{i\pi\mathcal{N}_c}{2\alpha_c}\sqrt{\frac{(E_{p_c}+m_c)}{E_{p_c}}\frac{(E_{p_c}+E_c)}{(E_c+m_c)}}\left(\frac{\vec{p}_c^{\,4}}{8\alpha_c{}^2}-\frac{5\vec{p}_c^{\,2}}{4\alpha_c}+\frac{15}{8}\right)\exp\left(-\frac{\vec{p}_c^{\,2}}{4\alpha_c}\right).$$

- *Using the momentum probability amplitudes for quarks and antiquarks we write the momentum profile function for meson as :*

$$\mathcal{G}_{B_c}(\vec{p}_b,\vec{p}_{\bar{c}}) = \sqrt{G_b(\vec{p}_b)\tilde{G}_{\bar{c}}(\vec{p}_{\bar{c}})}$$

MESON STATE & MESON NORMALIZATION

The meson state at definite momentum reflect the momentum distribution among constituent quark and antiquark

$$|\,B_c(\vec{P}, S_{B_c}\rangle = \hat{\Lambda}_{Bc}(\vec{P}, S_B)\ \ |(\vec{p_b}, \lambda_b);(\vec{p_c}, \lambda_c)\ \rangle$$

Where $|\,(\vec{p_b},\lambda_b);\,(\vec{p_c},\lambda_c)\rangle = \hat{b}^\dagger_b(\vec{p_b},\lambda_b)\tilde{b}^\dagger(\vec{p_c},\lambda_c)|0\rangle$

In the fockspace representation of the unbound quark and antiquark are

$$\hat{\Lambda}_{Bc}(\vec{P}, S_B) = \frac{\sqrt{3}}{\sqrt{N_{B_c}(\vec{P})}} \sum_{\delta_b,\delta_{\bar{c}}} \varsigma^{B_c}_{b,\bar{c}}(\lambda_b, \lambda_{\bar{c}}) \int d^3\vec{p_b}\, d^3\vec{p_{\bar{c}}}\ \ \delta^{(3)}(\vec{p}_b + \vec{p}_{\bar{c}} - \vec{P})\, \mathcal{G}_{B_c}(\vec{p}_b, \vec{p}_{\bar{c}})$$

Imposing Normalization condition

$$\left\langle B_c\left(\vec{P}'\right)\middle|B_c(\vec{P})\right\rangle = \delta^3(\vec{P} - \vec{P}')$$

$$N(\vec{P}) = \int d^3\vec{p}_b|\mathcal{G}_{B_c}(\vec{p}_b, \vec{p} - \vec{p}_b)|^2.$$

INPUT PARAMETERS

For ground state we take the quark masses, corresponding binding energies and potential parameters:

$$(a, V_0) \equiv (0.017166\text{GeV}^3, -0.1375\text{GeV})$$

$$(m_b, m_c, m_u) \equiv (4.77659, 1.49276, 0.07875)\text{GeV}$$

$$(E_b, E_c) \equiv (4.76633, 1.57951)\ \text{GeV}$$

1S

For 2s and 3s states potential parameter V_0 is -0.01545 GeV

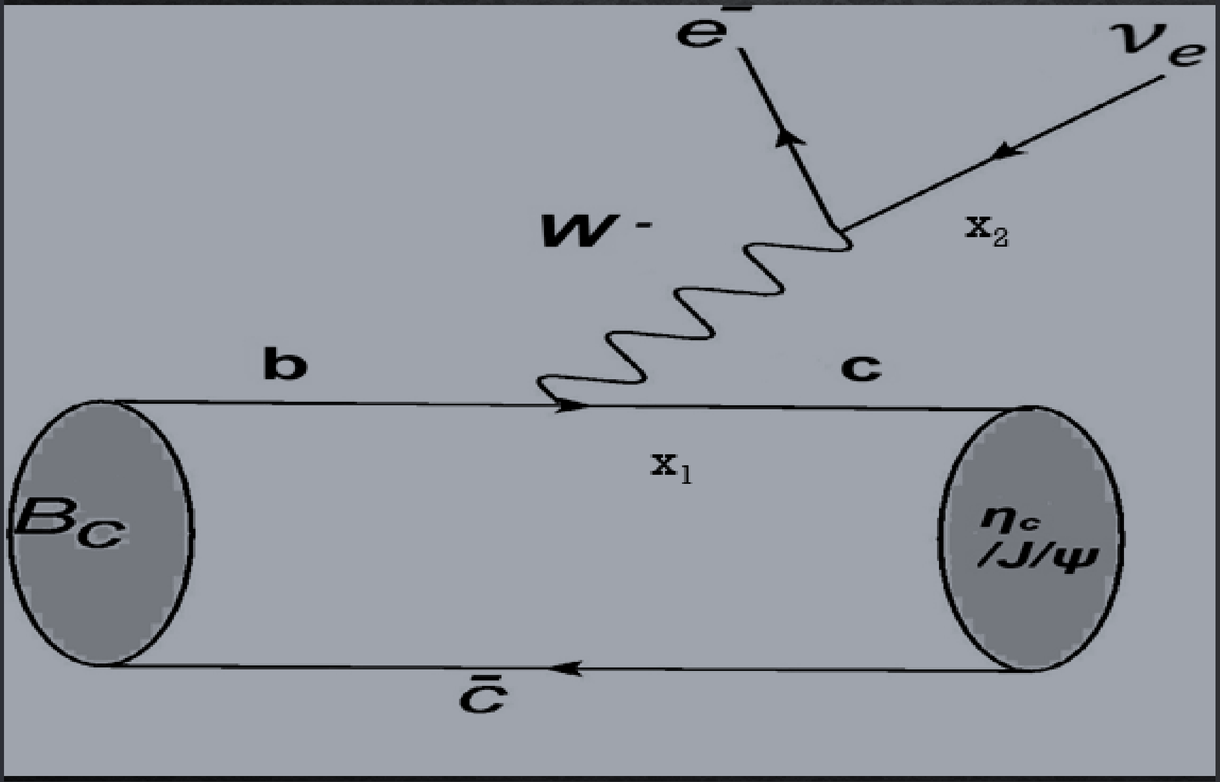
$$(E_u, E_c) \equiv (0.96221, 5.05366, 1.97015)\text{GeV}$$

