Trento Institute for
Fundamental Physics
and Applications

Istituto Nazionale di Fisica Nucleare

UNIVERSITY
OF TRENTO

Weekly Seminar
School of Particles and Accelerators
IPM, Tehran

THE ORIGIN OF MAGNETIC FIELDS DURING INFLATION

HELICITY AND A SAWTOOTH COUPLING

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OUTLINE

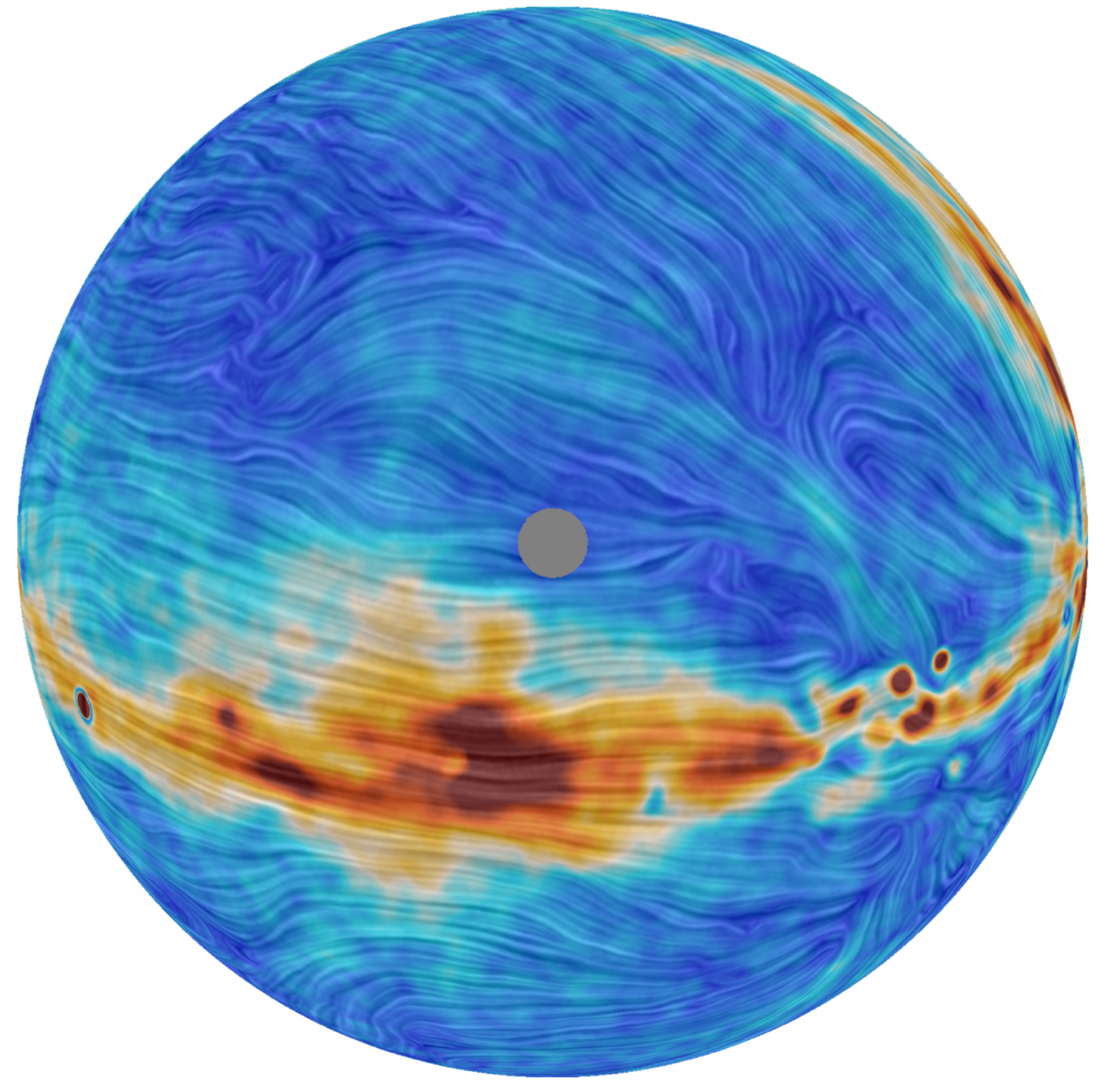
1. MAGNETIC FIELDS AND INFLATION

2. MODEL OF MAGNETOGENESIS

3. APPLICATION TO SCALE-INVARIANT GRAVITY

OBSERVATIONS: ASTROPHYSICAL AND COSMOLOGICAL SCALES

- Galaxies and clusters: $B_0 \sim \mu$G coherent over $\sim$ kpc scales
- Redshift-independent
- IGM: $10^{-16} \lesssim B_0 \lesssim 10^{-9}$ G on Mpc scales
  Upper bounds: CMB
  Lower bounds: blazar observations



Map of polarized microwave emission in the Galactic northern hemisphere measured by QUIJOTE. Credit: [1].

MAGNETOGENESIS: HINTS FOR MAGNETIC FIELDS' ORIGIN

ASTROPHYSICAL

- Seed generation via battery and subsequent amplification via galactic dynamo

CRITICISM: Unexplained universality in different environments

MAGNETOGENESIS: HINTS FOR MAGNETIC FIELDS' ORIGIN

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- Seed generation via battery and subsequent amplification via galactic dynamo

CRITICISM: Unexplained universality in different environments

PRIMORDIAL

- Relic from the Early Universe

CRITICISM: Causality scale confined within the Hubble radius

SMALL COHERENCE LENGTH ↔ HORIZON PROBLEM

CONFORMAL TIME

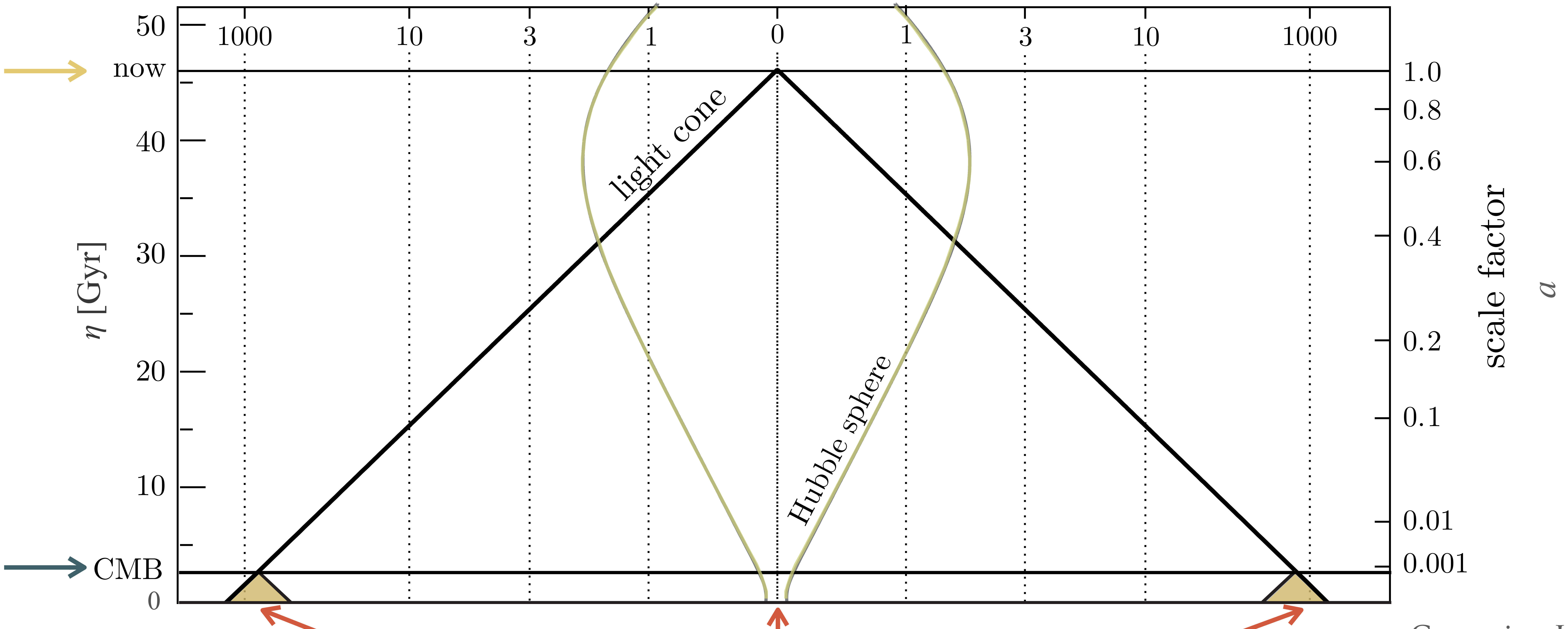
$$d\eta = \frac{dt}{a}$$

COMOVING HUBBLE RADIUS

$$d_H^c = \frac{1}{a(t)H(t)} = \frac{1}{\dot{a}}$$

HUBBLE PARAMETER

$$H(t) = \frac{\dot{a}}{a}$$



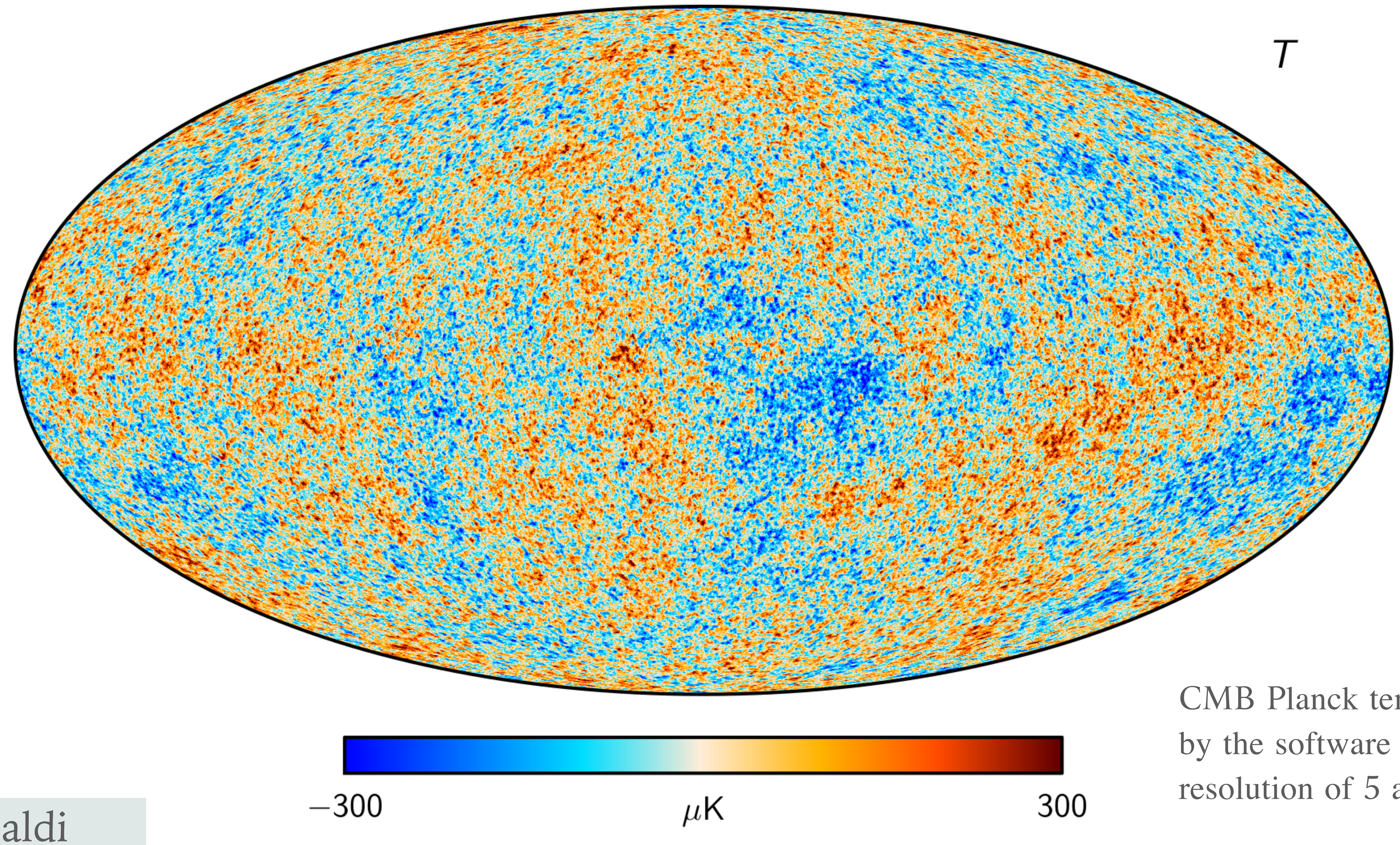
Small Hubble radius

Causally disconnected regions

Comoving Hubble radius in the Friedmann universe. Credit: [2].

MANIFEST HOMOGENEITY

- 10^5 causally disconnected patches at last scattering
- How to explain temperature fluctuations $\delta T \sim 10^{-5}\,\mathrm{K}$?



CMB Planck temperature map produced by the software COMMANDER with a resolution of 5 arcmin. Credit: [3].

