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Collective dynamics of polarized spin-half fermions in relativistic heavy-ion collisions

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Heavy-ion collisions and QGP

→ As per thermal history of our Universe, it is expected that Early Universe was in a state of quark-gluon plasma (QGP).

Annual Review of Nuclear and Particle Science 2006 56:1, 441-500

→ Then the phase transition happened, when $T_{universe} \approx 200\,MeV$ & $Age_{universe} \approx 10^{-6}\,sec$, from QGP to confined hadrons.

→ Properties of QGP and phase transition can be studied using relativistic heavy-ion collisions. This is crucial to understand the existence of nuclear matter and confinement.

Prog.Part.Nucl.Phys. 72 (2013) 99-154

→ Experimental evidences of QGP, provided by CERN and BNL, showed that it behaves as a strongly-coupled system whose evolution closely follows the dynamics of perfect fluid.

nucl-th/0002042, Nucl.Phys.A757:184-283,2005

→ This is due to very small kinematic shear viscosity obtained from the transverse momentum spectra of charged particles.

Nucl.Phys.A750:30-63,2005
Prog.Part.Nucl.Phys.62:48-101,2009

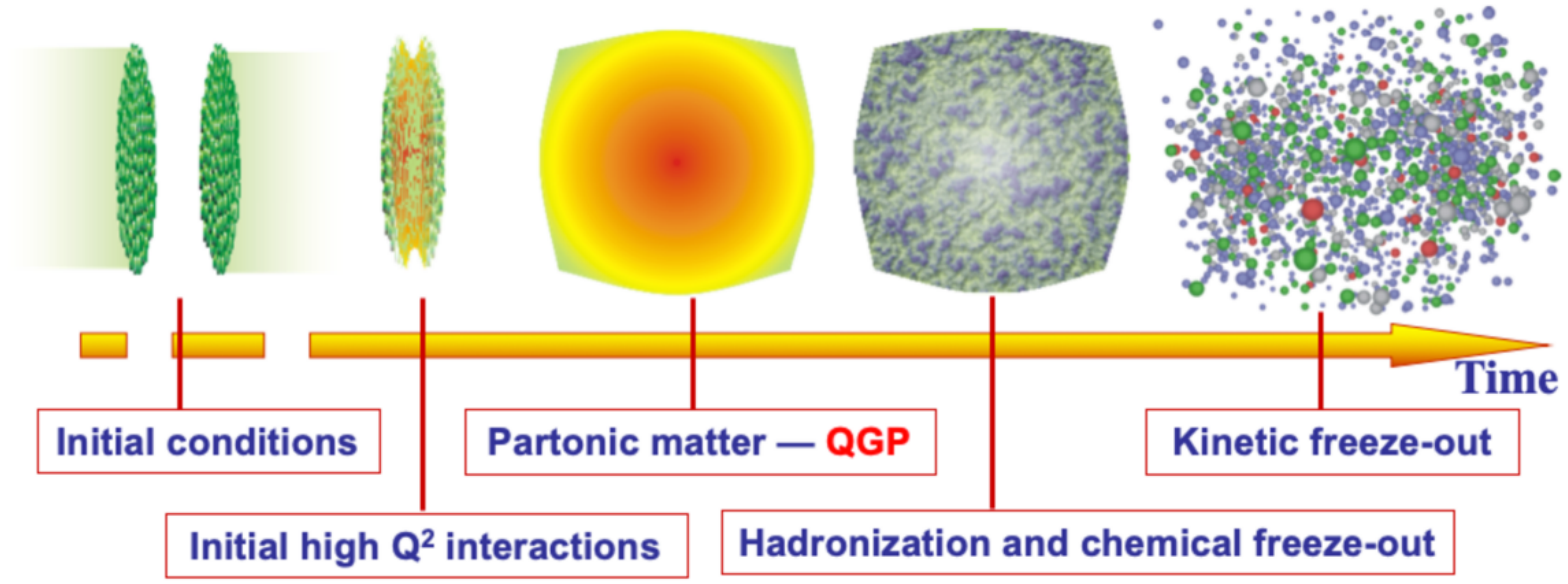


Figure: Evolution stages of ultra-relativistic heavy-ion collision. Time in the horizontal axis has the unit of fm/c and Q^2 denotes four-momentum transfer squared.
(Nucl.Phys.News 30 (2020) 2, 10-16)

Spin polarization - I

Phys.Rev.Lett. 94 (2005) 102301, Phys. Rev. C 77, 024906

→ Non-central ultra-relativistic heavy-ion collisions, due to spatial inhomogeneity, create large orbital angular momentum, $\boldsymbol{L}_{\text{initial}} \approx 10^5\,\hbar$.

Phys. Rev. Lett. 94 (2005) 102301

→ This orbital angular momentum is along y-axis (orthogonal to the reaction $(x - z)$ plane) and may polarize spin of the QGP constituents.

→ The spin polarization of QGP constituents is then transferred to the hadrons leading to their global spin polarization along y-axis.

→ Among various spin-polarizable hadrons, Lambda $(\Lambda(\bar{\Lambda}))$ hyperons are special as they are self-analyzing.

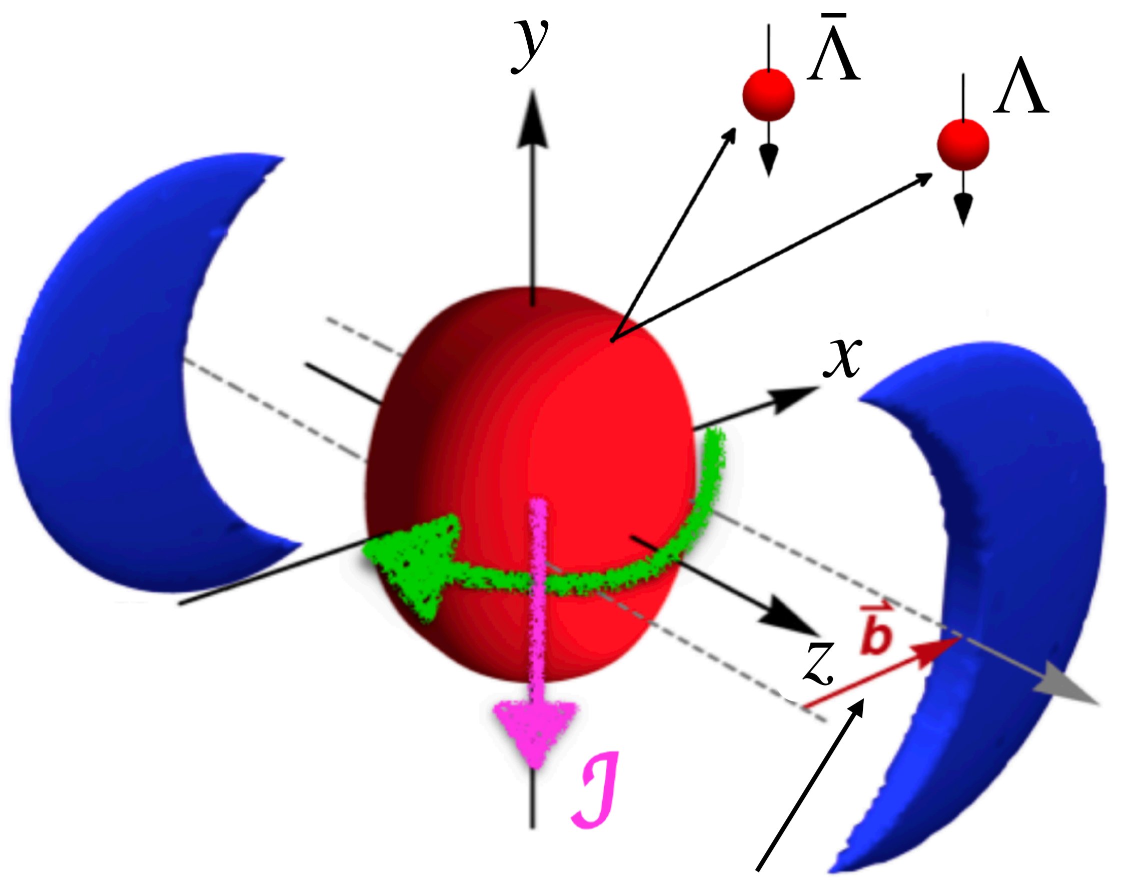


Figure: Schematic diagram of the initial angular momentum orientation in non-central heavy-ion collision.
(Prog.Part.Nucl.Phys. 108 (2019) 103709)