2S

$$(E_u, E_c) \equiv (0.1.29356, 5.21703, 2.22478)\text{GeV}$$

3S

$B_c \to \eta_c\,, J/\psi\; e^- \nu_e$



INVARIANT MATRIX AMPLITUDES

$$S_{fi} = (2\pi)^4 \delta^4(p - k - k_1 - k_2) \frac{(-M_{fi})}{\sqrt{(2\pi)^3 2M (2\pi)^3 2E_k (2\pi)^3 2E_{k_1} (2\pi)^3 2E_{k_2}}}$$

$$M_{fi} = \frac{-G_F}{\sqrt{2}} V_{q_1 q'_1} H^{\mu} L^{l}_{\mu}$$

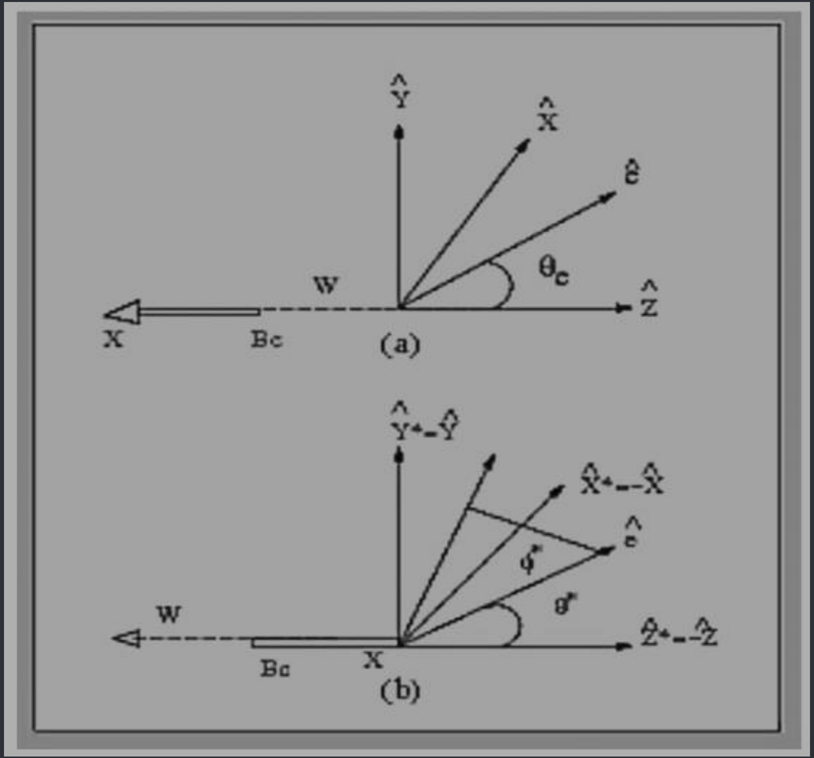
where $H^{\mu} = \sqrt{\frac{4ME_k}{N(\vec{k})N(\vec{p})}} \int \frac{d^3 p_1}{\sqrt{2E_{p_1} 2E_{(\vec{k}-\vec{P}+\vec{P_1})}}} \sum_{q_1 q'_1 q_2} \varsigma_{q'_1 q_2}(\lambda'_1 \lambda_2) \varsigma_{q_1 q_2}(\lambda_1 \lambda_2)$

$$\times\, \mathcal{G}_M(\vec{K} - \vec{P} + \vec{P}_1, \vec{P} - \vec{P}_1)\, \mathcal{G}_M(\vec{P}_1, \vec{P} - \vec{P}_1)$$

And $L^{l}_{\mu} = \bar{U}(k_e, \delta_e) \Gamma_{\mu} V(k_\nu, \delta_\nu)$

$$|M_{fi}|^2 = \frac{{G_F}^2}{2} V_{q_1 q'_1} H^{\mu\vartheta} L^{l}_{\mu\vartheta} \qquad L_{ij} = \frac{4\pi}{3} q^2 \delta_{ij}$$

FRAME OF REFERENCE



$$\vec{H} = H_+\hat{e}_+ + H_-\hat{e}_- + H_0\hat{e}_0$$

$$\hat{e}_\pm = \frac{1}{\sqrt{2}}(\mp\hat{x} - i\hat{y});\ \hat{e}_0 = \hat{z}$$

a. Lepton rest frame
b. Vector meson helicity frame

DECAY RATES & HADRONIC AMPLITUDES

$$\frac{d\tilde{\Gamma}}{dy} = \frac{1}{288\pi^3} {G_F}^2 \mid V_{Qq} \mid^2 M_{B_c}^2 K y \left[\mid \bar{H}_+ \mid^2 + \mid \bar{H}_- \mid^2 + \mid \bar{H}_0 \mid^2 \right]$$

Where

$$\bar{H}_\pm = [\, f(q^2) \mp 2M_{B_c} K g(q^2) \,],$$

$$\bar{H}_0 = \frac{M_{B_c}}{2M_X\sqrt{y}} \left[\left(1 - \frac{M_X^2}{M_{B_c}^2} - y\right) f(q^2) + 4K^2 a_+(q^2) \right].$$