INFLATIONARY SOLUTION

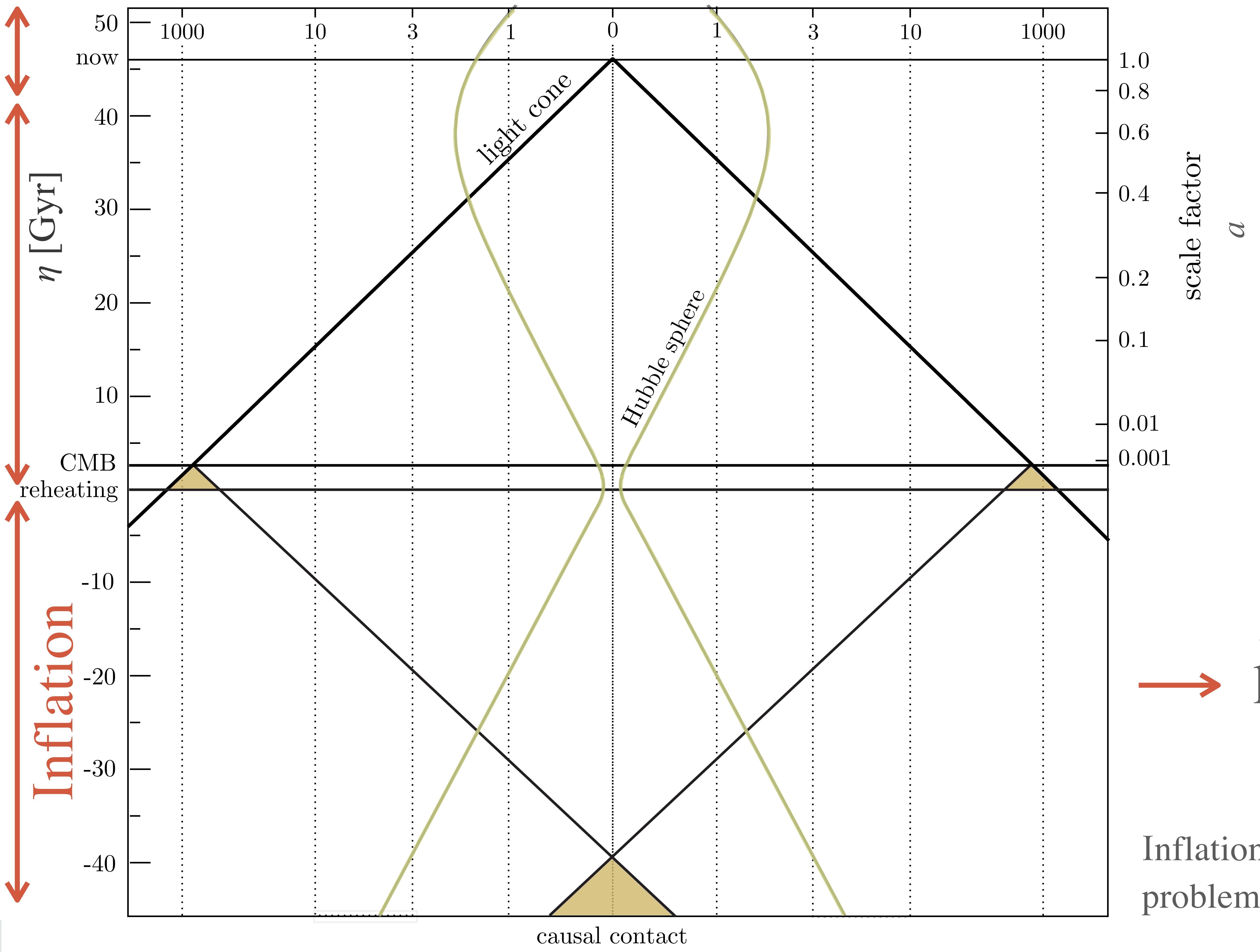
INFLATION: Stage of accelerated expansion of the Universe when gravity acts repulsively

Accelerated expansion

Decelerated expansion

Additional stage of accelerated expansion

Inflation



→ Large coherence lengths naturally emerge

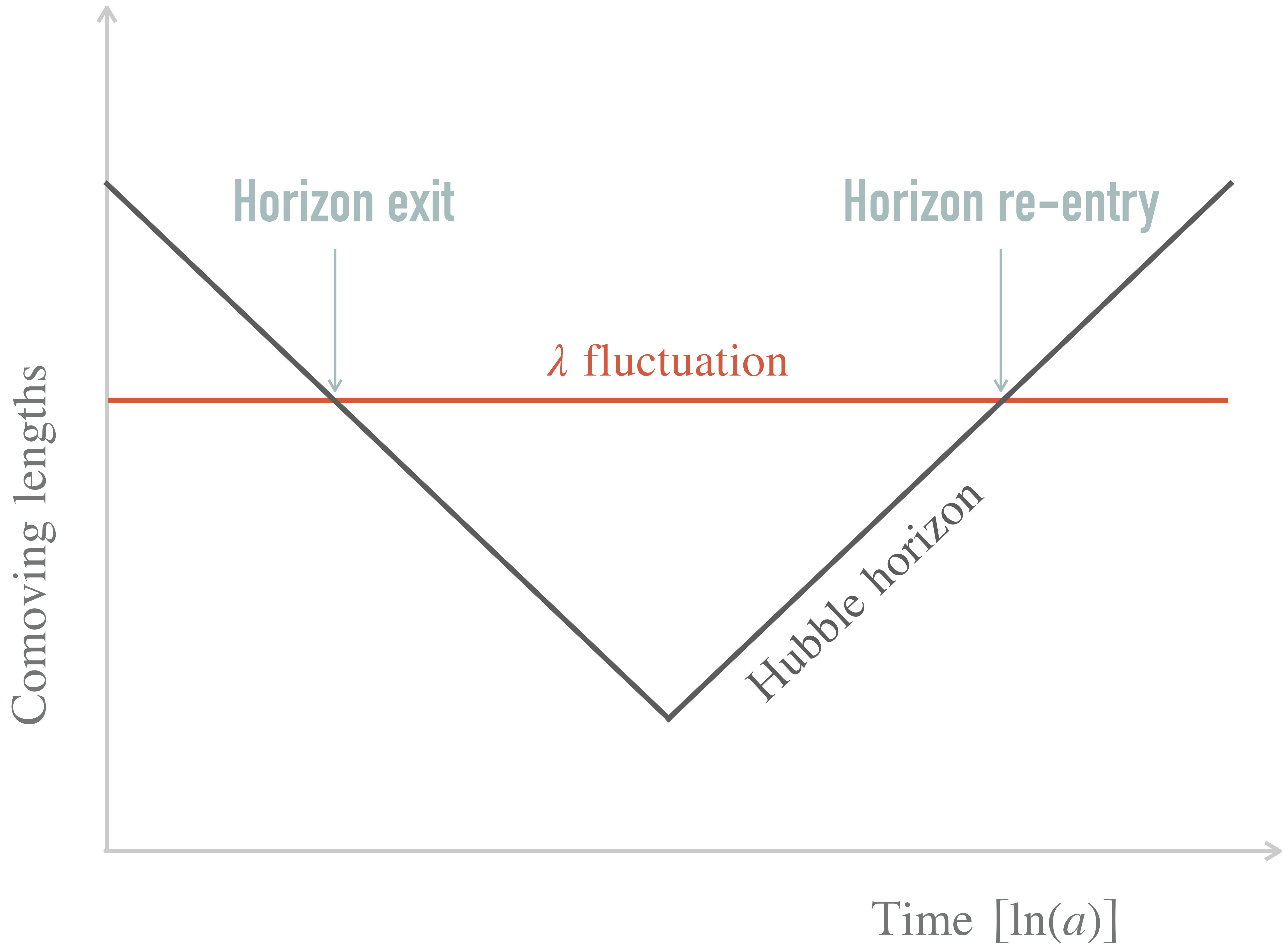
Inflationary solution to the horizon problem. Credit: [2].

QUANTUM FLUCTUATIONS DURING INFLATION

Sub-horizon sized quantum fluctuations are stretched during inflation, when $a \sim e^{Ht}$

- **SCALAR PERTURBATIONS (SPIN 0)**
  → large scale structure

- **TENSOR PERTURBATIONS (SPIN 2)**
  → GW



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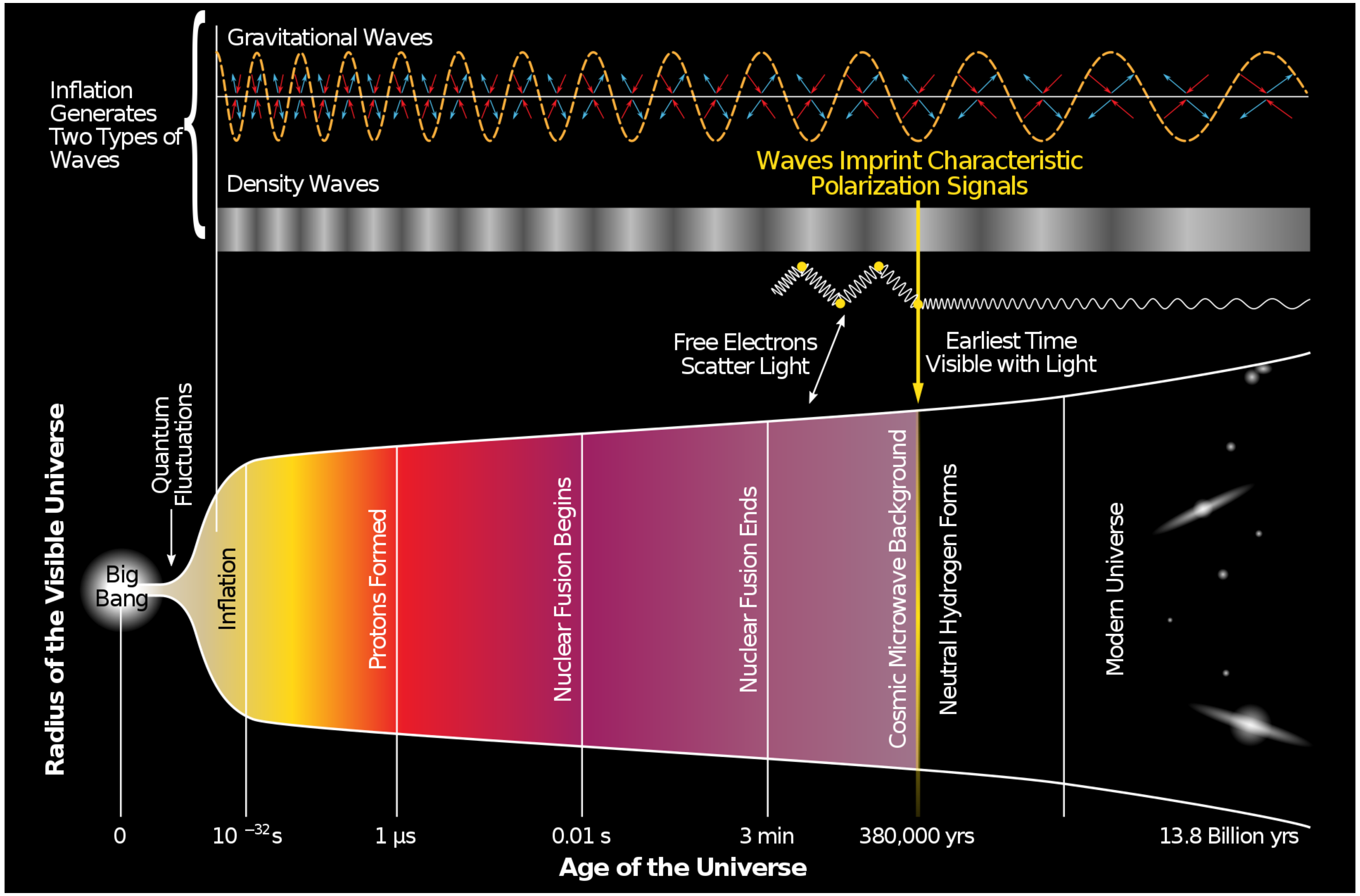

History of the Universe. Credit: [4].

AMPLIFICATION OF MAGNETIC FIELD PERTURBATIONS

Problem: conformal invariance $g^*_{\mu\nu} = \Omega^2(x) g_{\mu\nu}, \quad S^*_{EM} = S_{EM}, \quad g^{RW}_{\mu\nu} = a^2(\eta) g^{M4}_{\mu\nu}$

Electromagnetic fields in the expanding Universe behave as in flat static spacetime

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Electromagnetic fields in the expanding Universe behave as in flat static spacetime

- Magnetic flux is conserved
- Vector perturbations quickly vanish during inflation: $B \sim \dfrac{1}{a^2}$ $\quad B \to 0$ as $a \sim e^{Ht}$

AMPLIFICATION OF MAGNETIC FIELD PERTURBATIONS

Break conformal invariance to amplify vector perturbations

AMPLIFICATION OF MAGNETIC FIELD PERTURBATIONS

Break conformal invariance to amplify vector perturbations

A solution: introduce a time dependence $I^2(t) = I^2[\phi(t), \ldots]$

No source term: conductivity vanishes during inflation

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\left[I^2(\phi)\, F_{\mu\nu}F^{\mu\nu}\right] + \int d^4x\sqrt{-g}\,\mathscr{L}_\phi$$