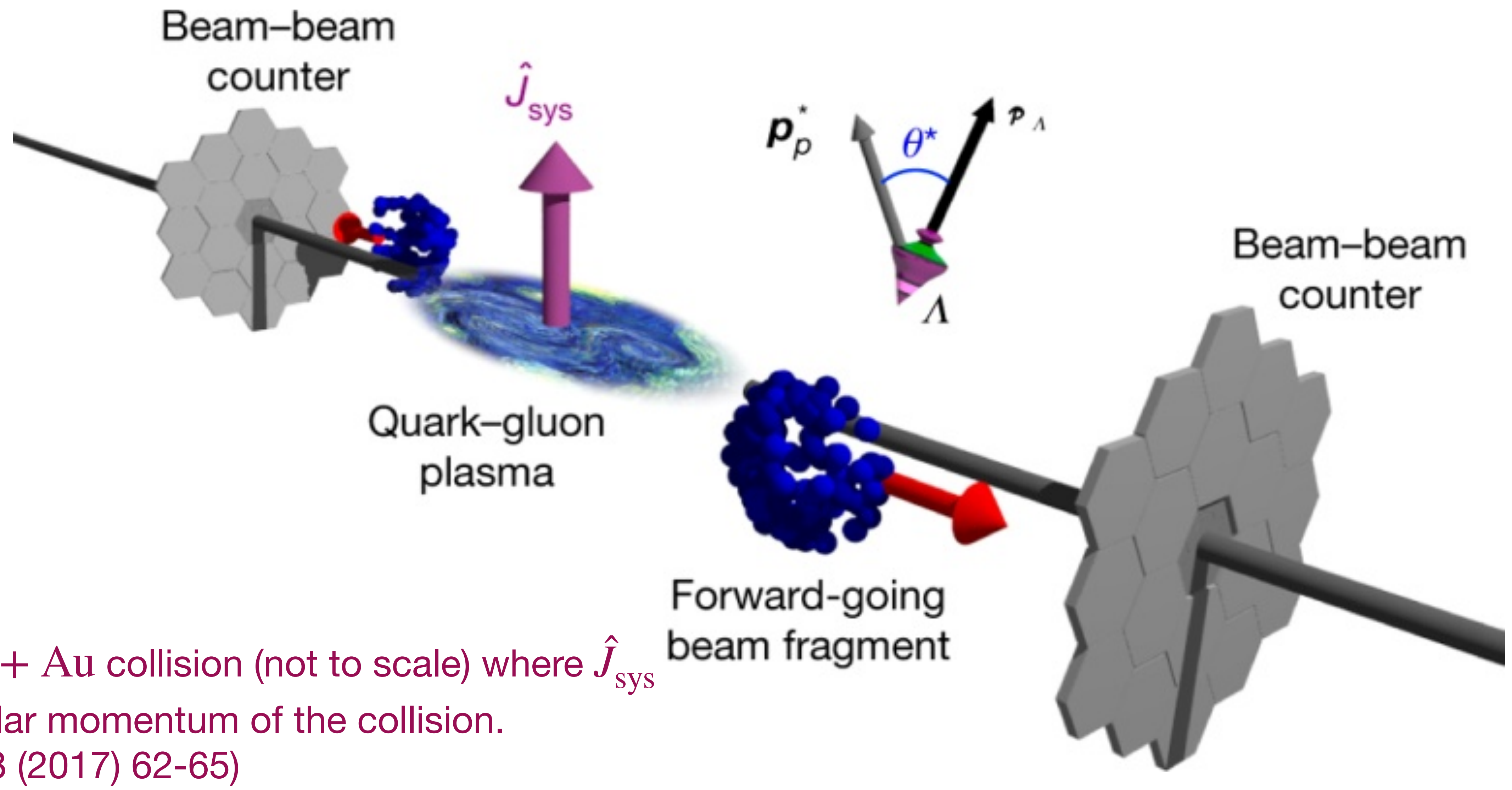


Figure: Schematic diagram of a $\mathrm{Au} + \mathrm{Au}$ collision (not to scale) where $\hat{J}_{\text{sys}}$ is the direction of the angular momentum of the collision.
(Nature 548 (2017) 62-65)

Spin polarization - II

- First observation of global spin polarization of $\Lambda(\bar{\Lambda})$ was by STAR collaboration, providing evidence of the vortical structure of QGP.

- It shows decreasing behavior with increase in collision energy.

- Differences between Λ and $\bar{\Lambda}$ polarization may be due to initial electromagnetic field caused during the collisions, however, we do not have clear explanation yet.

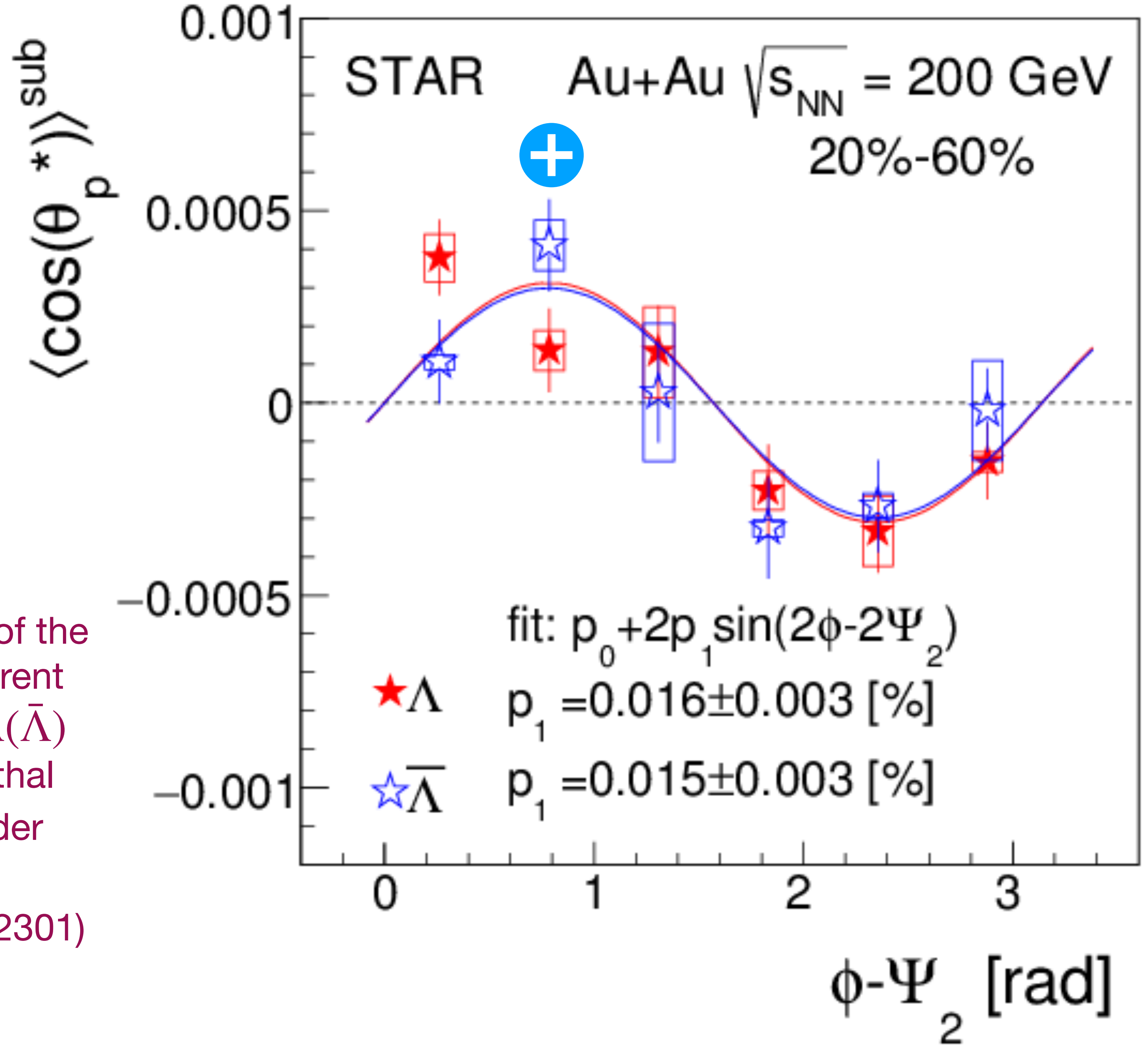


Figure: Cosine of the polar angle of the proton in the rest frame of its parent $\Lambda(\bar{\Lambda})$ that is averaged over all $\Lambda(\bar{\Lambda})$ particles as a function of azimuthal angle (ϕ) relative to second-order event plane (ψ). (Phys.Rev.Lett. 123 (2019) 13, 132301)