The decay rate in case of $(0^- \to 0^-)$ type transition is

$$\frac{d\tilde{\Gamma}}{dy} = \frac{{G_F}^2 \mid V_{Qq} \mid^2 K^3 M_{B_c}^2}{72\pi^3} \mid f_+(q^2) \mid^2$$

where

$$\bar{H}_\pm = 0; \quad \bar{H}_0 = -2\frac{K}{\sqrt{y}} f_+(q^2)$$

FORM FACTOR FOR $0^- \to 0^-$ TRANSITION

$$f_+ = \frac{1}{2M_{B_c}} \int d\vec{p}_b \mathcal{C}(\vec{p}_b)[(E_b(\vec{p}_b) + m_b)(E_c(\vec{p}_b + \vec{k}) + m_c + M_{B_c} - \tilde{E}_X) + \vec{p}_b{}^2]$$

$$\mathcal{C}(\vec{p}_b) = \sqrt{\frac{(E_b(\vec{p}_b) + E_b)(E_c(\vec{p}_b + \vec{k}) + E_c)}{N_{B_c}(0) N_X(\vec{k})}} \times \frac{\mathcal{G}_{B_c}(\vec{p}_b, -\vec{p}_b)\mathcal{G}_X(\vec{k} + \vec{p}_b, -\vec{p}_b)}{\sqrt{E_b(\vec{p}_b)E_c(\vec{p}_b + \vec{k})(E_c(\vec{p}_b + \vec{k}) + m_c)(E_b(\vec{p}_b) + m_b)}}$$

FORM FACTORS FOR $0^- \to 1^-$ TRANSITION

$$g = -\frac{1}{2M_{B_c}} \int d\vec{p}_b \mathcal{C}(\vec{p}_b)(E_b(\vec{p}_b) + m_b)$$

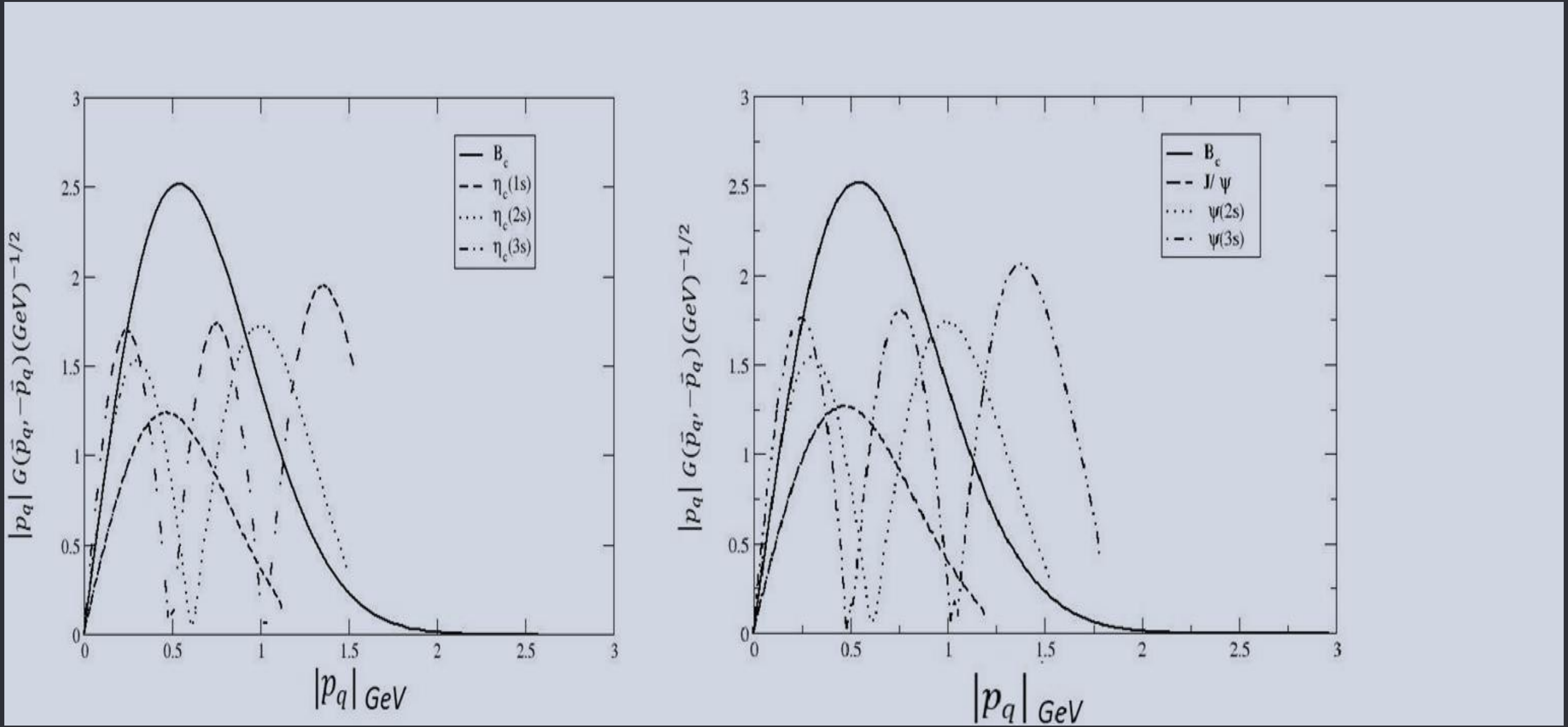
$$f = \frac{4Mm}{N_M(0)N_m(0)} \int \frac{d\vec{p}_b \mathcal{G}_m(\vec{p}_b, -\vec{p}_b)\mathcal{G}_M(\vec{p}_b, -\vec{p}_b)}{\sqrt{4E^0_{p_c} E_{p_b}(E^0_{p_c} + m_c)(E_{p_b} + m_b)}} [3(E^0_{p_c} + m_c)(E_{p_b} + m_b) - \vec{p}_b{}^2/3]$$