Test EM field

Inflationary background evolution

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AMPLIFICATION OF MAGNETIC FIELD PERTURBATIONS

Break conformal invariance to amplify vector perturbations

A solution: introduce a time dependence $I^2(t) = I^2[\phi(t), \ldots]$

REHEATING: conductivity jumps to large values

- Electric field is shorted out
- Magnetic field decays $B \sim a^{-2}$

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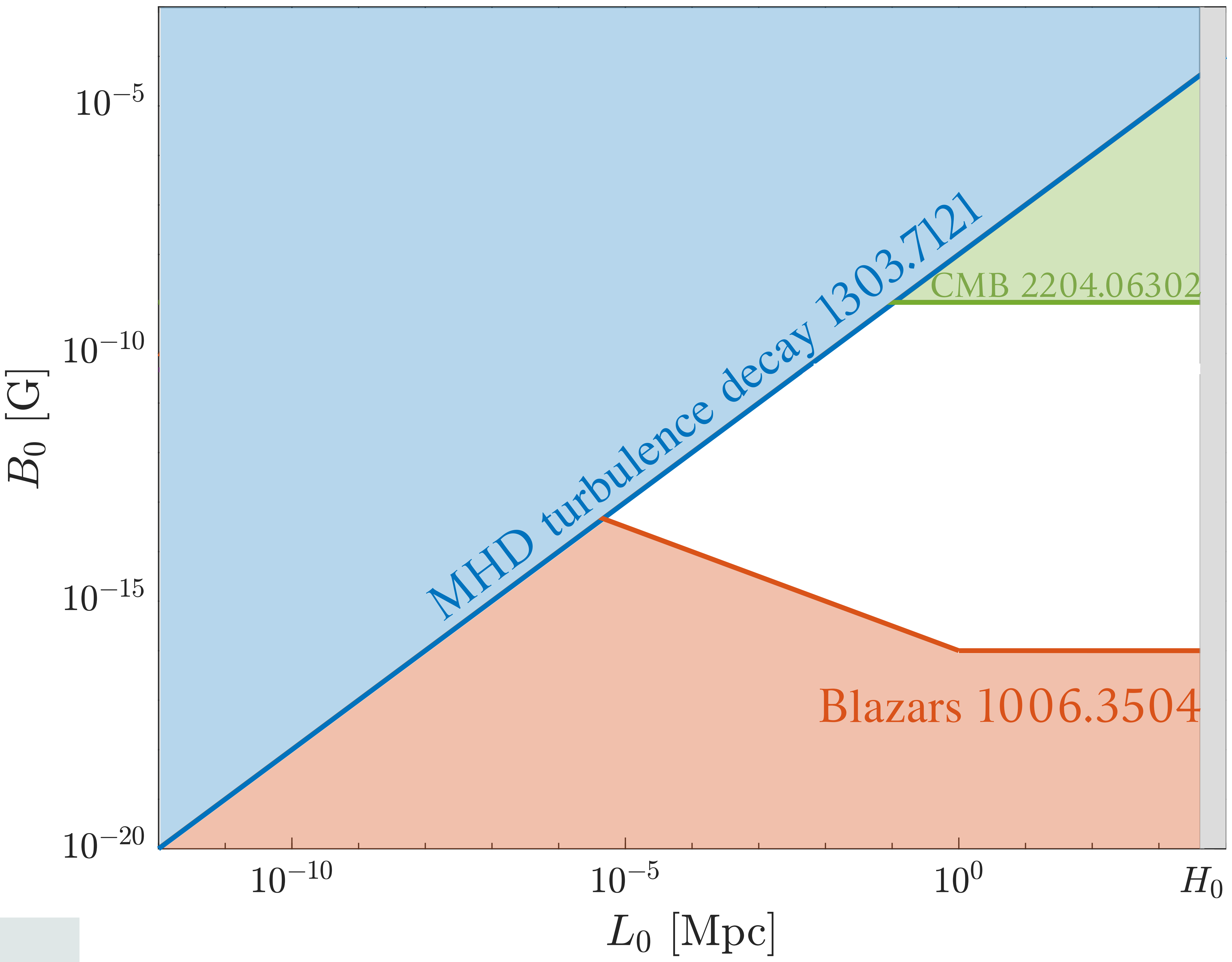
CONSTRAINTS ON MAGNETOGENESIS

The B field in gravitationally collapsed objects has been highly processed after inflation

1. Predict $\left(B_0, L_0\right)$ compatible with lower bounds in the IGM



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CONSTRAINTS ON MAGNETOGENESIS

2. Restore conformal invariance after inflation

   → $I_f \sim 1$ before adiabatic decay

$$S_{EM}\Big|_f = -\frac{1}{16\pi}\int\sqrt{-g}\,\mathrm{d}^4x\,I_f^2 F_{\mu\nu}F^{\mu\nu} = S_{Maxwell}$$

3. EM field should not spoil inflation

   → Avoid back reaction: $\rho_{EM} < \rho_\phi$

4. The theory should be in its perturbative regime

   → Avoid strong coupling: $g \sim I^{-1} \ll 1$

V. Demozzi, V. Mukhanov, & H. Rubinstein, JCAP (2009)

MODELLING THE COUPLING FUNCTION

$$\boxed{I(\phi) \to I(\eta) = a(\eta)^n}$$

$$\mathscr{A}''(\eta, k) + \left(k^2 - \frac{n(n+1)}{\eta^2}\right)\mathscr{A}(\eta, k) = 0$$

$$\boxed{\mathscr{A}(\eta, k) \equiv \frac{A(\eta, k)}{I}}$$

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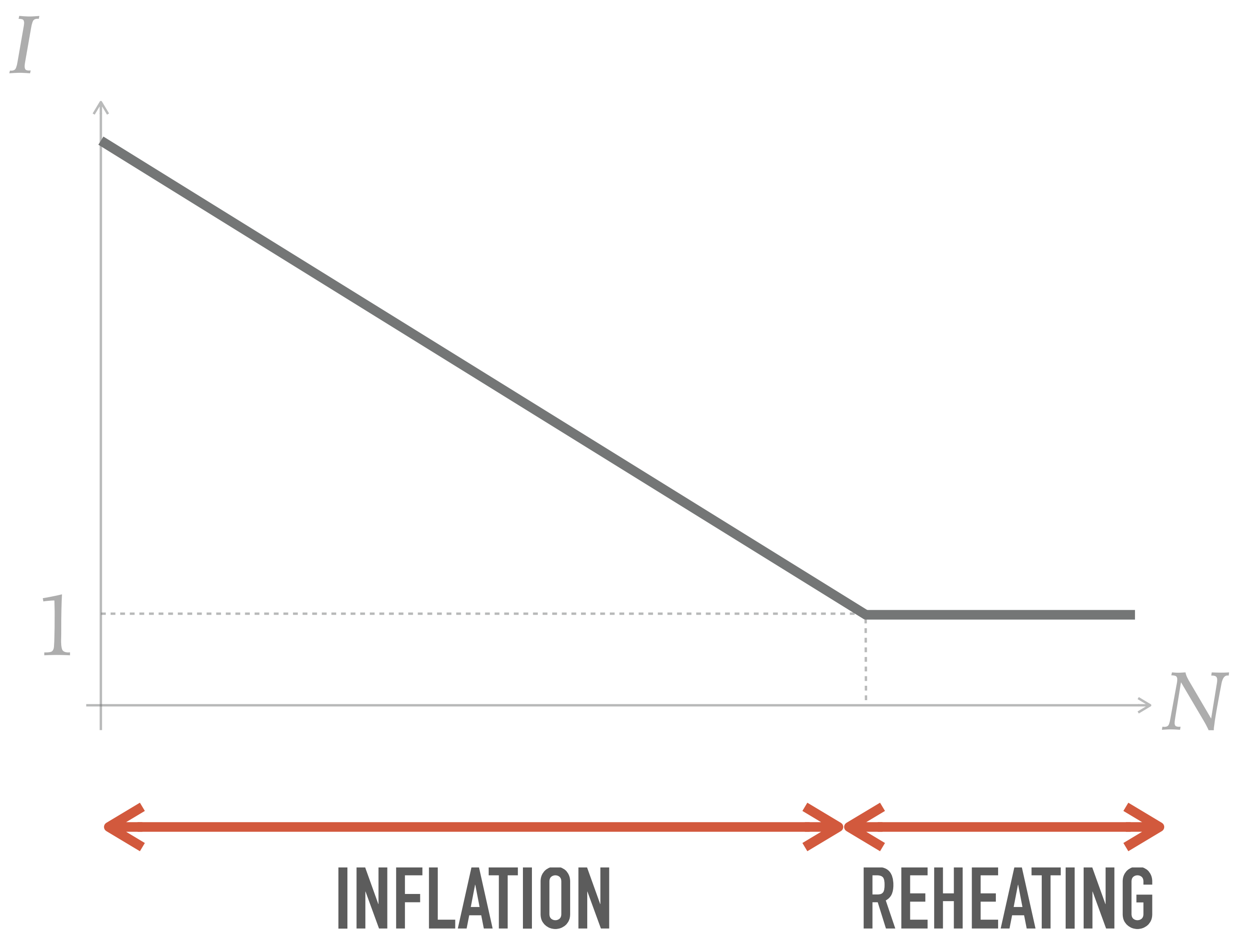
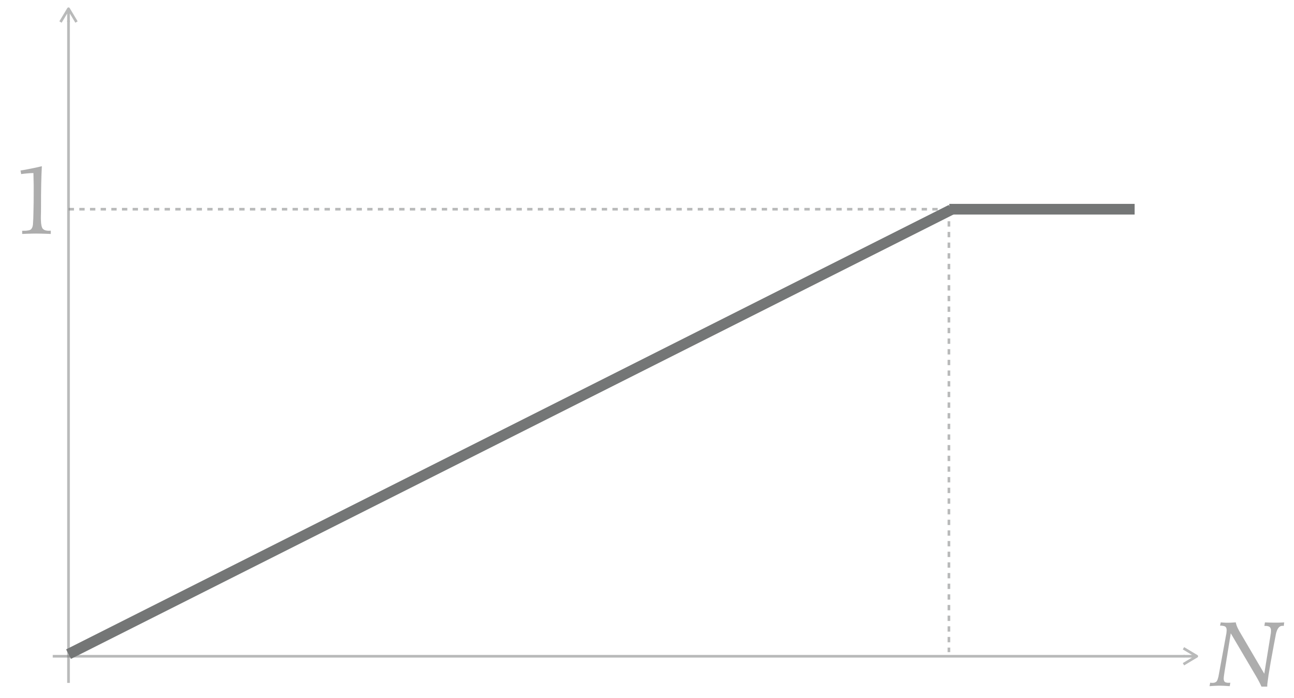


BACK-REACTION PROBLEM

- The EM field spoils inflation

$$\rho_{EM} > \rho_\phi$$



STRONG COUPLING PROBLEM

- Out of perturbative regime

$$I \sim g^{-1} < 1$$

$$e \gg 1$$

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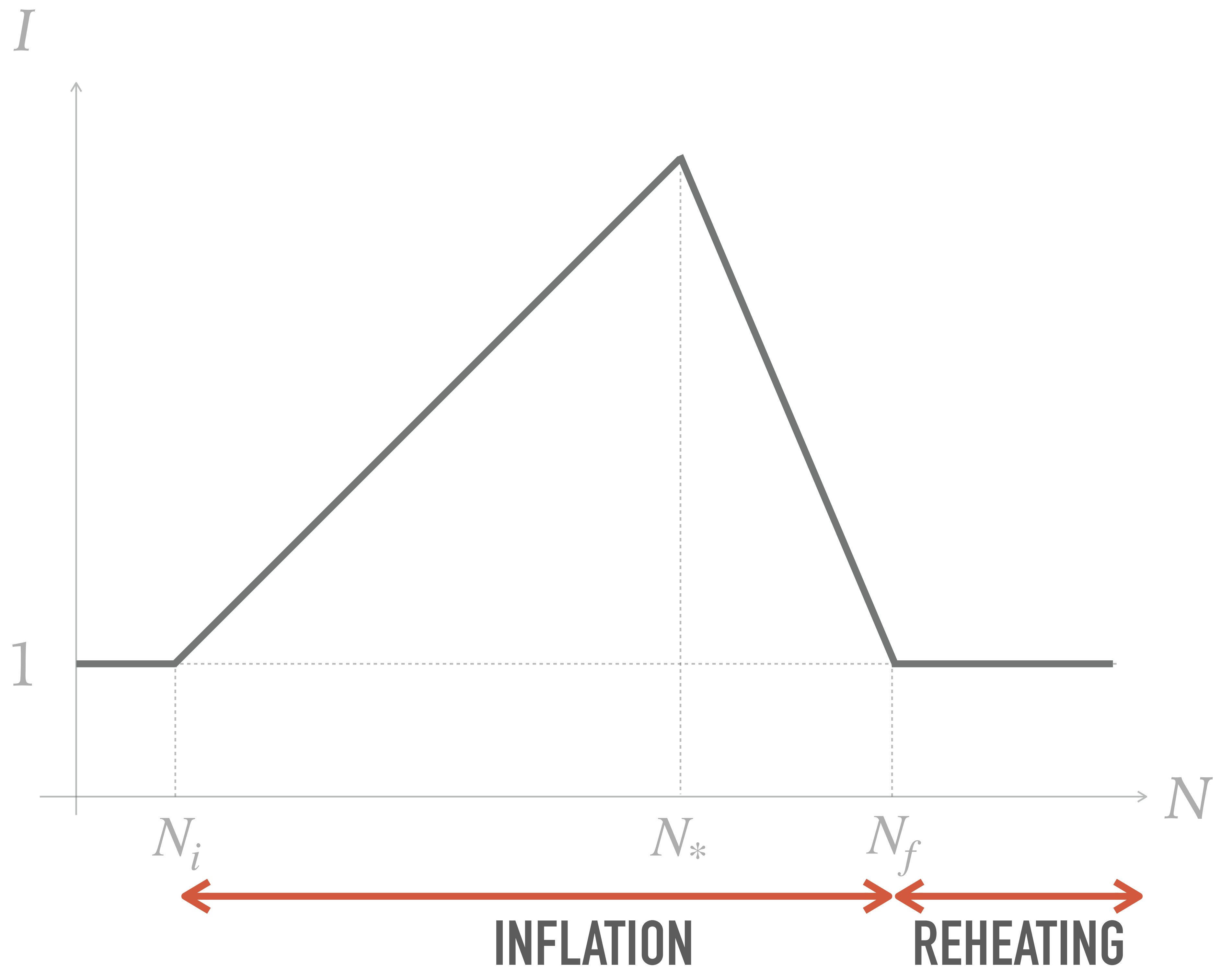
SAWTOOTH COUPLING