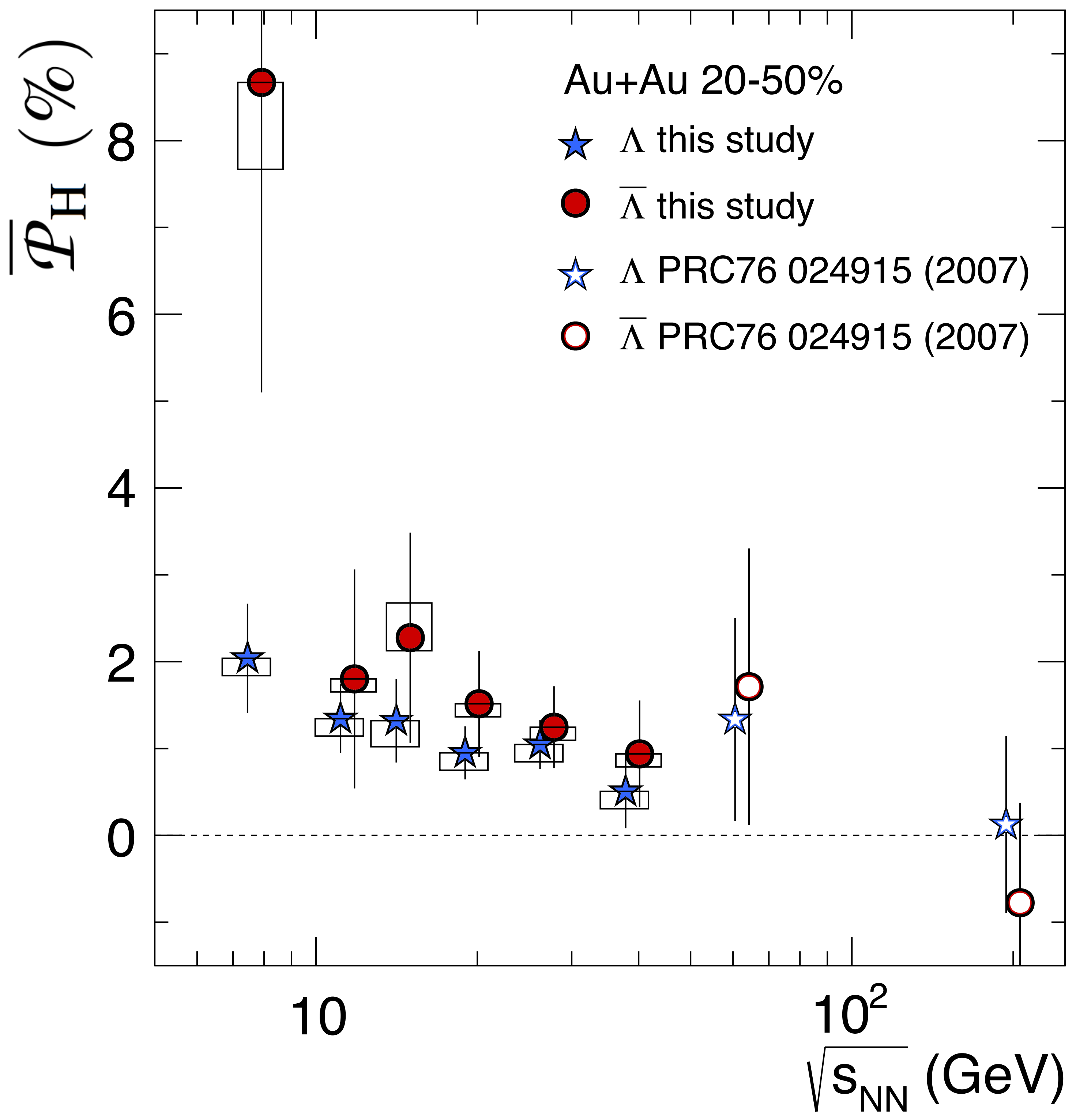


Figure: Average global spin polarization for $\Lambda(\bar{\Lambda})$ hyperons in 20-50% centrality Au + Au collisions as a function of collision energy. (Nature 548 (2017) 62-65)

- Besides the global spin polarization along y-axis, STAR also observed spin polarization of $\Lambda(\bar{\Lambda})$ along the beam direction (z) which may result from the flow structure in the transverse plane.

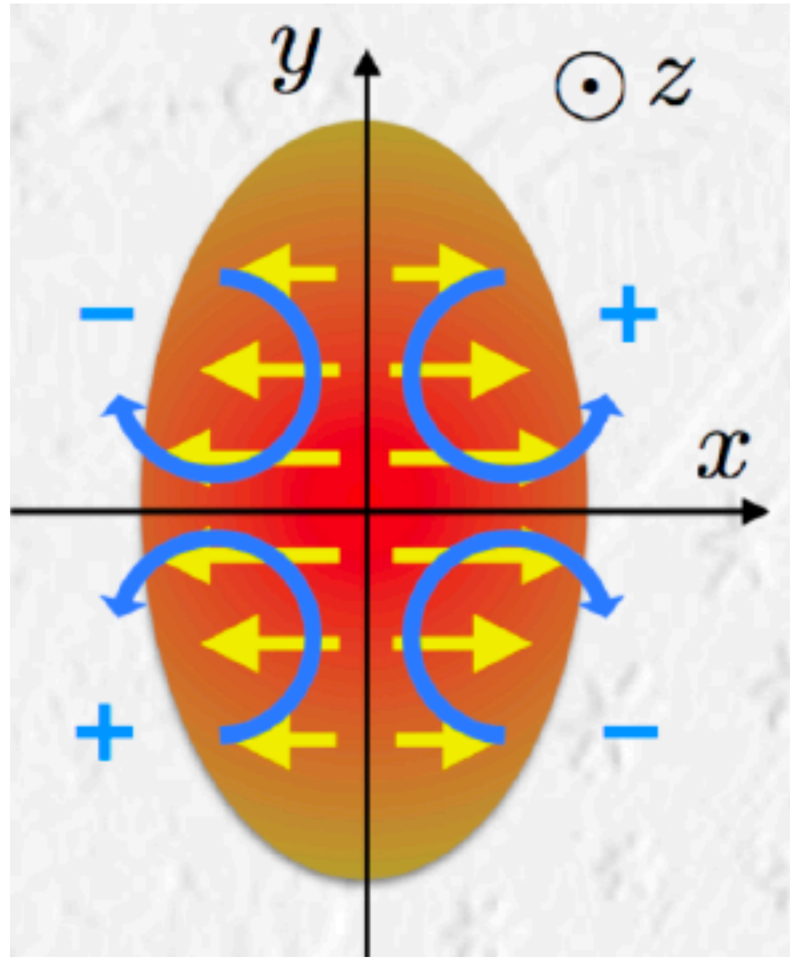


Figure: Schematic view of the flow structure in the transverse plane which may generate longitudinal polarization. (Phys.Rev.Lett. 120 (2018) 1, 012302) (EPJ Web Conf. 171 (2018) 07002)

Objective

→ Models that assume local thermodynamic equilibrium (LTE) of spin degrees of freedom are able to explain global spin polarization measurement.

→ However, they were unsuccessful to provide clear theoretical explanation for the azimuthal angle dependence of longitudinal polarization.

Phys.Rev.Lett. 127 (2021) 27, 272302, Phys.Rev.Lett. 127 (2021) 14, 142301

→ As QGP behaves as a perfect fluid so it is natural to use relativistic hydrodynamics to study its properties.

→ Lack of clear understanding of spin polarization motivates us to consider a new hydrodynamic approach where spin polarization is an independent dynamical variable.

→ Thus, we will discuss such an approach where spin degrees of freedom are incorporated into standard relativistic perfect fluid hydrodynamics.

Canonical currents

→ Being an effective theory, hydrodynamics is defined at a length scale larger than the mean free path of microscopic particles but smaller than the system size.

→ For formulating hydrodynamics with spin, we need to define energy-momentum ($T^{\mu\nu}$) and spin ($S^{\lambda,\mu\nu}$) currents as ensemble averages of their respective normal-ordered QFT operators

$$\boxed{T^{\mu\nu} = \langle :\hat{T}^{\mu\nu}: \rangle\,, \quad S^{\lambda,\mu\nu} = \langle :\hat{S}^{\lambda,\mu\nu}: \rangle}$$

→ For a system with spin we have

$$\hat{J}^{\lambda,\mu\nu} = \hat{L}^{\lambda,\mu\nu} + \hat{S}^{\lambda,\mu\nu} = x^{\mu}\hat{T}^{\lambda\nu} - x^{\nu}\hat{T}^{\lambda\mu} + \hat{S}^{\lambda,\mu\nu}$$

Conservation of total angular momentum

$$\partial_\lambda \hat{J}^{\lambda,\mu\nu} = \partial_\lambda \hat{L}^{\lambda,\mu\nu} + \partial_\lambda \hat{S}^{\lambda,\mu\nu} = \hat{T}^{\mu\nu} - \hat{T}^{\nu\mu} + \partial_\lambda \hat{S}^{\lambda,\mu\nu} = 0$$

gives

$$\partial_\lambda \hat{S}^{\lambda,\mu\nu} = \hat{T}^{\nu\mu} - \hat{T}^{\mu\nu}$$

We also have $\partial_\mu \hat{T}^{\mu\nu} = 0$

For massive Dirac particles:

$$\hat{T}^{\mu\nu}_{\rm Can} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi - g^{\mu\nu}\mathscr{L}_D$$

$$\hat{S}^{\lambda,\mu\nu}_{\rm Can} = \frac{i}{8}\bar{\psi}\left\{\gamma^{\lambda}, \left[\gamma^{\mu}, \gamma^{\nu}\right]\right\}\psi$$

ψ & $\bar{\psi}$ are Dirac field operators

$\mathscr{L}_D$ is Dirac Lagrangian

$g^{\mu\nu} = \{1, -1, -1, -1\}$

$\overleftrightarrow{\partial} \equiv \overrightarrow{\partial} - \overleftarrow{\partial}$

QFT, Itzykson and Zuber (Saclay 1980)

de Groot–van Leeuwen–van Weert pseudogauge

→ However, one can obtain new pair of $\hat{T}^{\mu\nu}$ and $\hat{S}^{\lambda,\mu\nu}$ using $\hat{T}^{\mu\nu}_{\rm Can}$ and $\hat{S}^{\lambda,\mu\nu}_{\rm Can}$ through pseudogauge transformation