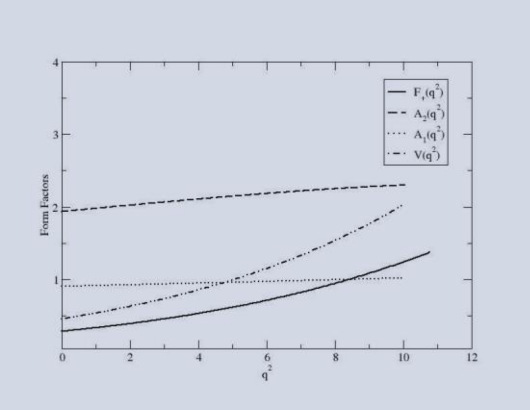
$$a_+ = -\frac{E_k(M+m)}{2M(M+2E_k)}[3(M - E_k)\frac{(I-f)(M-m)}{E_k{}^2 - m^2} - T]$$

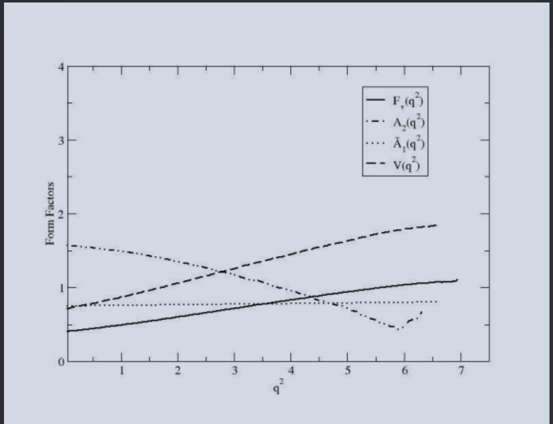
- *The weak form factors are expressed in the dimensionless form*

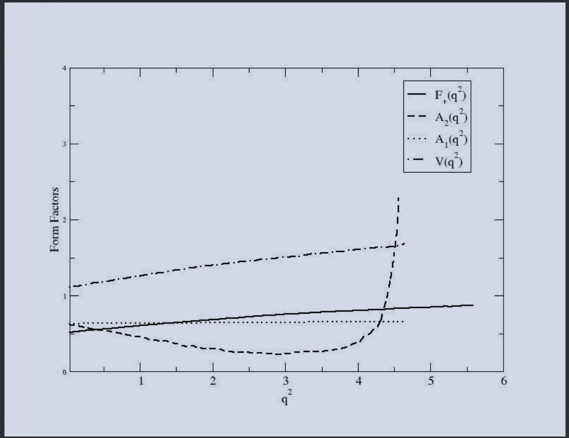
$$\begin{aligned} F_1(q^2) &= f_+(q^2) \\ V(q^2) &= (M_{B_c} + M_X)g(q^2) \\ A_1(q^2) &= (M_{B_c} + M_X)^{-1}f(q^2) \\ A_2(q^2) &= -(M_{B_c} + M_X)a_+(q^2) \end{aligned}$$

OVERLAPPING MOMENTUM DISTRIBUTION AMPLITUDE









q^2 dependence of form factors

TABLE I: **Predicted branching ratios (%) of $B_c^+ \to X e^+ \nu_e$ decay in comparison with other model predictions**

Transition	Our work	RQM	QCDPM	RVQM	ISGW2	QCDSR	NRQM
$B_c^+ \to \eta_c e^+ \nu_e$	0.375	0.81	0.55	0.42	0.67	1.64	0.48
$B_c^+ \to J/\psi e^+ \nu_e$	2.170	2.07	1.73	1.23	1.49	2.37	1.54
$B_c^+ \to \eta_c(2S) e^+ \nu_e$	0.223	...	0.07	0.03	...	...	...
$B_c^+ \to \psi(2S) e^+ \nu_e$	0.814	...	0.1	0.03	...	...	...
$B_c^+ \to \eta_c(3S) e^+ \nu_e$	0.179	...	...	5.5×10^{-4}	...	...	...
$B_c^+ \to \psi(3S) e^+ \nu_e$	0.333	...	...	5.7×10^{-4}	...	...	...

Table 2 The partial branching ratios (%) and polarization ratio: $\frac{\Gamma_L}{\Gamma_T}$ of $B_c^+ \to X e^+ \nu_e$ decays in different q^2 regions

Transition	Region-I	Region-II	Total Region
$B_c^+ \to J/\psi e^+ \nu_e$	0.937	1.358	2.299
$\frac{\Gamma_L}{\Gamma_T}$	0.596	0.444	0.503
$B_c^+ \to \psi(2S) e^+ \nu_e$	0.390	0.470	0.862
$\frac{\Gamma_L}{\Gamma_T}$	1.164	0.649	0.848
$B_c^+ \to \psi(3S) e^+ \nu_e$	0.190	0.162	0.353
$\frac{\Gamma_L}{\Gamma_T}$	2.108	0.732	1.276

- We have studied the behavior of radial momentum distribution amplitude and calculated the Lorentz-invarinat weak form factors from the overlap integral.

- We have predicted the q^2 dependence of weak form factors for decay modes in their physical kinematic range.

- As expected, our predicted BR are obtained in the following hierarchy:
  $BR(B_c \to \eta_c\ \psi(3S)) < BR(B_c \to \eta_c\ \psi(2S)) < BR(B_c \to \eta_c\ \psi(1S))$. This is due to tighter phase space and weaker q^2 dependence of form factors.

- Our predicted branching ratios for decays involving ground and radially excited S-wave charmonium states are in good agreement with other model predictions.

- We have also studied the longitudinal and transverse polarization contribution to the branching ratios of $B_c \to \Psi(nS)\ e\ \vartheta_e$ in the lower, higher and whole kinematic region.