R. Ferreira, R.K. Jain, & M. Sloth, JCAP10 (2013) 004
R. Sharma, S. Jagannathan, T. R. Seshadri & K. Subramanian, Phys Rev D 96 (2017)

- A possibility is to take advantage of both the increasing and the decreasing mode



$$I = \begin{cases} \mathscr{C}\left(\dfrac{a}{a_*}\right)^{\nu_1} & a_i > a > a_* \\ \mathscr{C}\left(\dfrac{a}{a_*}\right)^{-\nu_2} & a_* > a > a_f \end{cases}$$

$$\boxed{\nu_1, \nu_2 \geq 0}$$

- *Fix $\mathscr{C}$ and a_* to have $I_i = I_f = 1$*

ADDING HELICITY

C. Caprini & L. Sorbo JCAP 1410 (2014)

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\; I^2[\phi(t)]\left[F_{\mu\nu}F^{\mu\nu} - \gamma F_{\mu\nu}\tilde{F}^{\mu\nu}\right] + \int d^4x\sqrt{-g}\,\mathscr{L}_\phi$$

$$\mathscr{A}''_\pm(\eta,k) + \left(k^2 \pm \frac{2k\gamma n}{\eta} - \frac{n(n+1)}{\eta^2}\right)\mathscr{A}_\pm(\eta,k) = 0$$

Exponential amplification of
the positive helicity mode

WHY HELICITY?

- ➤ Observations
- ➤ Inverse-cascade evolution

SCALAR FIELD Φ

- How to model a finite stage of accelerated expansion in the Universe?

$$\boxed{\ddot{a} > 0}$$

- From the Friedmann equations

$$\ddot{a} = -\frac{4\pi}{3} G \left(\epsilon + 3p\right) a \xrightarrow{\qquad} \boxed{\epsilon + 3p < 0}$$

- Scalar field

$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V$$

$$p = \frac{1}{2}\dot{\phi}^2 - V$$

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SCALAR FIELD Φ

- How to model a finite stage of accelerated expansion in the Universe?

$$\downarrow$$

$$\boxed{\ddot{a} > 0}$$

- From the Friedmann equations

$$\ddot{a} = -\frac{4\pi}{3}G\left(\epsilon + 3p\right)a \longrightarrow \boxed{\epsilon + 3p < 0}$$

- Scalar field + slow roll

$$\begin{aligned}\epsilon &= \cancel{\frac{1}{2}\dot{\phi}^2} + V \\ p &= \cancel{\frac{1}{2}\dot{\phi}^2} - V\end{aligned} \longrightarrow \boxed{0 < \epsilon = -p}$$

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INFLATIONARY EVOLUTION

MODIFIED GRAVITY THEORIES

- A scalar condensate can be imitated within gravity
- Einstein gravity $\mathscr{L}_{EH} = R - 2\Lambda$ as a low-curvature limit of a more complicated theory

JORDAN FRAME $\xrightarrow{\text{Conformal transformation}}$ **EINSTEIN FRAME**

$$S = \frac{M_{pl}^2}{2}\int d^4x\sqrt{-g}\, f(R)$$

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\underbrace{\frac{M_{pl}^2}{2}\tilde{R}}_{\text{Einstein gravity}} + \underbrace{\mathscr{L}_\phi}_{\text{Scalar field}}\right]$$

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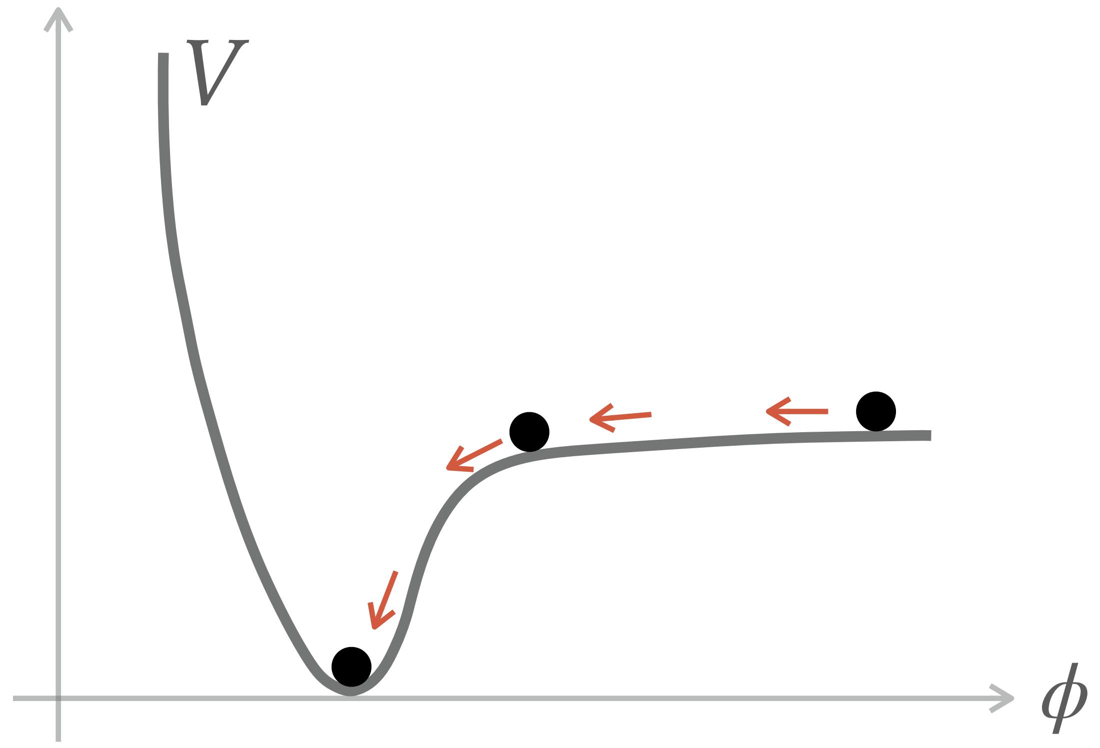
JORDAN FRAME —— Conformal transformation ⟶ **EINSTEIN FRAME**

$$S = \frac{M_{pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\, f(R)$$

$$S_E = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\frac{M_{pl}^2}{2}\tilde{R} + \mathscr{L}_\phi\right]$$

STAROBINSKY'S MODEL

$$f(R) = R + \frac{R^2}{6M^2}$$



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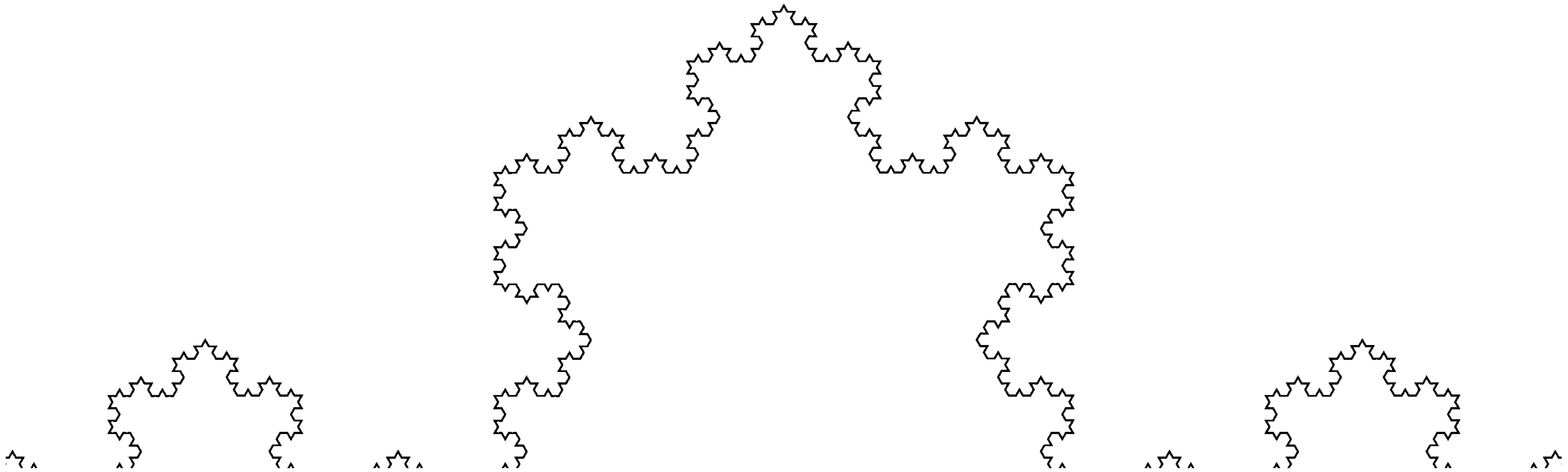
FUNDAMENTAL SCALE INVARIANCE

Scale symmetry could have shaped the Universe at high energies

C. Wetterich , Nuclear Physics B, 115326 (2021)
A. Strumia & A. Salvio, J High Energ Phys, 6 (2017)

- ➤ Nearly scale-invariant spectrum of perturbations on the CMB
- ➤ Naturally flat inflationary potentials
- ➤ Standard Model
- ➤ Dynamical mass generation
- ➤ Dark matter candidates



SCALE-INVARIANT MODIFIED GRAVITY MODEL

M. Rinaldi and L. Vanzo PR D 94 (2016)

- $\mathscr{L}_{EH} \longrightarrow f(R, \phi)$

$$\mathscr{L} = \sqrt{-g}\left[\underbrace{\frac{\alpha}{36}R^2}_{\text{Higher order term in R}} + \frac{\xi}{6}\phi^2 R \underbrace{- \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4}_{\text{Scalar field}}\right], \qquad \alpha, \lambda, \xi > 0$$

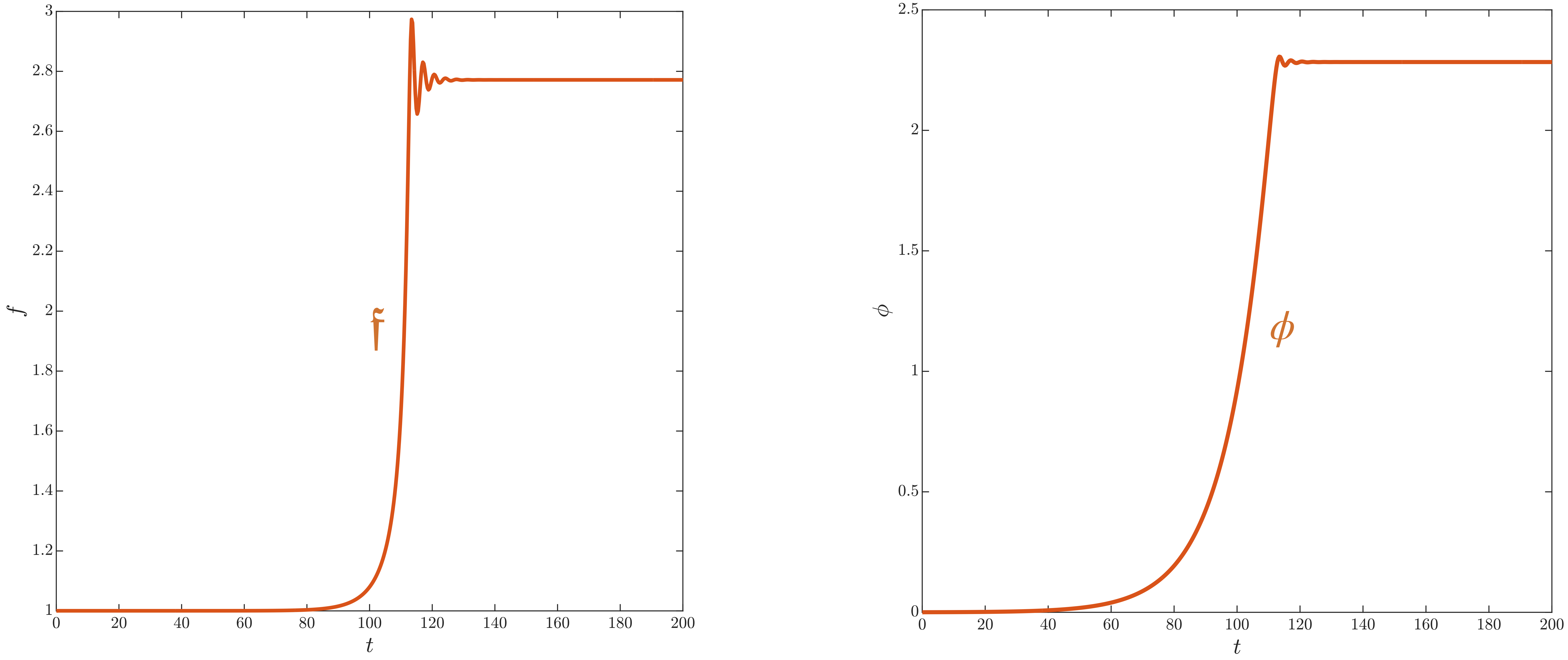
- Scale symmetry (dilations)
  - $\bar{g}_{\mu\nu}(x) = g_{\mu\nu}(\ell x)$
  - $\bar{\phi}(x) = \ell\phi(\ell x)$

$$\bar{\mathscr{L}} = \mathscr{L}$$

SCALE-INVARIANT MODIFIED GRAVITY MODEL

1. JORDAN FRAME → EINSTEIN FRAME

The theory has two dynamical scalar fields: are we in a multi-field theory?