Rept.Math.Phys. 9 (1976) 55-82,

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{\rm Can} + \frac{1}{2}\partial_\lambda(\hat{\Pi}^{\lambda,\mu\nu} + \hat{\Pi}^{\nu,\mu\lambda} + \hat{\Pi}^{\mu,\nu\lambda})$$

$$\hat{S}^{\lambda,\mu\nu} = \hat{S}^{\lambda,\mu\nu}_{\rm Can} - \hat{\Pi}^{\lambda,\mu\nu} + \partial_\rho \hat{\Upsilon}^{\mu\nu,\lambda\rho}$$

$$\hat{\Pi}^{\lambda,\mu\nu} = -\hat{\Pi}^{\lambda,\nu\mu}$$

$$\hat{\Upsilon}^{\mu\nu,\lambda\rho} = -\hat{\Upsilon}^{\nu\mu,\lambda\rho} = -\hat{\Upsilon}^{\mu\nu,\rho\lambda}$$

$$\sigma^{\mu\nu} = (i/2)\left[\gamma^\mu, \gamma^\nu\right]$$

→ One may have several choices of $\hat{\Pi}^{\lambda,\mu\nu}$ & $\hat{\Upsilon}^{\mu\nu,\lambda\rho}$, however, we choose

$$\hat{\Pi}^{\lambda,\mu\nu} = \frac{i}{4m}\bar{\psi}(\sigma^{\lambda\mu}\overleftrightarrow{\partial}^{\nu} - \sigma^{\lambda\nu}\overleftrightarrow{\partial}^{\mu})\psi$$

$$\hat{\Upsilon}^{\mu\nu,\lambda\rho} = 0$$

$$\hat{T}^{\mu\nu}_{\rm GLW} = -\frac{1}{4m}\bar{\psi}\overleftrightarrow{\partial}^{\mu}\overleftrightarrow{\partial}^{\nu}\psi$$

$$\hat{S}^{\lambda,\mu\nu}_{\rm GLW} = \bar{\psi}\left[\frac{\sigma^{\mu\nu}}{4} - \frac{1}{8m}\left(\gamma^\mu\overleftrightarrow{\partial}^{\nu} - \gamma^\nu\overleftrightarrow{\partial}^{\mu}\right)\right]\gamma^\lambda\psi + {\rm h.c}$$

S. De Groot, W. Van Leeuwen, and C. Van Weert, Relativistic Kinetic Theory. Principles and Applications. North Holland, 1, 1980

Prog.Part.Nucl.Phys. 108 (2019) 103709

Relativistic hydrodynamics with spin

Prog.Part.Nucl.Phys. 108 (2019) 103709

→ Using Wigner function (in equilibrium) formalism and ansatz for local equilibrium distribution functions

$$f^+_{rs}(x,p) = \frac{1}{2m}\bar{\mathscr{U}}_r(p)\, X^+\, \mathscr{U}_s(p)$$
$$f^-_{rs}(x,p) = -\frac{1}{2m}\bar{\mathscr{V}}_s(p)\, X^-\, \mathscr{V}_r(p)$$

$$X^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu\right]\left[1 \pm \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]$$
$$\Sigma^{\mu\nu} = (i/4)\left[\gamma^\mu,\gamma^\nu\right], \quad \xi(x) = \mu_B/T, \quad \beta_\mu(x) = U^\mu/T$$
$$\mathscr{C} = \cosh(\xi), \quad \Delta^{\mu\nu} = g^{\mu\nu} - (U^\mu U^\nu)/(U\cdot U)$$
$$\mathscr{B}_{(0)} = -\frac{2}{z^2}\frac{\mathscr{E}_{(0)}+\mathscr{P}_{(0)}}{T}, \quad \mathscr{A}_{(0)} = 2\mathscr{N}_{(0)} - 3\mathscr{B}_{(0)}$$
$$z = m/T$$

One can derive the <u>constitutive relations</u> for

- Net baryon current

$$N^\alpha(x) = \mathscr{N}\, U^\alpha$$

with

$$\mathscr{N} = 4\sinh(\xi)\,\mathscr{N}_{(0)}(T)$$
$$\mathscr{N}_{(0)}(T) = \frac{T^3}{2\pi^2}z^2K_2(z)$$

- Energy-momentum tensor

$$T^{\mu\nu}_{\rm GLW}(x) = (\mathscr{E}+\mathscr{P})\,U^\mu U^\nu - \mathscr{P}\,g^{\mu\nu}$$

with

$$\mathscr{E} = 4\cosh(\xi)\,\mathscr{E}_{(0)}(T)$$
$$\mathscr{P} = 4\cosh(\xi)\,\mathscr{P}_{(0)}(T)$$
$$\mathscr{E}_{(0)}(T) = \frac{T^4}{2\pi^2}z^2\left[zK_1(z) + 3K_2(z)\right]$$
$$\mathscr{P}_{(0)}(T) = T\mathscr{N}_{(0)}(T)$$

- Spin tensor

$$S^{\alpha,\beta\gamma}_{\rm GLW} = U^\alpha\left(\mathscr{A}_1\,\omega^{\beta\gamma} + \mathscr{A}_2\,U^{[\beta}\,\omega^{\gamma]}{}_\delta\,U^\delta\right) + \mathscr{A}_3\left(U^{[\beta}\,\omega^{\gamma]\alpha} + g^{\alpha[\beta}\,\omega^{\gamma]}{}_\delta\,U^\delta\right)$$

with

$$\mathscr{A}_1 = \mathscr{C}\left(\mathscr{N}_{(0)} - \mathscr{B}_{(0)}\right)$$
$$\mathscr{A}_2 = \mathscr{C}\left(\mathscr{A}_{(0)} - 3\mathscr{B}_{(0)}\right)$$
$$\mathscr{A}_3 = \mathscr{C}\,\mathscr{B}_{(0)}$$

→ $$\boxed{\partial_\alpha N^\alpha = 0\,,\ \partial_\alpha T^{\alpha\beta}_{\rm GLW} = 0\,,\ \partial_\alpha S^{\alpha,\beta\gamma}_{\rm GLW} = 0.}$$

Dispersion relation of spin-wave velocity - I

$\kappa \cdot U = \omega \cdot U = 0$

$\kappa_\mu = \omega_{\mu\alpha} U^\alpha , \quad \omega_\mu = (1/2)\, \epsilon_{\mu\alpha\beta\gamma}\, \omega^{\alpha\beta} U^\gamma$

→ $\omega_{\mu\nu}$ is an antisymmetric tensor of rank 2: $\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta$

→ We introduce a basis formed by a set of mutually orthogonal four-vectors: $U, X, Y, \& \, Z$

$$\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha$$

Thus

$$\omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$$

where

$$\boldsymbol{C_\kappa} = (C_{\kappa X}, C_{\kappa Y}, C_{\kappa Z})$$

$$\boldsymbol{C_\omega} = (C_{\omega X}, C_{\omega Y}, C_{\omega Z})$$

are spin components

→ In an unpolarized fluid at rest, $U^\mu = (1{,}0{,}0{,}0) \,\&\, \omega^{\mu\nu} = 0$. Considering small perturbations along z, we look for oscillations in $\omega_{\mu\nu}$.

→ Perfect fluid background leads to well-known sound speed

$$c_s^2 = \left(\frac{\partial \mathscr{P}}{\partial \mathscr{E}}\right)_{\mathscr{N}} + \frac{\mathscr{N}}{\mathscr{E} + \mathscr{P}} \left(\frac{\partial \mathscr{P}}{\partial \mathscr{N}}\right)_{\mathscr{E}}$$

→ We find $\partial_t C_{\kappa Z} = \partial_t C_{\omega Z} = 0$ and remaining components propagate as transverse waves.

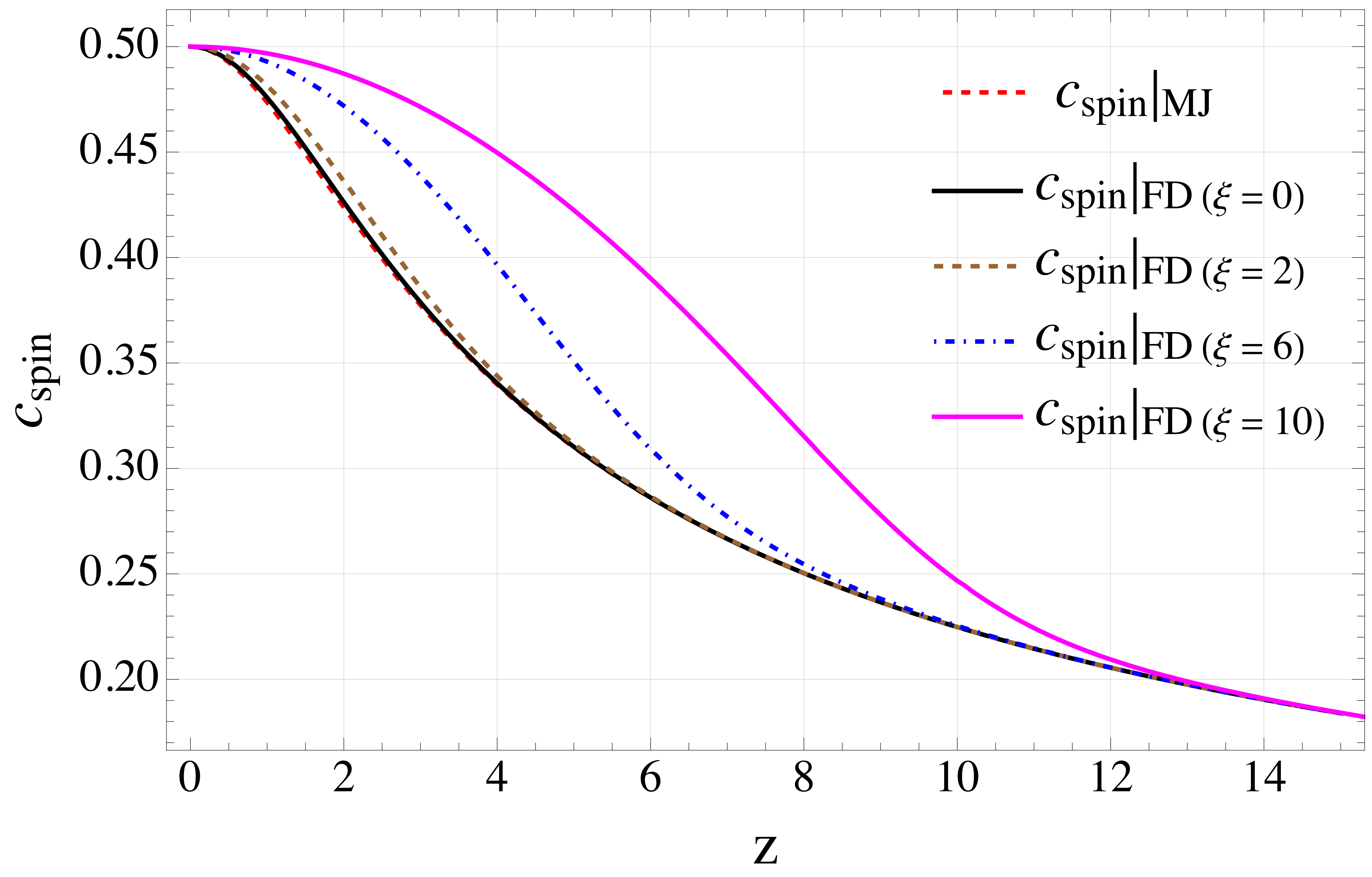
$$\boxed{c_{\rm spin}^2 = \frac{1}{4} \frac{\left(\partial \mathscr{E}/\partial T\right)_\xi - z^2 \left(\partial \mathscr{N}/\partial \xi\right)_T}{\left(\partial \mathscr{E}/\partial T\right)_\xi + \frac{z^2}{2} \left(\partial \mathscr{N}/\partial \xi\right)_T}}$$