Lepton Mass Effects in

$B_c \to \eta_c, J/\psi l\, \vartheta$, $B_c \to D\,(D^*) l\, \vartheta,$ $B_c \to X(2S/3S), l, \vartheta_l$

Differential decay rates considering angular distribution with the momentum transfer squared q^2

$$\frac{d\Gamma}{dq^2 d(\cos\theta)} = \frac{1}{(2\pi)^3} \sum \frac{G_F^2}{12 M_p^2} |V_{bc}|^2 \frac{(q^2 - m_e^2)}{q^2} |\vec{K}|\; L^{\mu\vartheta} H_{\mu\vartheta}$$

$$L^{\mu\vartheta} H_{\mu\vartheta} = \frac{2}{3}(q^2 - m_e^2)\left[\frac{8}{3}(1+\cos^2\theta)\widehat{H}_U + \frac{3}{4}\sin^2\theta \widehat{H}_L - \frac{3}{4}\cos\theta \widehat{H}_p + \frac{m_e^2}{2q^2}\left(\frac{3}{4}\sin^2\theta \widehat{H}_U + \frac{3}{2}\cos^2\theta \widehat{H}_L + 3\cos\theta \widehat{H}_{SL} + \frac{1}{2}\widehat{H}_S\right)\right.$$

Where

$$H_U = |H_+|^2 + |H_-|^2$$
$$H_L = |H_0|^2$$
$$H_P = |H_+|^2 - |H_-|^2$$
$$H_S = 3|H_t|^2$$
$$H_{SL} = Re(H_0 H_t)$$

$H_U = \mathrm{Re}(H_+ H_+^\dagger) + \mathrm{Re}(H_- H_-^\dagger)$: Unpolarized-transversed

$H_L = \mathrm{Re}(H_0 H_0^\dagger)$: Longitudinal

$H_P = \mathrm{Re}(H_+ H_+^\dagger) - \mathrm{Re}(H_- H_-^\dagger)$: Parity-odd

$H_S = 3\mathrm{Re}(H_t H_t^\dagger)$: Scalar

$H_{SL} = \mathrm{Re}(H_t H_0^\dagger)$: Scalar-Longitudinal Interference

DECAY & HELICITY RATES

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_U}{dq^2} + \frac{d\Gamma_L}{dq^2} + \frac{d\tilde{\Gamma}_U}{dq^2} + \frac{d\tilde{\Gamma}_L}{dq^2} + \frac{d\tilde{\Gamma}_S}{dq^2}$$

Where , $\frac{d\Gamma_i}{dq^2} = \frac{1}{(2\pi)^3} \sum \frac{G_F^2}{12 M_p^2} |V_{bc}|^2 \frac{(q^2 - m_l^2)}{q^2} |\vec{K}| H_i$

and $\frac{d\tilde{\Gamma}_i}{dq^2} = \frac{m_l^2}{2q^2} \frac{d\Gamma_i}{dq^2}$, $i = U, L, P, S$

Integrating over q^2 we get

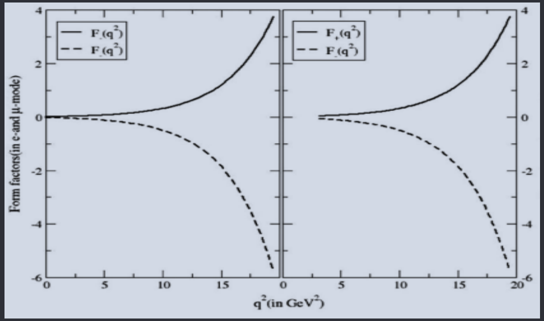
$$\Gamma = \Gamma_U + \Gamma_L + \tilde{\Gamma}_U + \tilde{\Gamma}_L + \tilde{\Gamma}_S$$

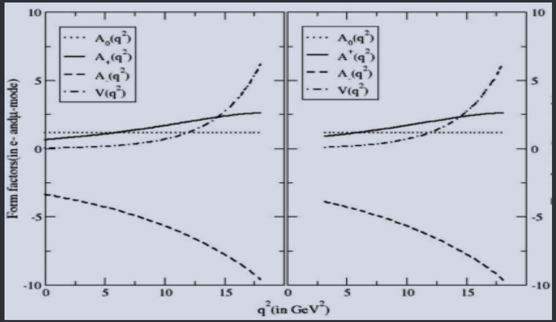
For Pseudoscalar Meson In Final state

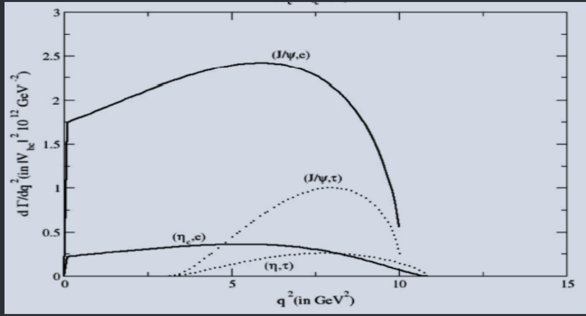
$$= \Gamma_L + \tilde{\Gamma}_L + \tilde{\Gamma}_S$$

For Vector meson In Final State

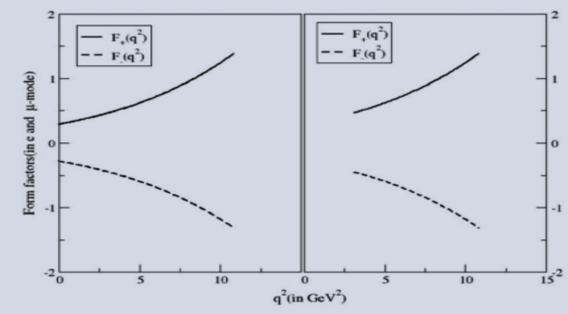
$$\Gamma_U + \Gamma_L + \tilde{\Gamma}_U + \tilde{\Gamma}_L + \tilde{\Gamma}_S$$