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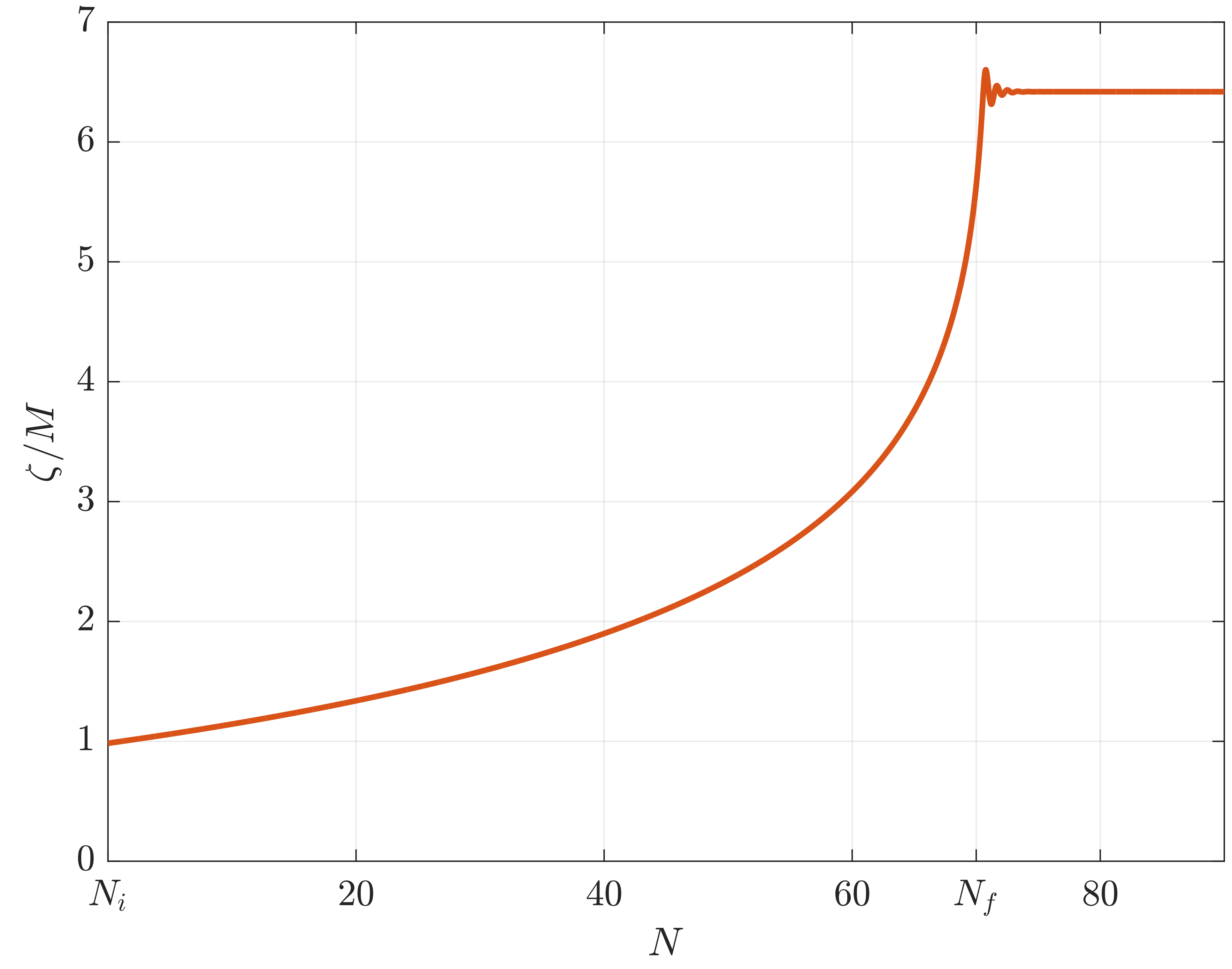
SCALE-INVARIANT MODIFIED GRAVITY MODEL

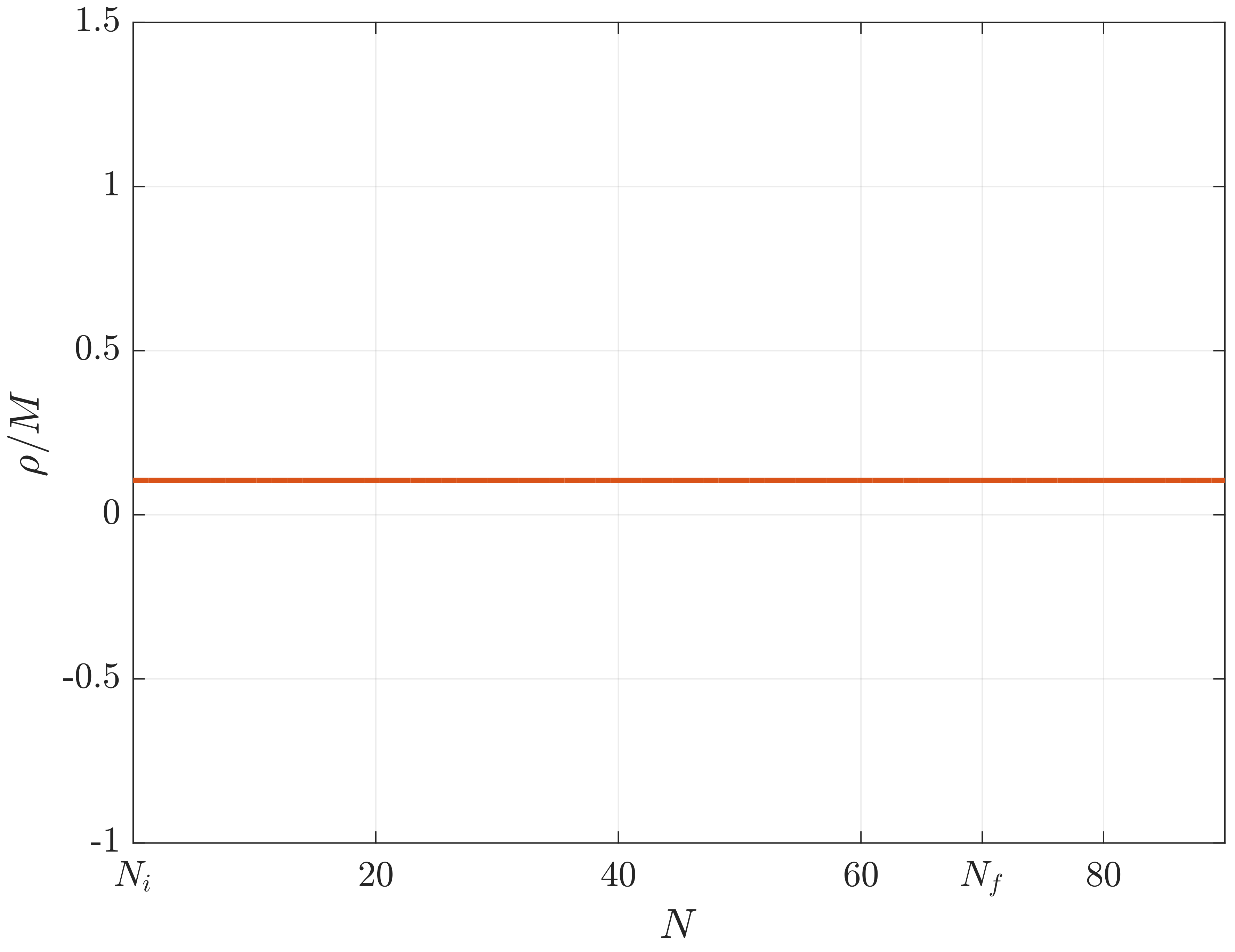
2. SCALE SYMMETRY ⟶ FIELDS REDEFINITION

G. Tambalo & M. Rinaldi Gen Relativ Gravit 49 (2017)

$$\zeta = g(\mathfrak{f}, \phi)$$



$$\rho = f(\mathfrak{f}, \phi)$$



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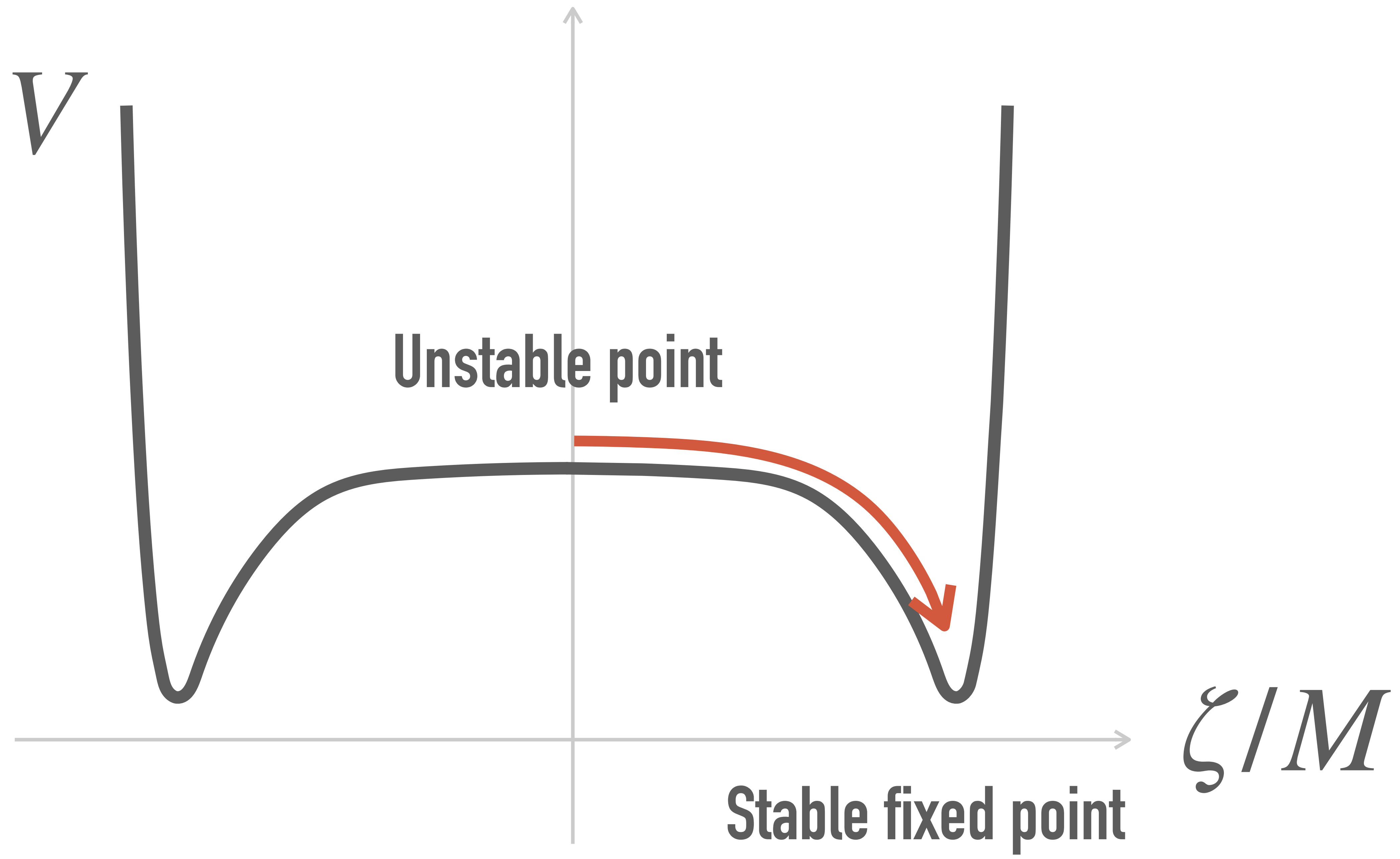
SCALE-INVARIANT MODIFIED GRAVITY MODEL