Dispersion relation of spin-wave velocity - II

→ Ideal-gas limit: $$c^2_{\rm spin}\Big|_{\rm MJ} = \frac{1}{4}\left[\frac{K_3(z)}{K_3(z)+\frac{z}{2}K_2(z)}\right]$$

→ Fermi-Dirac gas limit:

$$c^2_{\rm spin}\Big|_{\rm FD} = \frac{1}{4}\,\frac{\sum_{\ell=1}^{\infty}\frac{(-1)^{\ell+1}}{\ell}\cosh(\ell\xi)K_3(\ell z)}{\sum_{\ell=1}^{\infty}\frac{(-1)^{\ell+1}}{\ell}\cosh(\ell\xi)\left[K_3(\ell z)+\frac{\ell z}{2}K_2(\ell z)\right]}$$



→ Linearly polarized solutions:

$$C_{\boldsymbol{\kappa}} = C_0\,\mathrm{Re}\left[e^{-ik\left(c_{\rm spin}t - z\right)}\right]\left(\hat{\boldsymbol{e}}_1\cos(\theta)+\hat{\boldsymbol{e}}_2\sin(\theta)\right)$$

$$C_{\boldsymbol{\omega}} = 2\,c_{\rm spin}\,C_0\,\mathrm{Re}\left[e^{-ik\left(c_{\rm spin}t - z\right)}\right]\left(\hat{\boldsymbol{e}}_1\sin(\theta)-\hat{\boldsymbol{e}}_2\cos(\theta)\right)$$

where analogy to the EM waves is evident since $C_{\boldsymbol{\omega}} = 2c_{\rm spin}\hat{\boldsymbol{n}}\times C_{\boldsymbol{\kappa}}$

$kc_{\rm spin}$ is the angular frequency

$\xi = \mu_B/T\,,\quad z = m/T$

θ is the inclination angle with respect to x-axis

$\hat{\boldsymbol{n}} = \hat{\boldsymbol{e}}_3$ being the direction of the wave propagation

C_0 is real amplitude of the wave

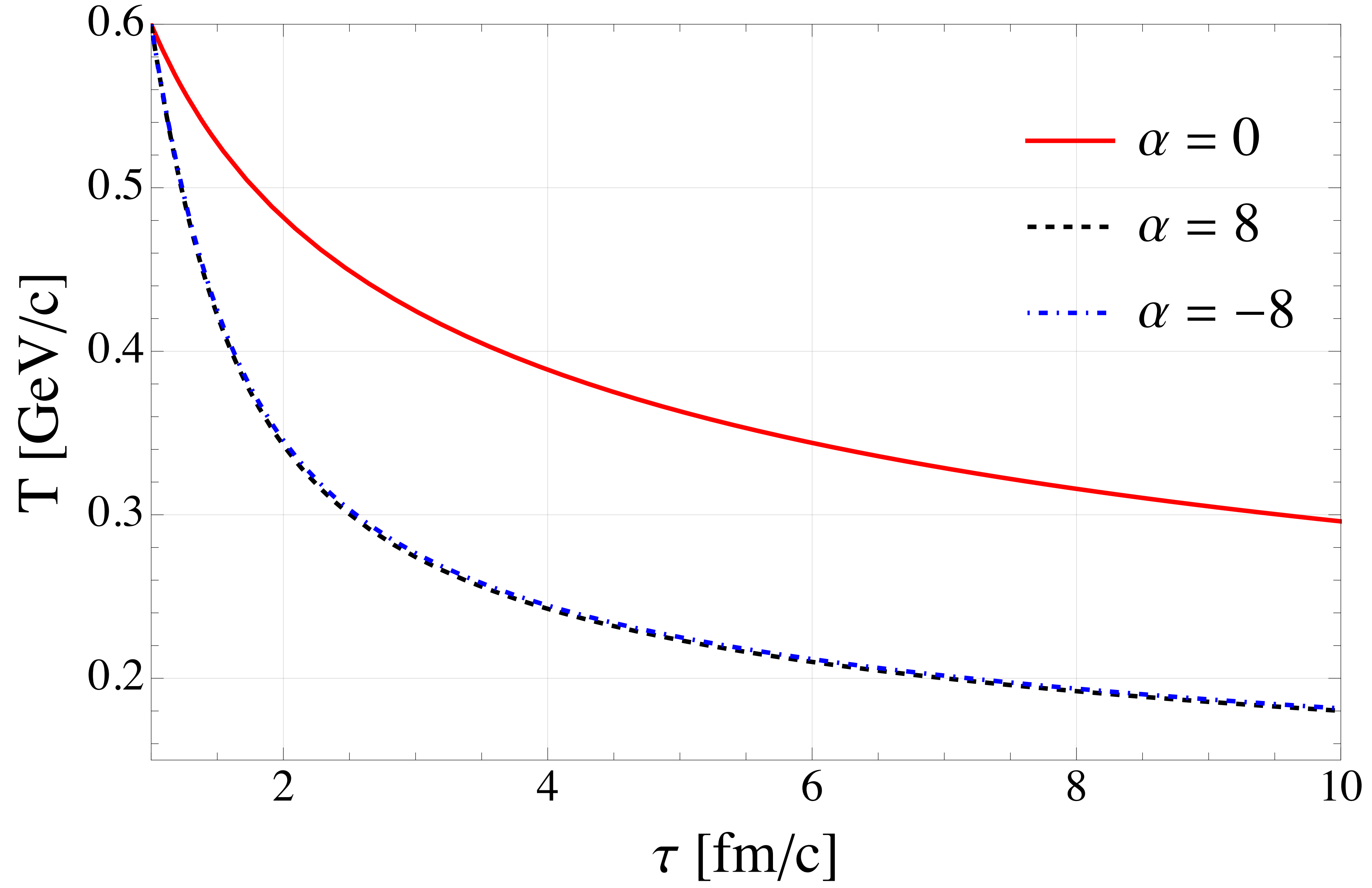
Modeling of the spin polarization dynamics - I