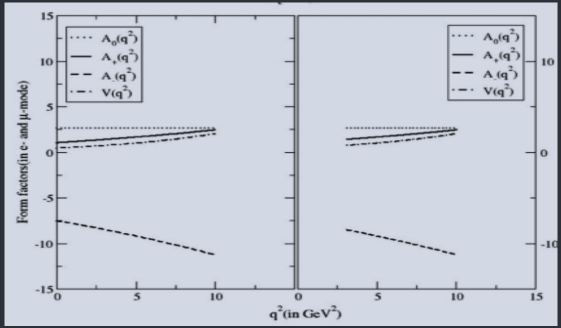


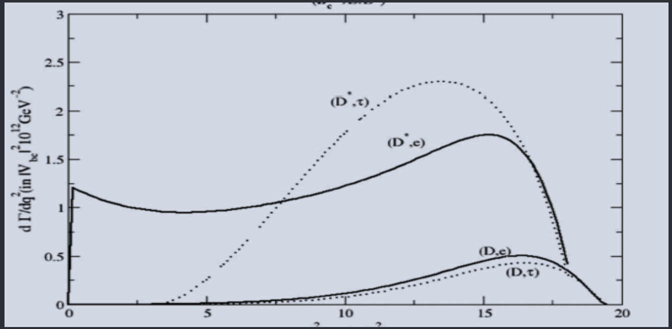




q^2-dependence of form factors







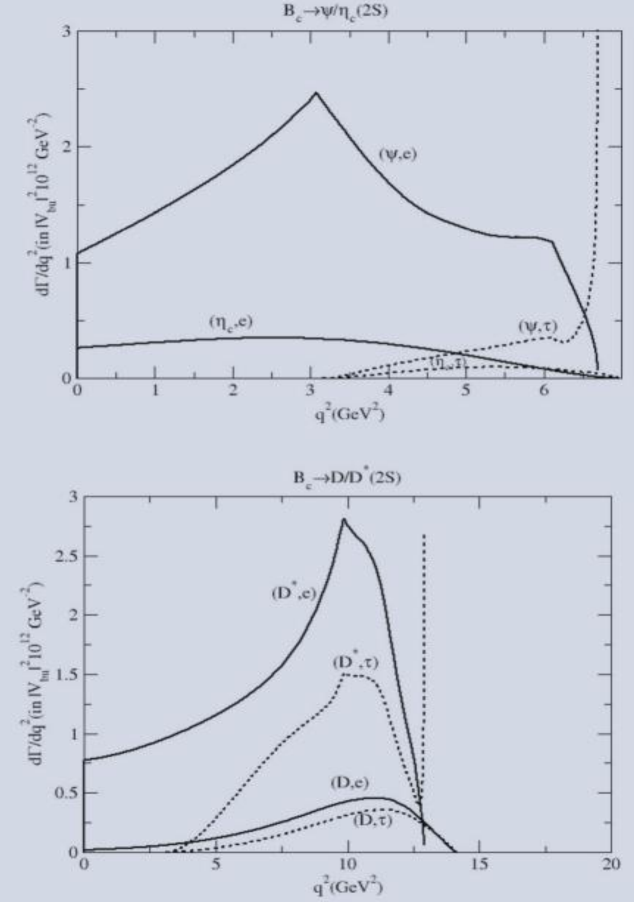


Fig. 8 q^2-spectrum of semileptonic decay rates $B_c \to \eta_c(2S)/\psi(2S)$ and $B_c \to D(2S)/D^*(2S)$ decays

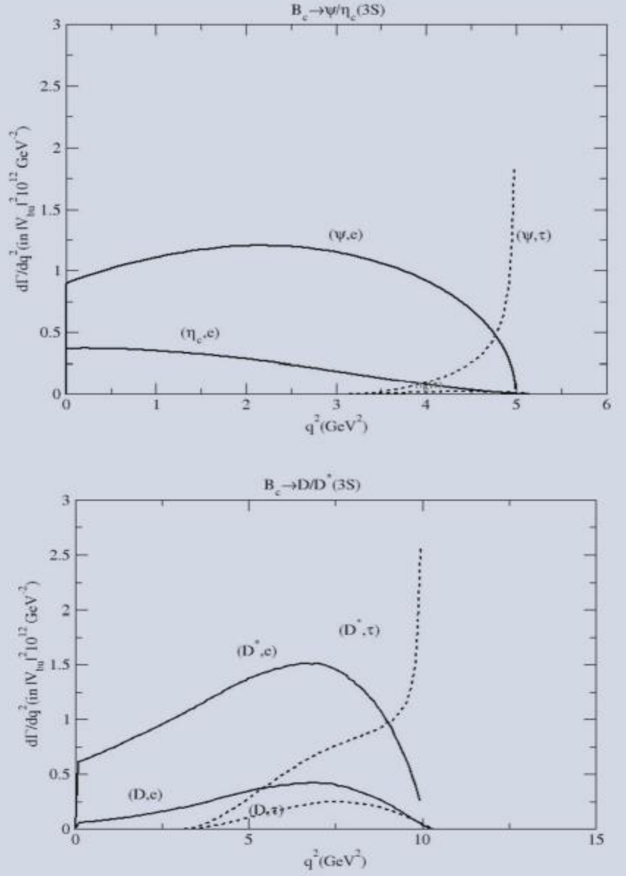


Fig. 9 q^2-spectrum of semileptonic decay rates for $B_c \to \eta_c(3S)/\psi(3S)$ and $B_c \to D(3S)/D^*(3S)$ decays

PREDICTIONS OF HELICITY RATES

Decay mode	U	$\tilde{U}$	L	$\tilde{L}$	P	S	$\tilde{S}$	$\widetilde{SL}$	Γ
$B_c^- \to \eta_c e^- \nu_e$			4.844	4.432×10^{-7}			15.397×10^{-7}	4.712×10^{-7}	4.844
$B_c \to \eta_c \tau^- \nu_\tau$			0.756	0.172			1.194	0.253	2.122
$B_c^- \to J/\psi e^- \nu_e$	18.634	6.052×10^{-7}	16.283	27.813×10^{-7}	8.368	1.188	66.653×10^{-7}	22.856×10^{-7}	34.918
$B_c^- \to J/\psi \tau^- \nu_\tau$	3.823	0.846	1.922	0.437	1.704	0.614	0.307	0.197	7.336
$B_c^- \to D e^- \nu_e$			0.047	4.611×10^{-10}			1.072×10^{-9}	4.038×10^{-10}	0.047
$B_c^- \to D \tau^- \nu_\tau$			0.028	0.003			0.007	0.0027	0.038
$B_c^- \to D^* e^- \nu_e$	0.2439	4×10^{-9}	0.078	7.760×10^{-9}	0.169	0.081	4.092×10^{-8}	3.648×10^{-9}	0.322
$B_c^- \to D^* \tau^- \nu_\tau$	0.113	0.015	0.0156	0.0021	0.092	0.046	0.151	0.0094	0.297