- ➤ Scale symmetry breaking
- ➤ Non-vanishing potential at the minima

ρ	**GOLDSTONE BOSON**
ζ	**INFLATON**



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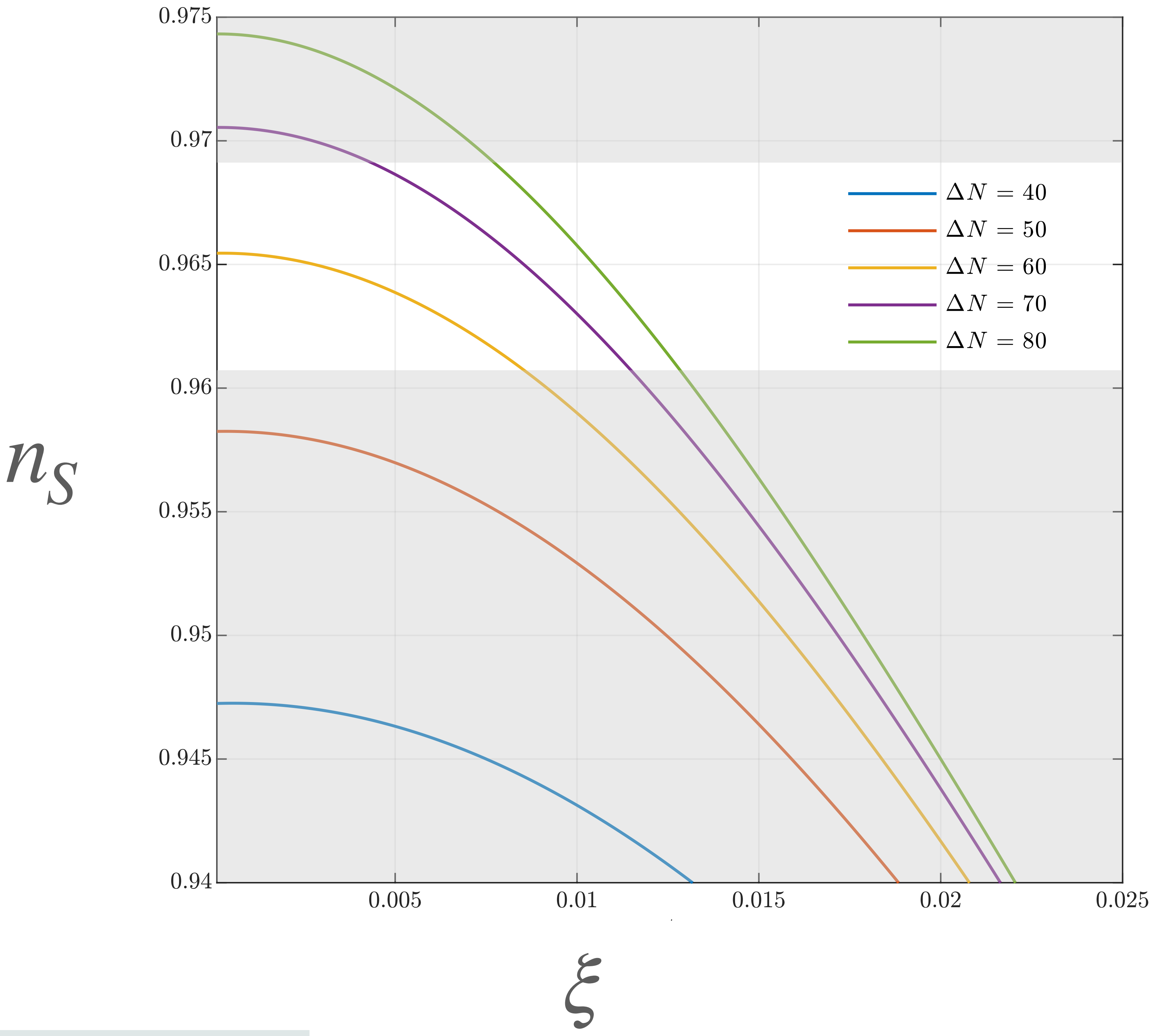
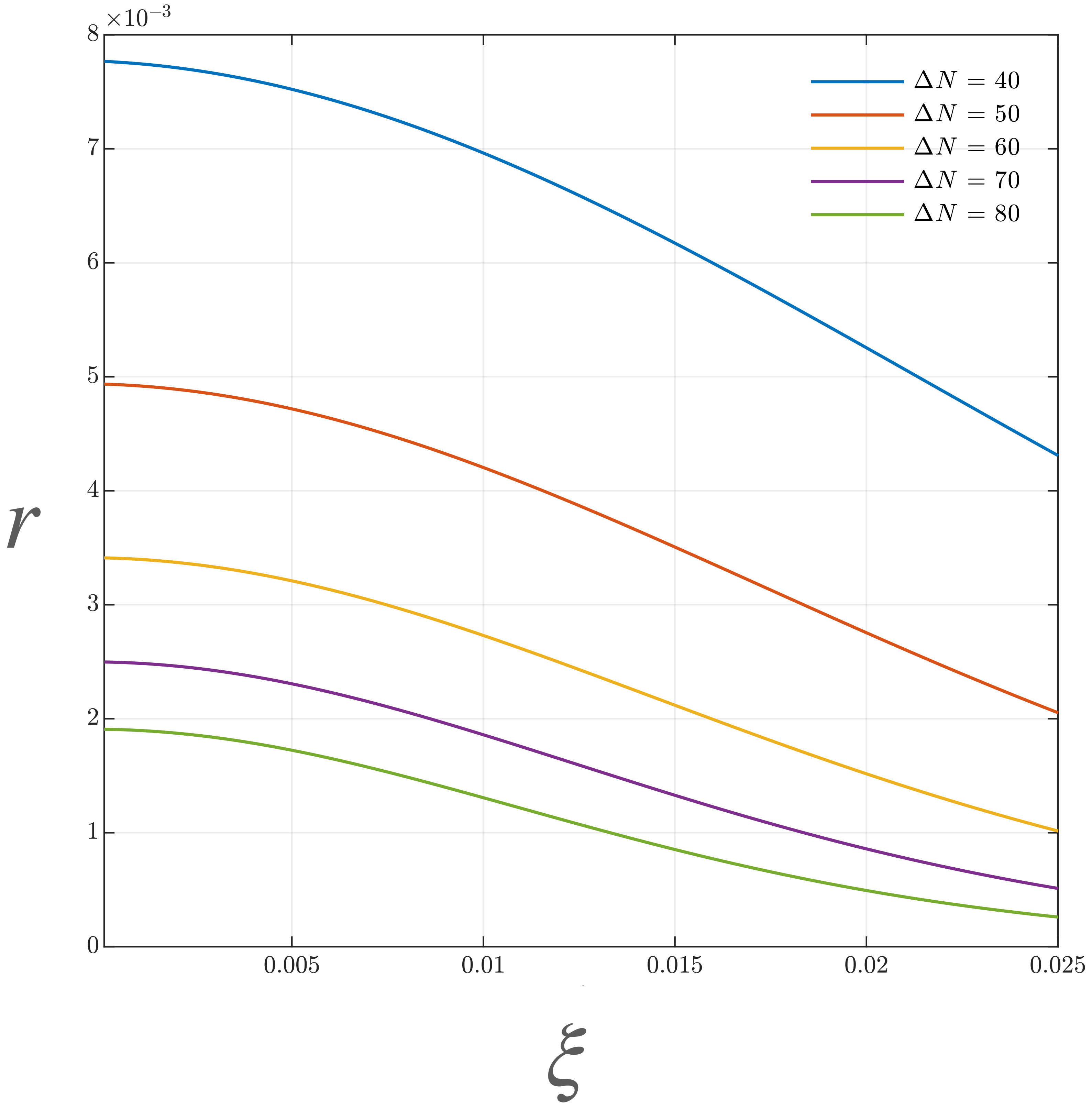
SCALE-INVARIANT MODIFIED GRAVITY MODEL

SPECTRAL INDICES

- $\Omega = \alpha\lambda + \xi^2 \lesssim 1.15\,\xi^2$
- $\alpha \gtrsim 2 \times 10^{10}$
- $\xi \lesssim 1.3 \times 10^{-2}$
- $\Delta N \gtrsim 55$





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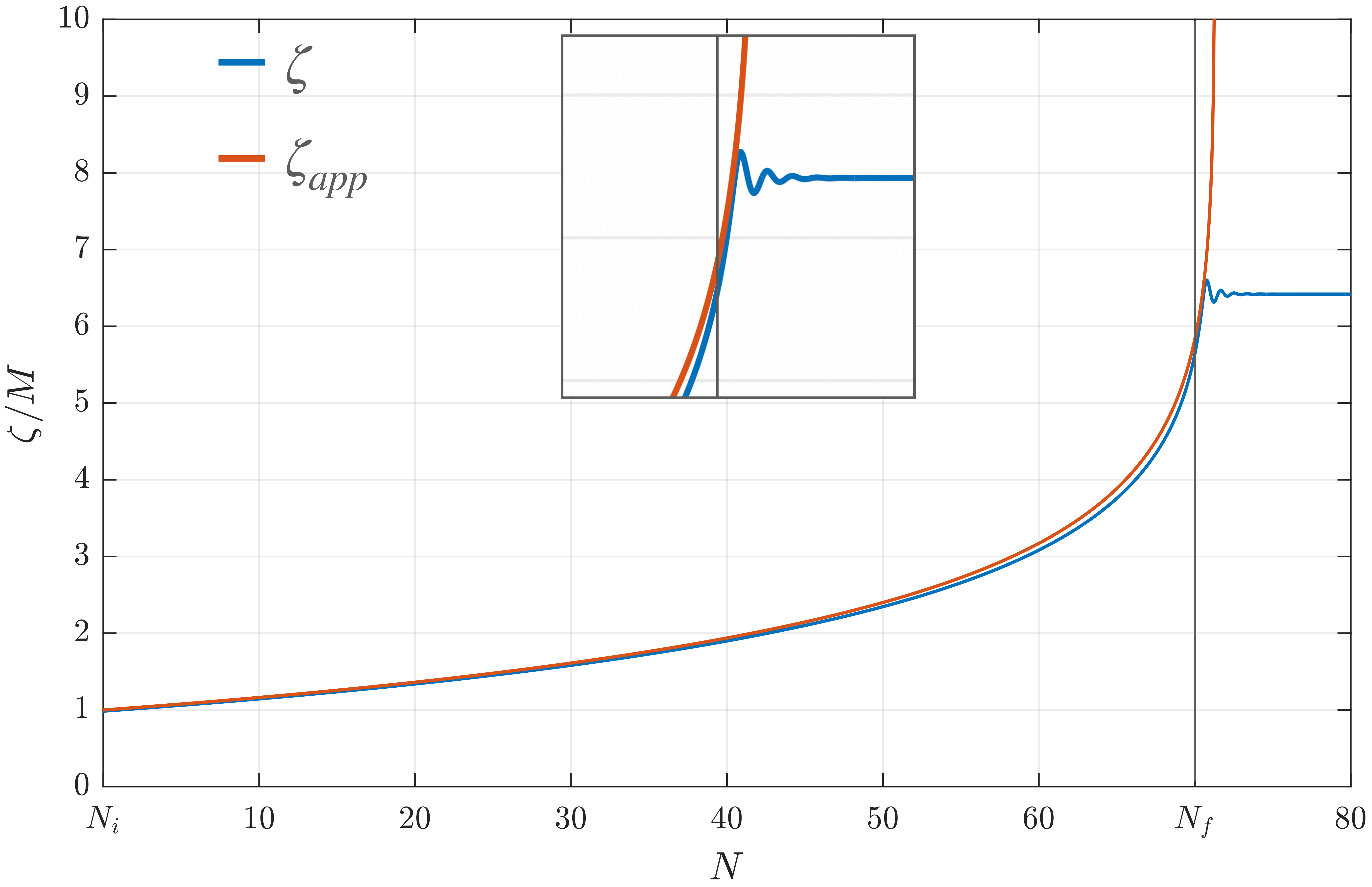
EVOLUTION OF THE INFLATON

APPROXIMATIONS

- Slow-roll
- $\xi \ll 1$
- $\Omega \rightarrow \xi^2$

$$\zeta(a) \simeq \sqrt{6}\, M \mathrm{ArcTanh}\left[\mathscr{C}\, a^{\frac{4}{3}\xi}\right]$$

$$\boxed{I(\zeta) = \propto \tanh\left[\frac{\zeta}{\sqrt{6}M}\right]^{\pm\frac{3}{4\xi}\nu_1} \simeq a^{\pm\nu_i}}$$



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RESULTS

FIX THE EXPONENTS ν_i

- Scale-invariant magnetic power spectrum is natural during inflation (scalar & tensor)

Impossible to achieve a scale-invariant magnetic power spectrum throughout inflation

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FIX THE EXPONENTS ν_i

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Impossible to achieve a scale-invariant magnetic power spectrum throughout inflation

CMB observations are only sensitive to the first stage of inflation

RESULTS

FIX THE EXPONENTS ν_i

- Scale-invariant magnetic power spectrum is natural during inflation (scalar & tensor)

$$\boxed{\nu_1 = 2}$$

FIRST STAGE

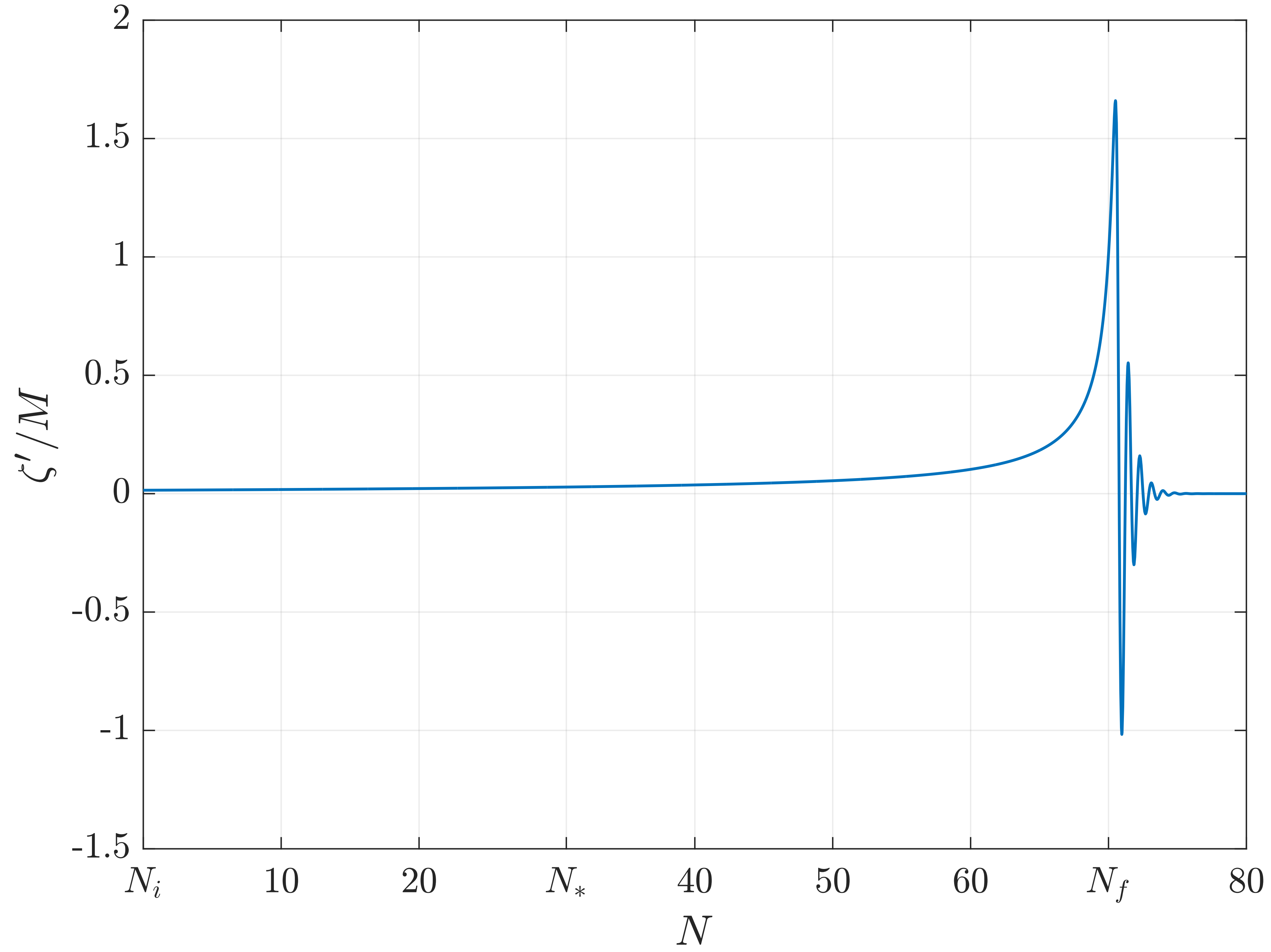
$$\rightarrow \frac{d\rho_{B,I}}{d\ln k}^{\pm} \text{ is scale-invariant}$$