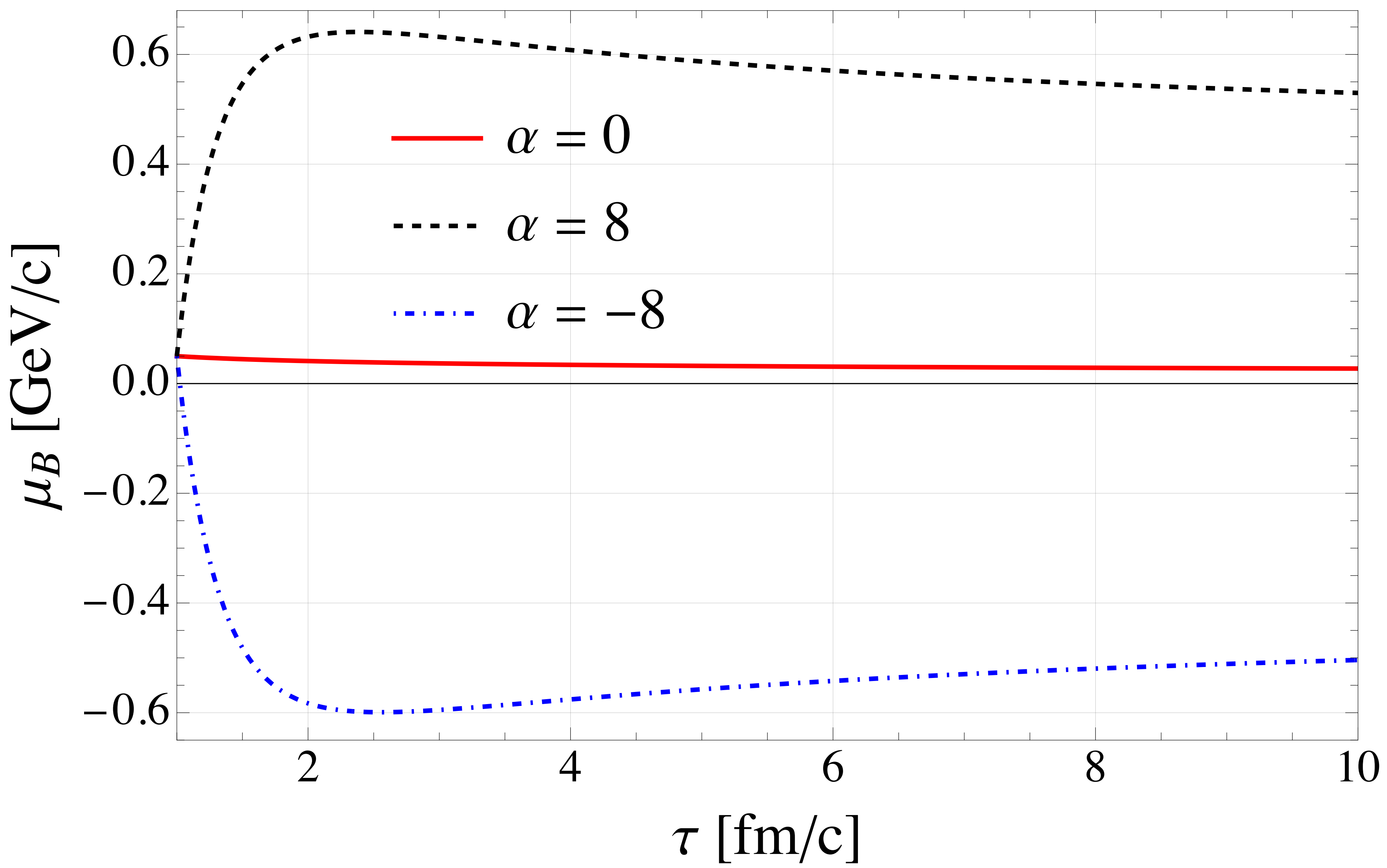
→ Boost-invariant and transversely homogeneous w/ & w/o external electric field

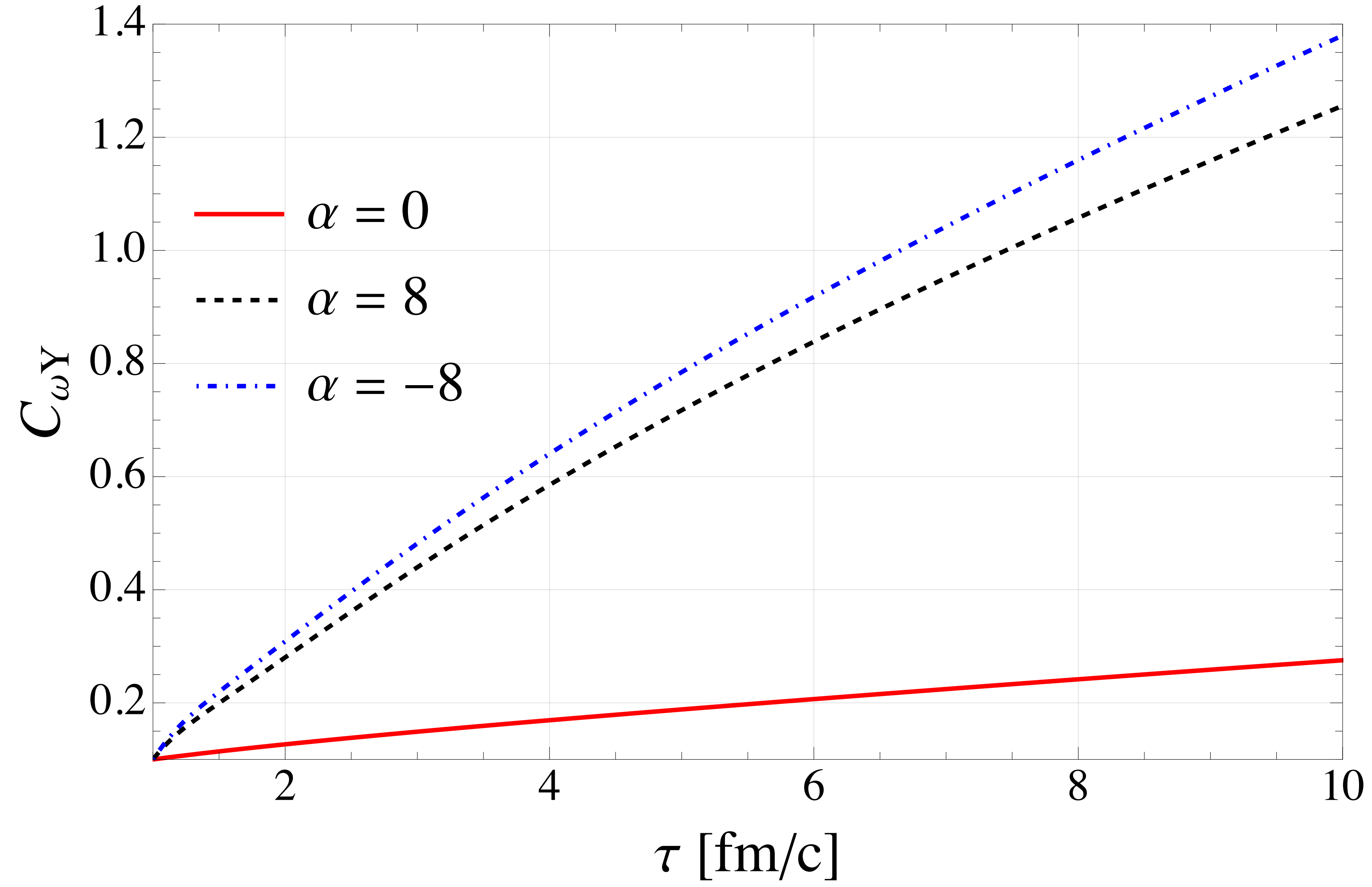
m = 1.116 GeV T_0 = 0.6 GeV μ_{B0} = 0.05 GeV τ_0 = 1 fm/c

$C_{\omega Y_0} = 0.1$

$E_0 = m_\pi^2/e$







→ Mean spin polarization per particle

$$\langle \pi_\mu \rangle_p = \frac{E_p \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

$$E_p \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p} = -\frac{1}{(2\pi)^3 m} \int \cosh(\xi)\, \Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}\, (\overset{\star}{\omega}_{\mu\beta} p^\beta)^*$$

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4}{(2\pi)^3} \int \cosh(\xi)\, \Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}$$

Freeze-out hyper-surface element

Modeling of the spin polarization dynamics - II

→ Non-boost-invariant and transversely homogeneous

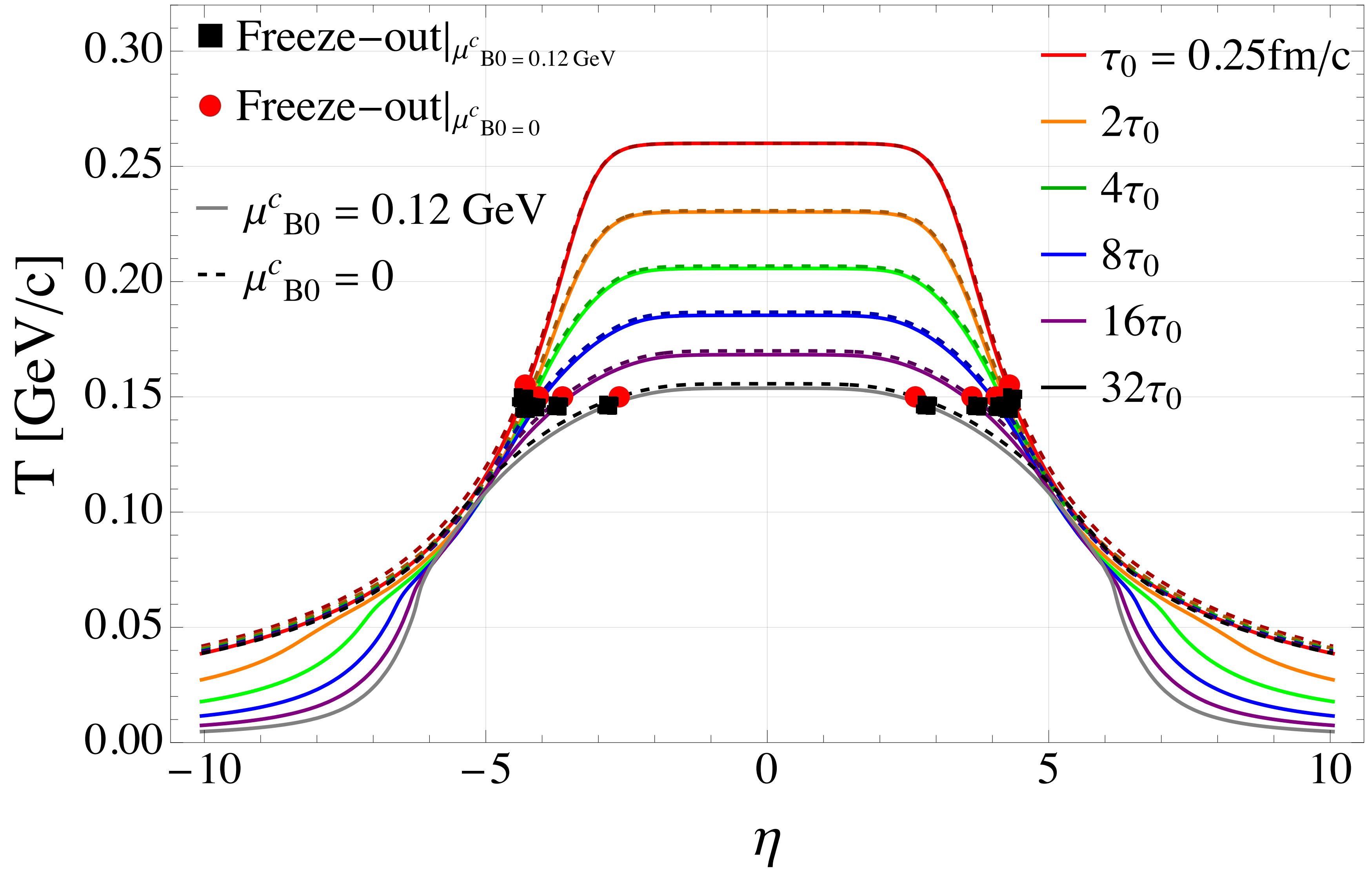
$$\mathscr{E}_0(\eta) = \frac{\mathscr{E}_0^c}{2}\left[\Theta(\eta)\left(\tanh(a-\eta\, b)+1\right) + \Theta(-\eta)\left(\tanh(a+\eta\, b)+1\right)\right]$$