PREDICTIONS OF FLAVOR OBSERVABLES IN RIQM

Ratio of Branching Fractions(R)	This work	QCD	RP	PQCD
$R_{\eta_c} = \frac{\mathcal{B}(B_c \to \eta_c l \nu)}{\mathcal{B}(B_c \to \eta_c \tau \nu)}$	2.312	3.96	3.68	3.2
$R_{J/\psi} = \frac{\mathcal{B}(B_c \to J/\psi l \nu)}{\mathcal{B}(B_c \to J\psi \tau \nu)}$	4.785	4.18	4.22	3.4
$R_D = \frac{\mathcal{B}(B_c \to D l \nu)}{\mathcal{B}(B_c \to D \tau \nu)}$	1.275	1.57	1.67	1.42
$R_{D^*} = \frac{\mathcal{B}(B_c \to D^* l \nu)}{\mathcal{B}(B_c \to D^* \tau \nu)}$	1.091	1.76	1.72	1.66

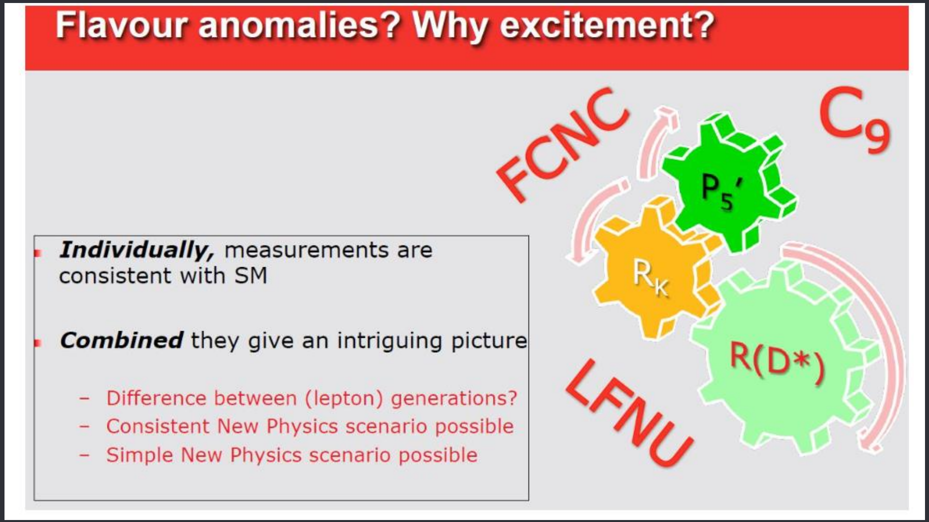
CONCLUSION AND OUTLOOK

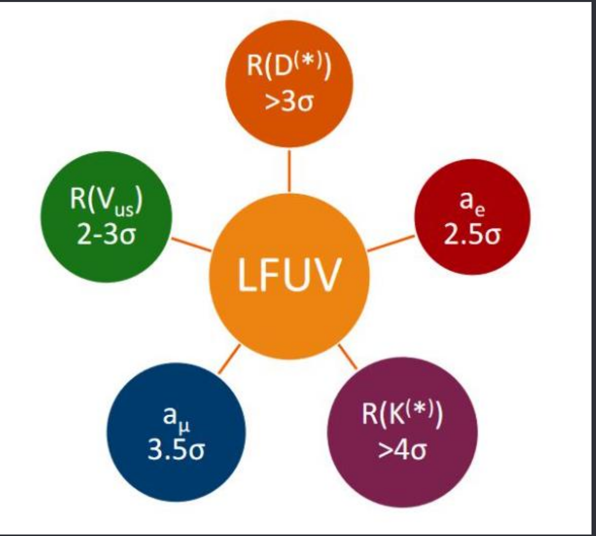
- *Predicted branching ratios and investigated the longitudinal and transverse polarization ratios in the low, high q^2region as well as in the whole kinematic region.*
- *The τ − phase space as compared to e^- and μ^- cases is considerably reduced and shifted to large q^2 region.*
- *Predicted q^2-dependence of helicity amplitude, partial helicity rates and partial decay rates.*
- *Predicted the q^2 dependence of the form factors in the physical kinematic region.*
- *Our predicted observable R for $B_c \to \eta_c(nS)$ and $B_c \to \psi(nS)$ is found comparable to other SM predictions.*
- *RIQ model does not incorporate any new physics bounds to explain these anomalies*
- *However, in the absence of predicted data from established model approaches in the literature for B_c decays to charm meson states, our predictions: $R_{D(2S)}$, $R_{D(3S)}$, $R_{D*(2S)}$ and $R_{D*(3S)}$ can be useful to identify the B_c-channels characterized by clear experimental signature.*

(PHYS. REV. D 104, 036012 (2021))

(EUR. PHYS. J. C 82, 750 (2022))

FUTURE PROSPECTS





- *There are other possible anomalies by different processes related to LFUV i.e. anomalous magnetic moment of electron and muon by Fermilab, mass of W boson by CDF collaboration, the ckm puzzle*
- *Therefore flavor anomalies are the strongest hints for BSM physics*

- *Moreover, with the start of the Run 3 data taking, period (2022-2025) at the LHC, the data collections are expected to boom approximately three times larger in three years. This will certainly increase the event statistics which will further reduce the statistical and systematic uncertainties*

- *On the theoretical side, we should comprehensively report the results from existing models and welcome more NP models to explain these anomalies for testing on the experimental predictions.*

- *As quoted by https://ncatlab.org/nlab/show/flavour+anomaly#CrivellinEtAl18*
  It is expected that if the ongoing evaluation of the data of LHC's Run 2 confirms the measurements of Run 1, then the statistical significance of the effect in each decay channel separately is expected to have reached 5 σ.