$$\rightarrow \frac{d\rho_{E,I}}{d\ln k}^{\pm} \text{ is scale-invariant}$$

SECOND STAGE

$$\rightarrow \frac{d\rho_{B,II}}{d\ln k}^{\pm} \propto k^4$$

$$\rightarrow \frac{d\rho_{E,II}}{d\ln k}^{\pm} \propto k^2$$



- Departure from scale invariance: inflaton ↔ EM field

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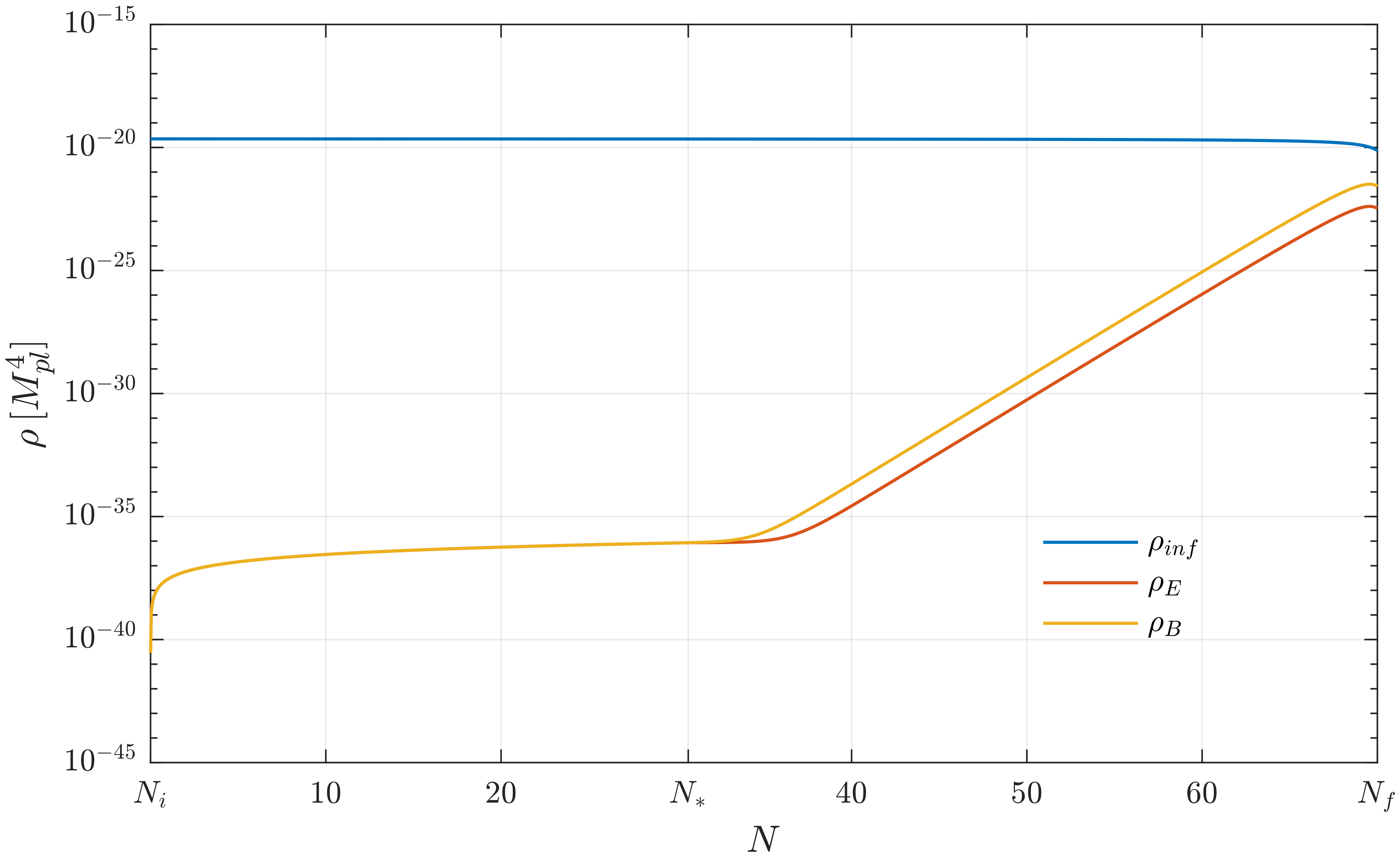
RESULTS

BACK-REACTION

- Explicit evaluation of the modified Klein-Gordon equations
- $\rho_{EM} \ll \rho_\zeta$



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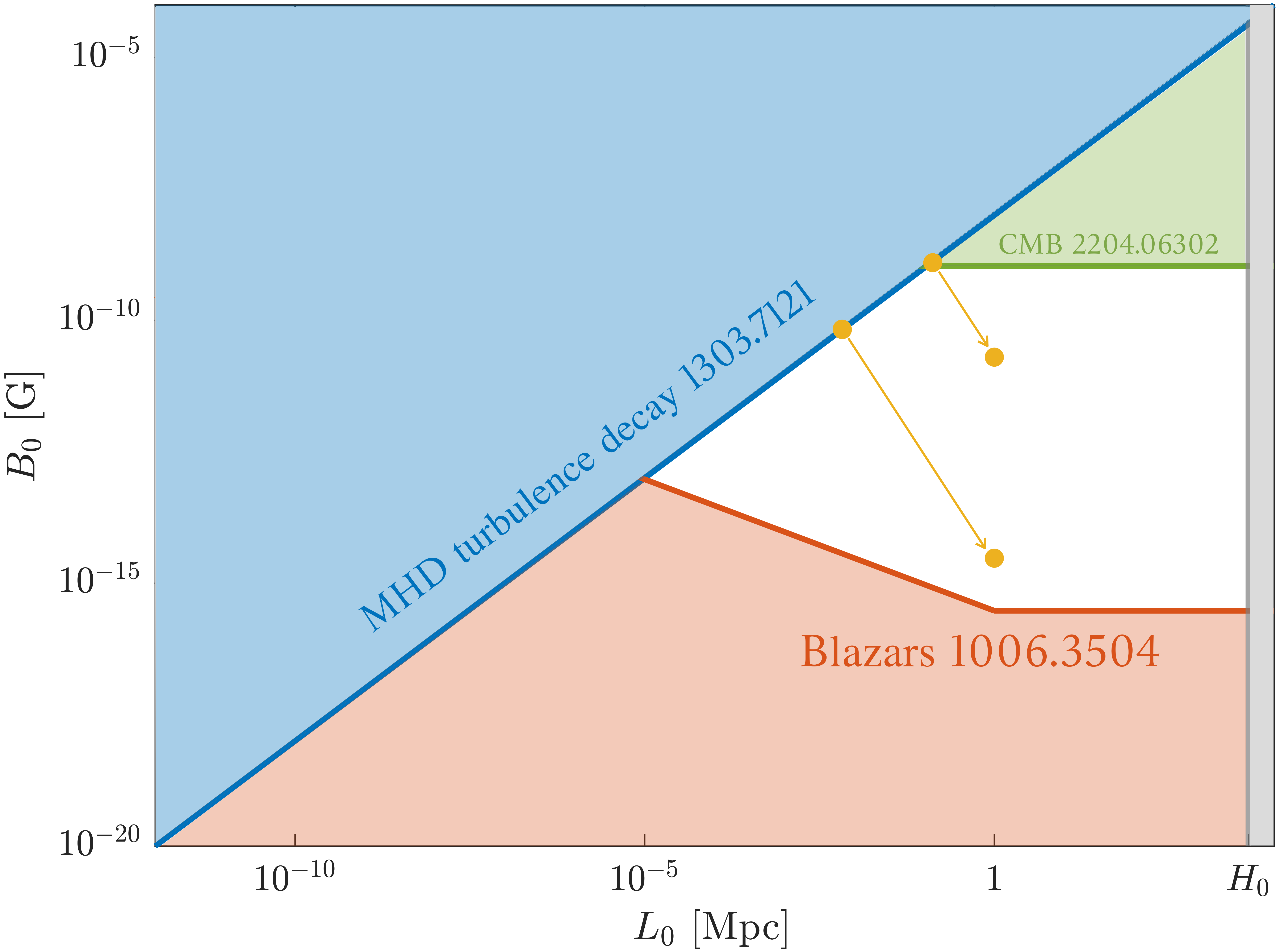
RESULTS

- Present-day (B_0, L_0) compatible with bounds within all the allowed region
- Valid for all the inflationary energy scales

$$10^{16}\,\text{GeV} \lesssim \rho_{inf} \lesssim 246\,\text{GeV}$$



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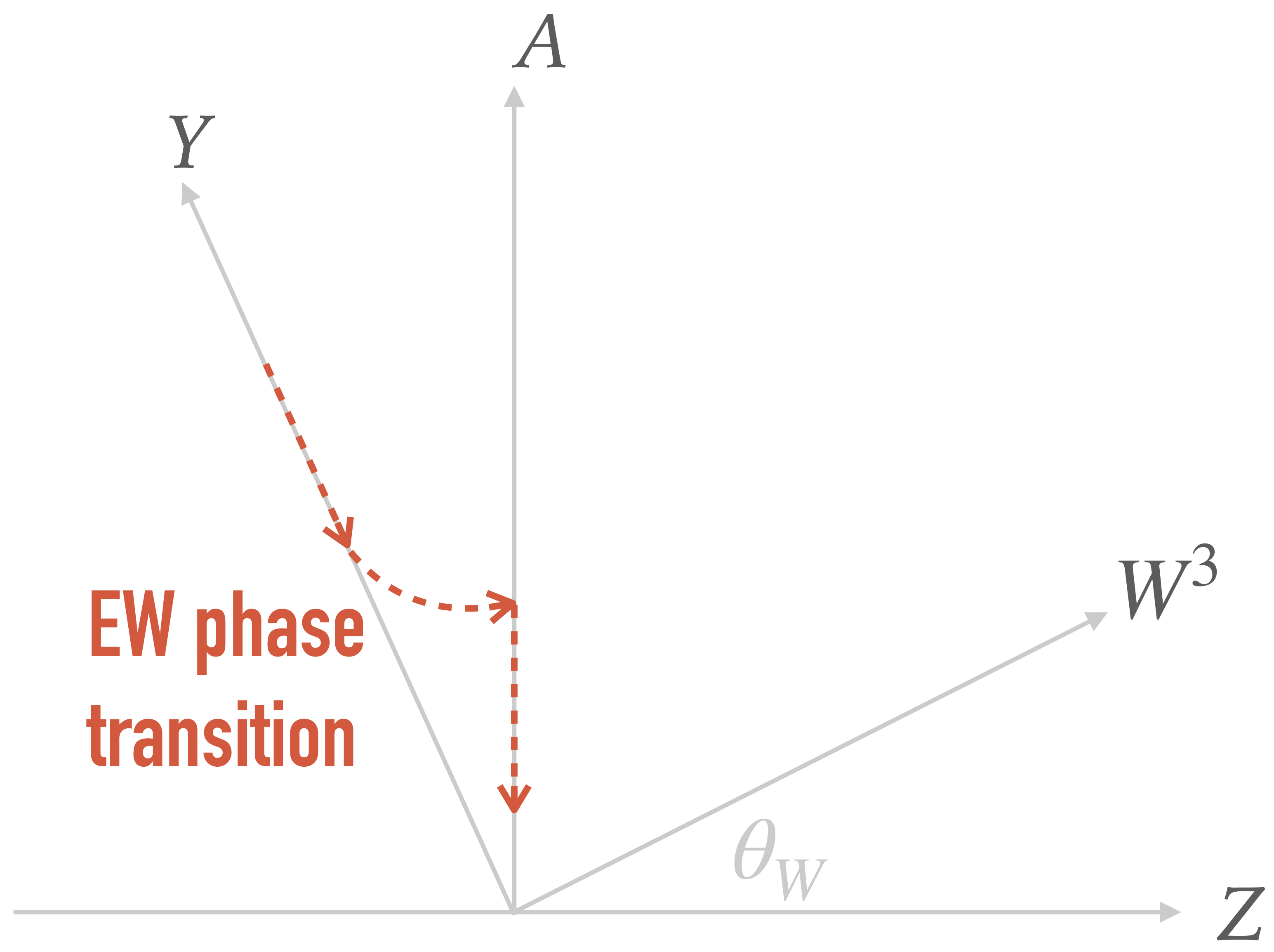
BARYOGENESIS