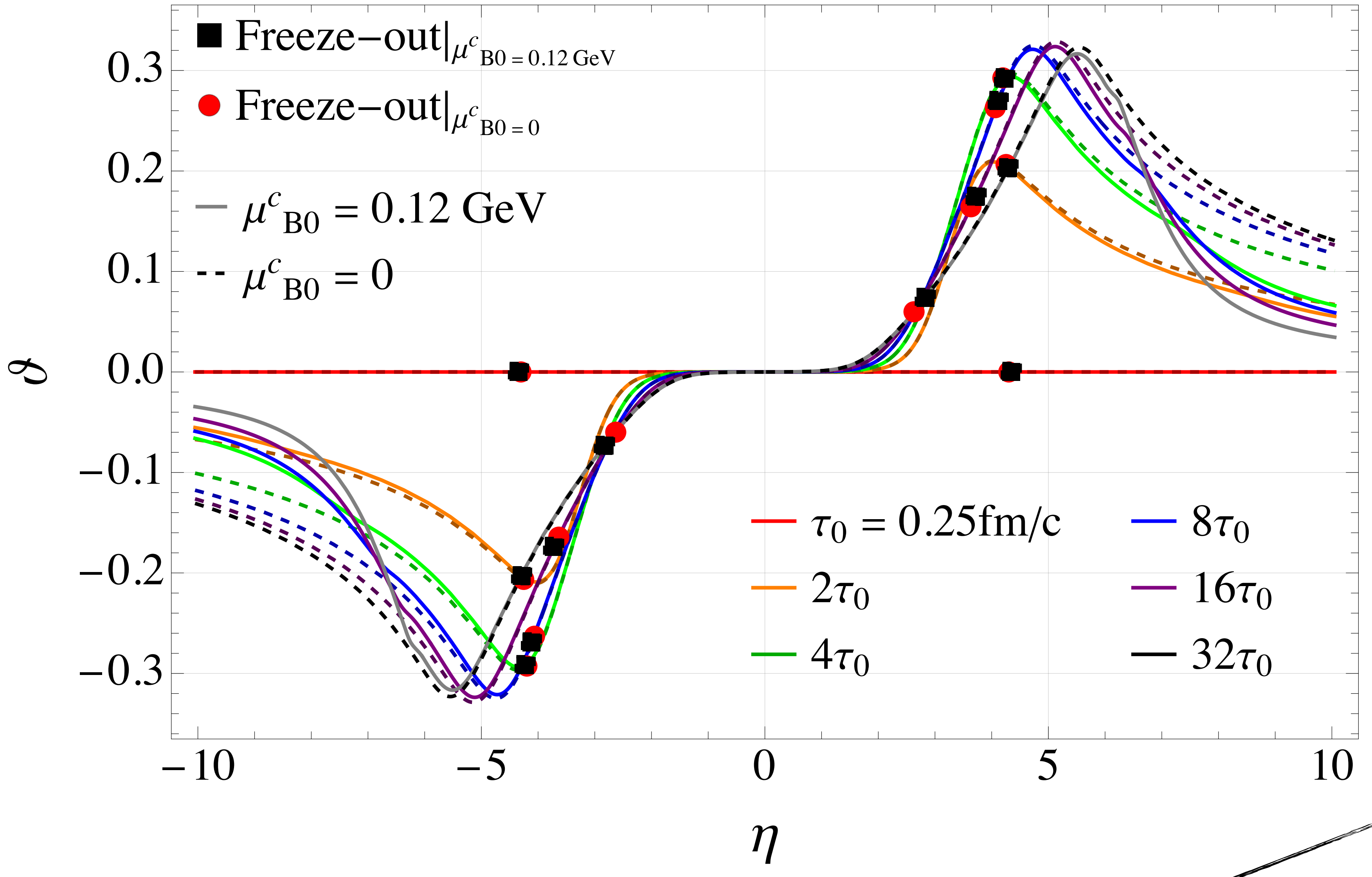
$a = 6.2\,, b = 1.9$

$T_0^c = 0.26$ GeV

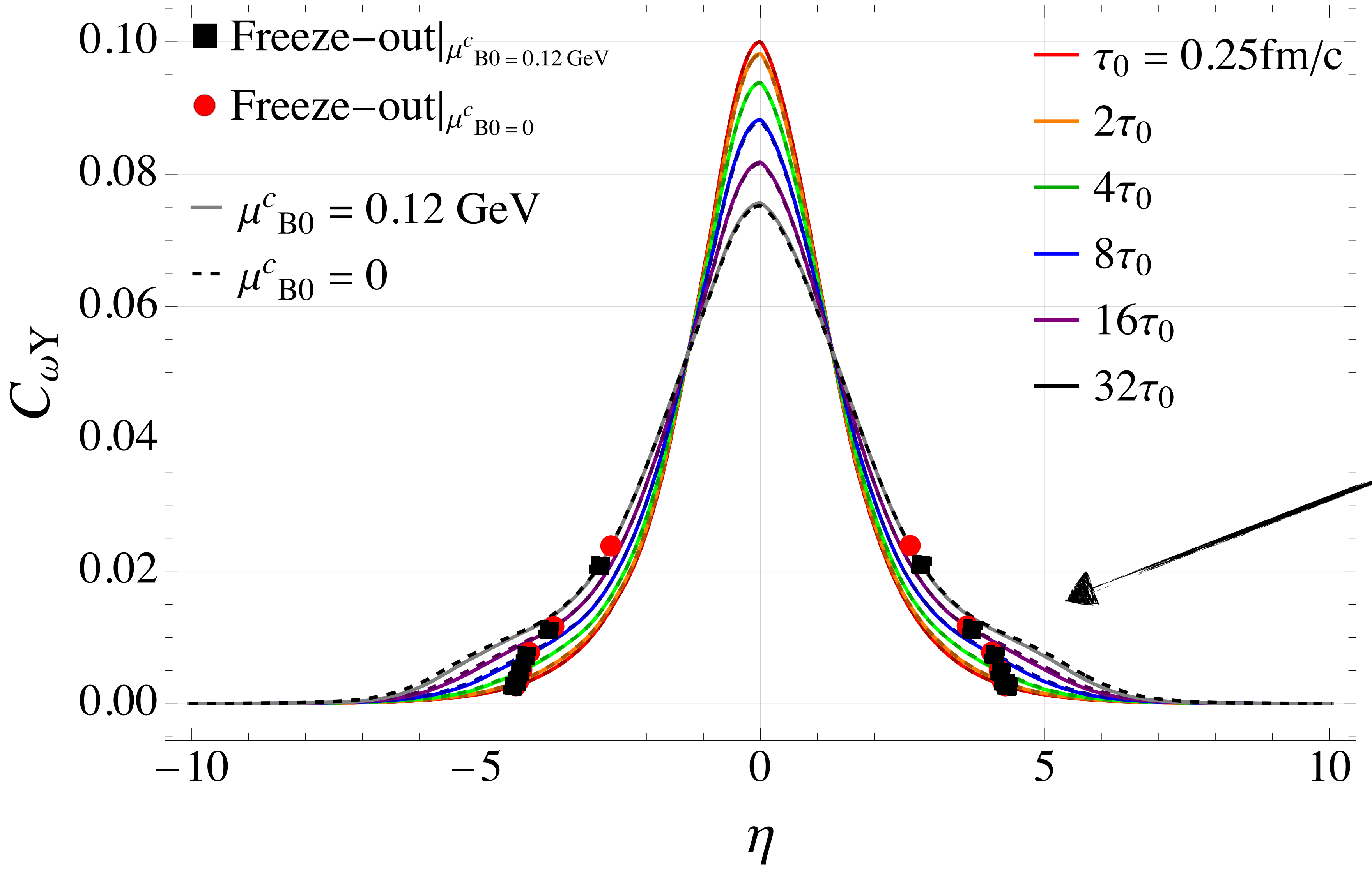
$\mathscr{E}_0^c = \mathscr{E}(T_0^c, \mu_{B0}^c)$