  LHCb Collaboration 18 predicts a statistical significance between 6 and 10 σ by the year 2025 https://ncatlab.org/nlab/show/flavour+anomaly#LKLR19

- *If the LUV anomalies stay then in no time there will be some remarkable evidence, unraveling the NP in the flavor community*

FUTURE RESEARCH WORK

To study the decays of $s \to u$ transitions involving ($K \to \pi \mu \nu_\mu$) and ($K \to \pi e\, \nu_e$) and other possible K decays. To measure the size of direct CP violation relative to the indirect CP violation (ϵ'/ϵ). These decays are considered golden modes in flavor physics as they are much sensitive to NP signals.

To study the decays of $b \to u$ transition involving $B \to \rho K^*$ and investigate its branching fraction, longitudinal and transverse polarization.

To study the decays of $b \to s$ transition involving $B \to K\,(K^*)l^+l^-$, where ($l = e\ \mu\ \tau$) and $B \to K(K^*)\vartheta\bar{\vartheta}$ and possible $b \to d$ decays. We will revisit the calculation for $R\,(K)$ and $R\,(K^*)$ using updated data by LHCb and Belle.

To study the decays of $b \to c$ transition involving $B \to D\,(D^*)l^+l^-$ and $B \to D(D^*)\ l_1 l_2$ where ($l = e\ \mu\ \tau$) and reinvestigate $R(D)$ and $R(D^*)$ and various other kinematic observables concerning LFU tests which will unravel the anomalies and help us to determine deviations from SM.

LIST OF PUBLICATIONS

- *Nov 2022 : A light shed on lepton flavor universality in B decays,*
  Sonali Patnaik, and Rajeev Singh https://arxiv.org/abs/2211.04348

- *Jan 2021 - Lepton mass effects in exclusive semileptonic B_c meson decays; Lopamudra Nayak, Sonali Patnaik, P. C. Dash, Susmita Kar and N. Barik;* ***Phys. Rev. D,*** *104, 036012*

- *Jan 2020 - Semileptonic Bc meson decays to S- wave charmonium states; Sonali Patnaik, Lopamudra Nayak, P. C. Dash, Susmita Kar and N. Barik;* ***Eur. Phys. J. Plus*** *135 (936)*

- *March 2018 - Electromagnetic transitions of (b $\bar c$) bound system; Sonali Patnaik, P. C. Dash, Susmita Kar and N. Barik;* ***Phys. Rev. D****; 97 056025*

- *Dec 2017 - Magnetic Dipole Transitions of Bc and B∗ c mesons in the relativistic independent quark model; Sonali Patnaik, P. C. Dash, Susmita Kar and N. Barik;*
  Phys. Rev. D*; 96 116010*


Thank you

- BACK UP

- *In my foregoing study we have presented a study of certain aspects of low energy hadronic phenomena relating to weak and electromagnetic decays of B_C meson*

- *Due to the complicacies arising from non-abelian and non-perturbative characteristics of QCD. Therefore we have adopted an otherwise successful phenomenological model for hadrons i.e "Relativistic Independent Quark Model"*

- *The dynamics of core constituents inside the hadron is represented through an effective zeroth order Lagrangian density with an interaction potential in equally mixed scalar-vector harmonic form.*

- *The potential parameters, the quark masses and their corresponding binding energies have already been fixed in this model from hadron spectroscopy in earlier works of model application.*

- *In view of the recent prediction of excited B_C meson states by ATLAS and CMS experiments, for the first time, we have extended the model application to study hadronic phenomena in radially excited states (2S and 3S).*

- *The outcomes of all the investigations are almost in agreement with other model predictions. Of course in some cases there have been disagreements with other model predictions, which is due to inherent constraints of all model dependent (phenomenological) studies. The future experiments (LHCb, Z-factory, Tevatron etc) can only infer which model is closer to truth.*

PREDICTIONS FOR RADIALLY EXCITED CHARMONIUM STATES

Decay mode	This work	[23]	[36]	[37]	[38]	[39,40]
$B_c \to \eta_c(2S)e\nu$	0.22	0.056	0.046	0.07	0.0496	0.03
$B_c \to \eta_c(2S)\tau\nu$	0.03	–	–	–	0.0025	–
$B_c \to \psi(2S)e\nu$	1.341	0.112	0.014	0.1	0.081	0.03
$B_c \to \psi(2S)\tau\nu$	0.114	–	–	–	0.0408	–
$B_c \to \eta_c(3S)e\nu$	0.14	–	–	–	0.00414	5.5×10^{-4}
$B_c \to \eta_c(3S)\tau\nu$	0.003	–	–	–	0.0043×10^{-2}	5.0×10^{-7}
$B_c \to \psi(3S)e\nu$	0.658	–	–	–	0.0109	5.7×10^{-4}
$B_c \to \psi(3S)\tau\nu$	0.0511	–	–	–	0.010×10^{-2}	3.6×10^{-6}

Ratio	RIQM	[79]	[80, 81]	[82]	[83]	[84]
$\mathcal{R}_{\eta_c}(2S)$	7.33	18.4	-	14.5	1.35	35.38
$\mathcal{R}_{\eta_c}(3S)$	46.67	96.24	1.1×10^3	7.36×10^2	33.33	
$\mathcal{R}_{\psi}(2S)$	11.76	1.98	-	14.3	-	14
$\mathcal{R}_{\psi}(3S)$	12.876	109	158.33	947.4		