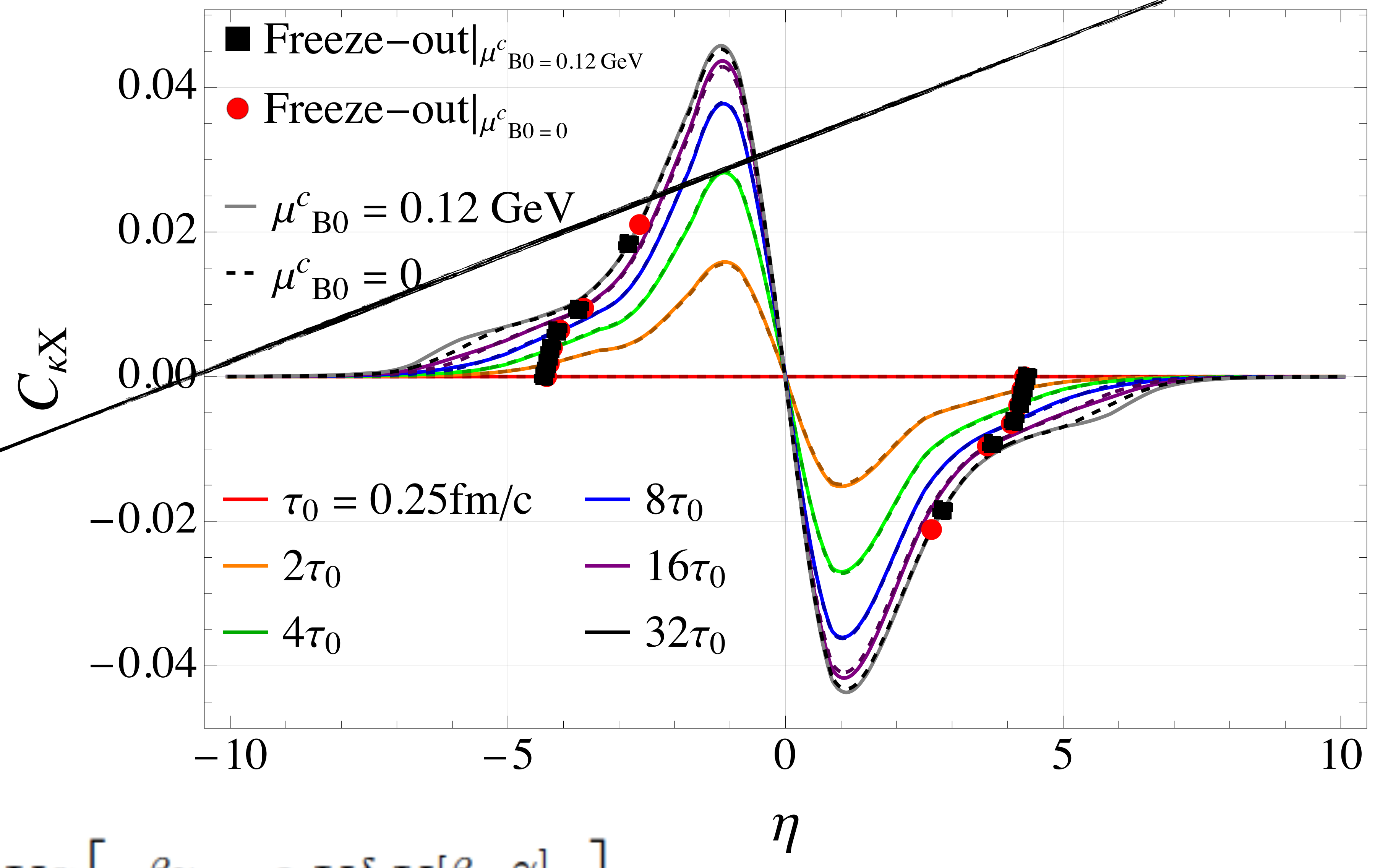
$C_{\omega Y_0} = 0.1\ \mathrm{sech}(\eta)$





Pattern is opposite in comparison to $\eta = 0$





$$\langle \pi_\mu \rangle = \frac{\int \mathrm{dP}\, \langle \pi_\mu \rangle_p\, E_p \frac{d\mathcal{N}(p)}{d^3p}}{\int \mathrm{dP}\, E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

$$\langle \pi_\mu(p_T) \rangle = \frac{\frac{1}{2\pi}\int d\phi_p\, \sin(2\phi_p)\, E_p \frac{d\Pi^*_\mu(p)}{d^3p}}{\int d\phi_p\, E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

$$\langle \pi_\mu(\phi_p) \rangle = \frac{\int p_T\, dp_T\, E_p \frac{d\Pi^*_\mu(p)}{d^3p}}{\int d\phi_p\, p_T\, dp_T\, E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

$$S^{\alpha,\beta\gamma}_{z\gg 1} = \cosh(\xi)\, \mathcal{N}_{(0)}\, U^\alpha \left[\omega^{\beta\gamma} + 2\, U^\delta\, U^{[\beta}\omega^{\gamma]}{}_\delta\right]$$

Modeling of the spin polarization dynamics - III

$\mu_B \neq 0$ plots not shown as they are qualitatively similar

→ Non-boost-invariant and transversely homogeneous

Conformal symmetry of perfect-fluid hydrodynamics with spin

⟶ Gubser-expanding perfect fluid background

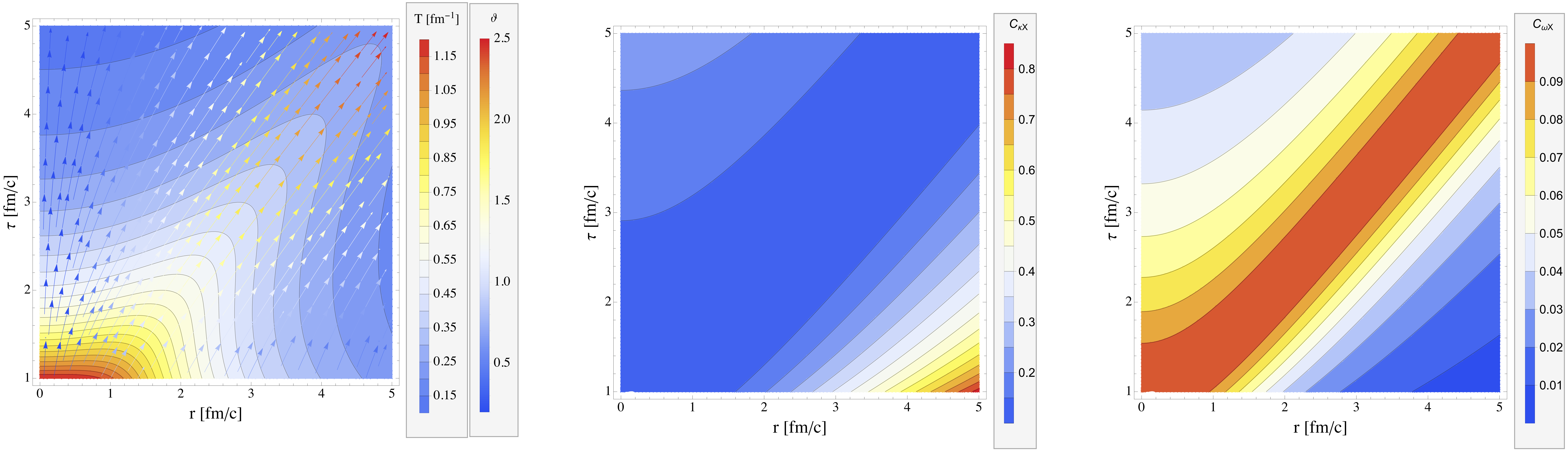
d_α is covariant derivative

$\Omega_{\rm cf} = \tau$

$$d_\alpha N^\alpha = \Omega_{\rm cf}^4\, \hat{\hat{d}}_\alpha \hat{\hat{N}}^\alpha$$

$$d_\alpha T^{\alpha\beta} = \Omega_{\rm cf}^6 \left[\hat{\hat{d}}_\alpha \hat{\hat{T}}^{\alpha\beta} - \hat{\hat{T}}^\lambda{}_\lambda\, \hat{\hat{g}}^{\beta\delta}\partial_\delta\varphi\right]$$

$$d_\alpha S^{\alpha\beta\gamma} = \Omega_{\rm cf}^6 \left[\hat{\hat{d}}_\alpha \hat{\hat{S}}^{\alpha\beta\gamma} - (\hat{\hat{S}}_\lambda{}^{\lambda\gamma}\, \hat{\hat{g}}^{\beta\sigma} + \hat{\hat{S}}^{\alpha\beta}{}_\alpha\, \hat{\hat{g}}^{\sigma\gamma})\partial_\sigma\varphi\right]$$



$T(\tau_0 = 1\,{\rm fm}, r = 0) = 1.2\,{\rm fm}^{-1} = 0.24\,{\rm GeV}$

$C_{\kappa X}(\tau_0 = 1{\rm fm}, r = 0) = C_{\omega X}(\tau_0 = 1{\rm fm}, r = 0) = 0.1$


Thank you for listening!
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IM. HENRYKA NIEWODNICZAŃSKIEGO
POLSKIEJ AKADEMII NAUK