K. Kamada and A. J. Long PRD 94 (2016)

- If helical PMF existed before the EW transitions, baryon asymmetry is generated
- Decaying hyper magnetic helicity→ (B+L) asymmetry



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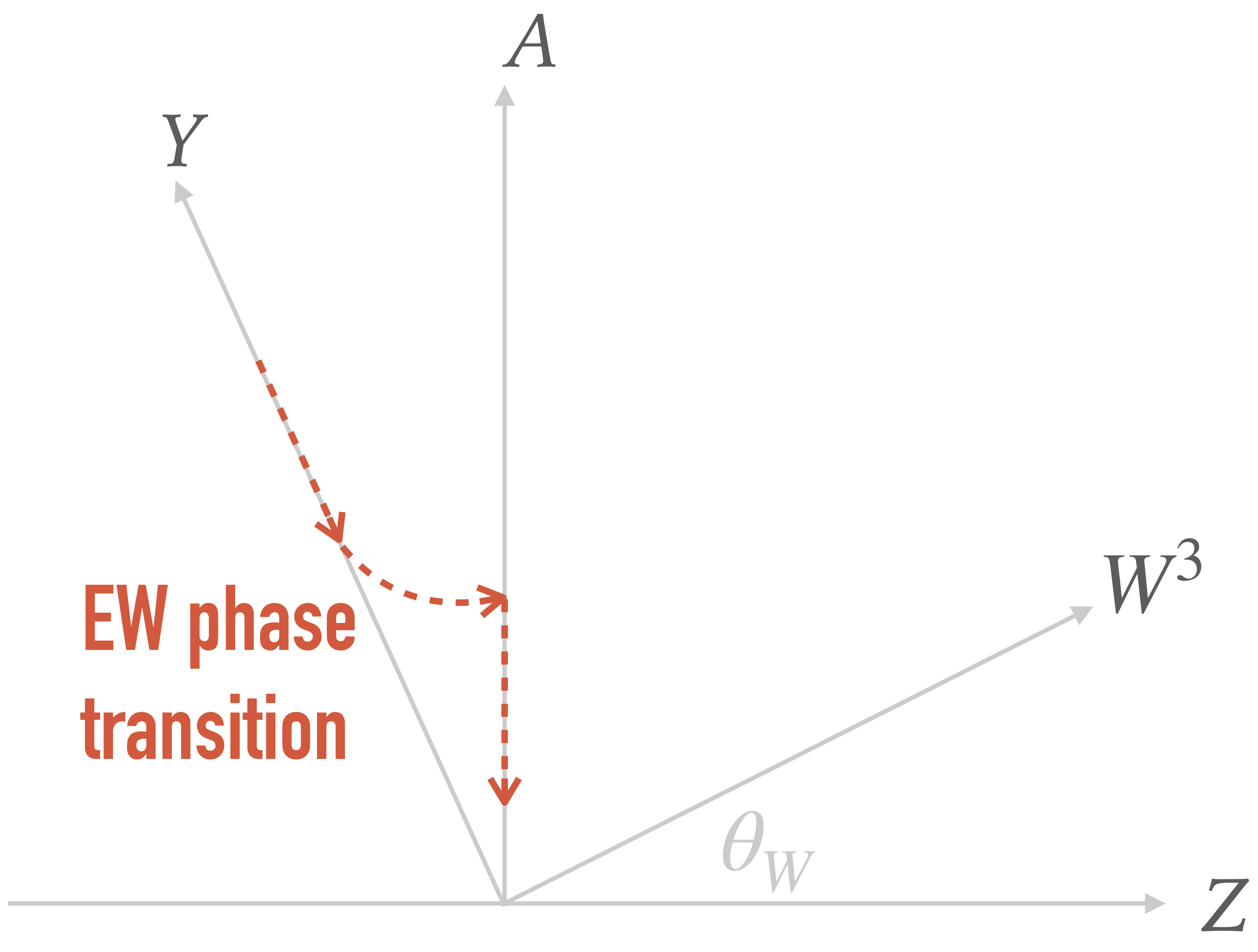


$\rho_{inf}^{1/4}$ [GeV]	ΔN	ν_1	ν_2	γ
2×10^7	55	3.4	0.3	0.2

$$\rightarrow L_0 = 4.6 \times 10^{-8} \text{ Mpc}$$

$$\rightarrow B_0(L_0) = 4.6 \times 10^{-7} \text{ nG}$$

$$\rightarrow B_0(\ell = 1\,\text{Mpc}) = 1.8 \times 10^{-11} \text{ nG}$$

- ➤ Observed BAU
- ➤ Seed for dynamo

SUMMARY

- Inflation is a natural setting for primordial magnetic fields generation
- Amplification of magnetic field perturbations requires breaking conformal invariance
- Model of magnetogenesis: sawtooth coupling + helicity
- Model of inflation: scale-invariant modified gravity
- Promising results for magnetic fields in the IGM

IMAGE CREDITS

1.QUIJOTE Collaboration

2. Baumann, D. & McAllister, L. Inflation and string theory(Cambridge University Press, Cambridge, 2015)

3.https://wiki.cosmos.esa.int/planck-legacy-archive/index.php/CMB_maps

4.National Science Foundation (NASA, JPL, Keck Foundation, Moore Foundation, related) — Funded BICEP2 Program; modifications by E. Siegel

BACKUP SLIDES

CONFORMAL INVARIANCE

- The EM action is conformal invariant

$$\begin{tikzcd}
S = -\frac{1}{16\pi}\int\sqrt{-g}\, d^4x\, g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} \arrow[d] \\
g^*_{\mu\nu} = \Omega^2 g_{\mu\nu} \arrow[d] \\
A^*_\mu = A_\mu, S^* = S
\end{tikzcd}$$

- Spatially flat FLRW models are conformally flat

$$ds^2_{FLRW} = \tilde{\Omega}^2(x)(d\eta^2 - \delta_{ik}dx^i dx^k)$$

$$\begin{tikzcd}
d\eta = \frac{dt}{a} \arrow[d] \\
g^{FLRW}_{\mu\nu} = \tilde{\Omega}^2(x)\eta_{\mu\nu}
\end{tikzcd}$$

- Maxwell equations in spatially flat FLRW are equivalent to the Minkowski ones:

$$B \sim \frac{1}{a^2} \quad \longrightarrow \quad B \to 0 \text{ as } a \sim e^{Ht}$$

STRONG COUPLING PROBLEM

V. Demozzi, V. Mukhanov, & H. Rubinstein JCAP 2009

- Possibly $I(\phi) \sim 1$ at the end of inflation to recover conformal invariance

Lagrangian density of the vector field coupled with a charged fermion ⟶

$$\mathscr{L} = -\frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi$$

Rescaling $A_{\mu} \rightarrow gA_{\mu}$ ⟶

$$\mathscr{L} = -\frac{1}{16\pi}\frac{F_{\mu\nu}F^{\mu\nu}}{g^2} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + iA_{\mu})\psi$$

$I[\phi(t)]$ IS NATURALLY INTERPRETED AS AN INVERSE COUPLING CONSTANT

- $I(\phi)$ should be large throughout inflation to have a trustable theory

SCALE-INVARIANT MODIFIED GRAVITY MODEL

EINSTEIN FRAME $g^*_{\mu\nu} = \Omega^2 g_{\mu\nu}$

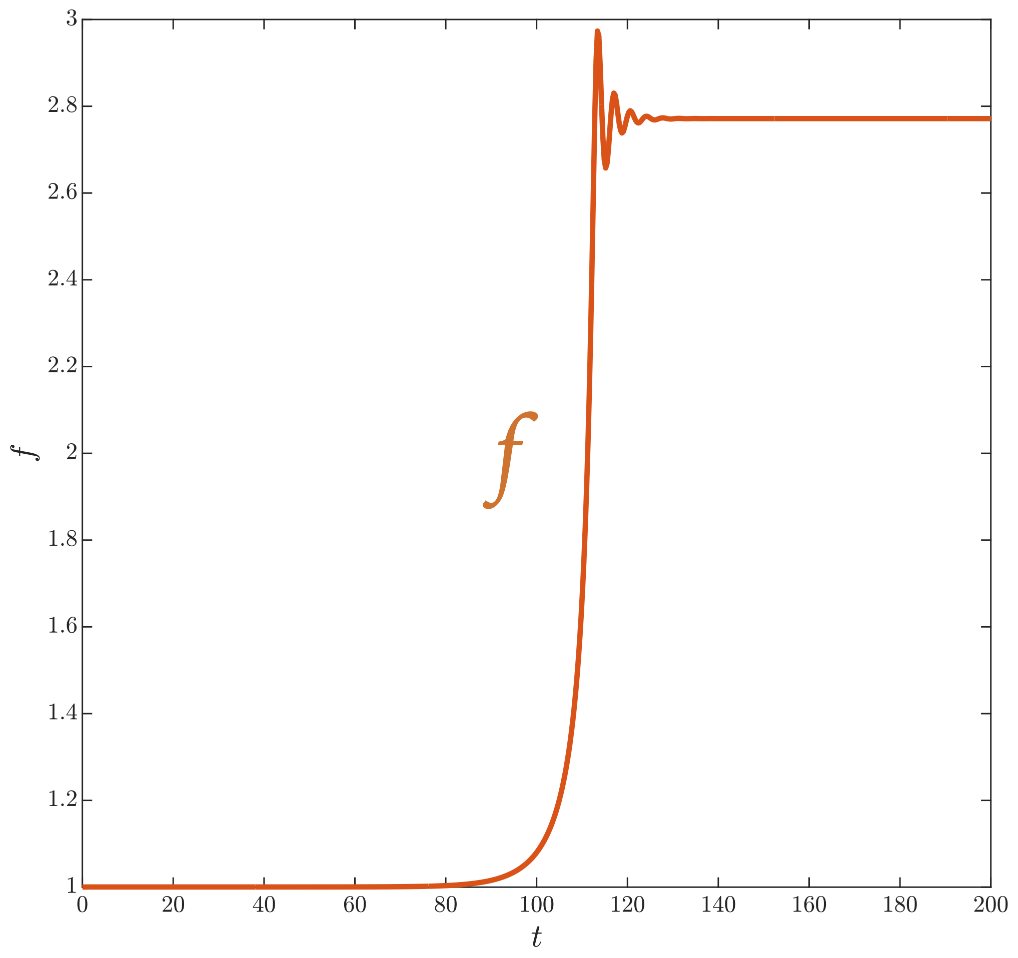
REDUNDANT MASS SCALE

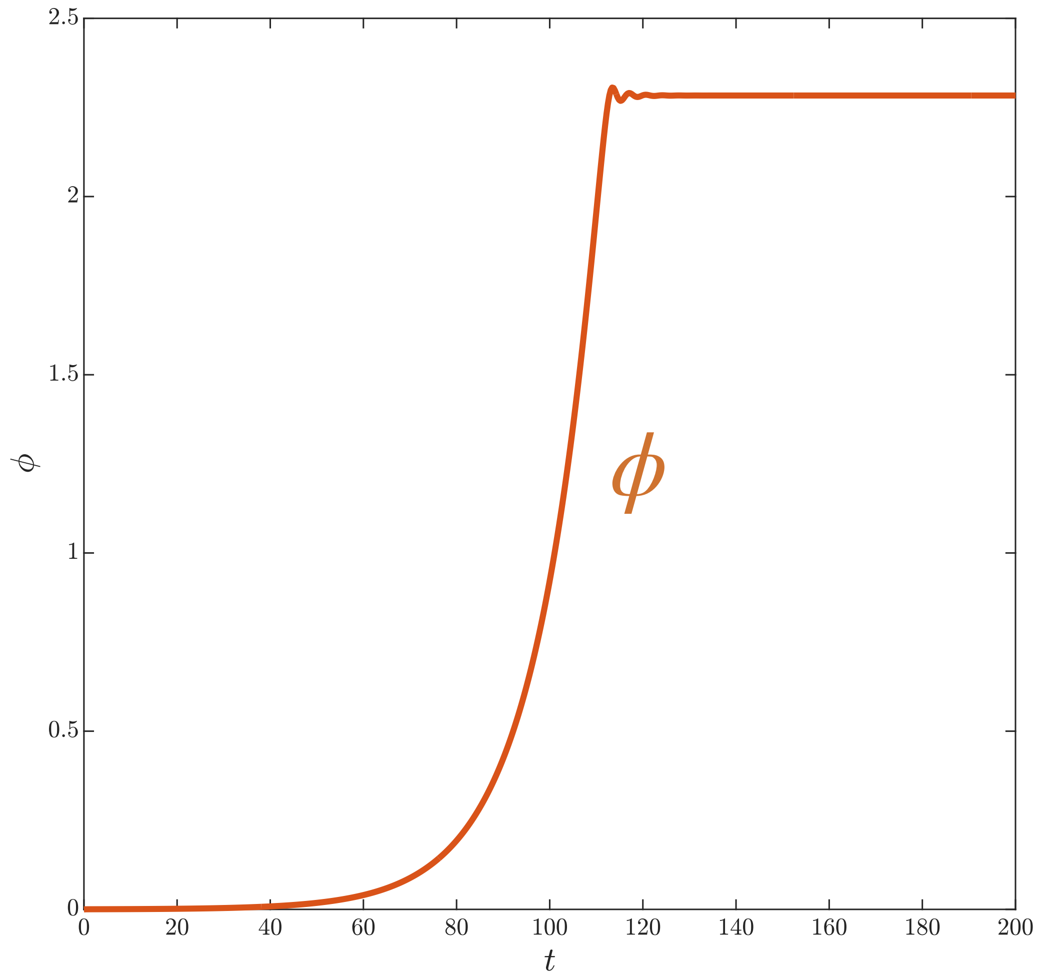
$$\mathscr{L}_E = \sqrt{-g}\left[\frac{M^2}{2}R - \frac{3M^2}{f^2}(\partial f)^2 - \frac{f^2}{2M^2}(\partial\phi)^2 - V(f,\phi)\right]$$

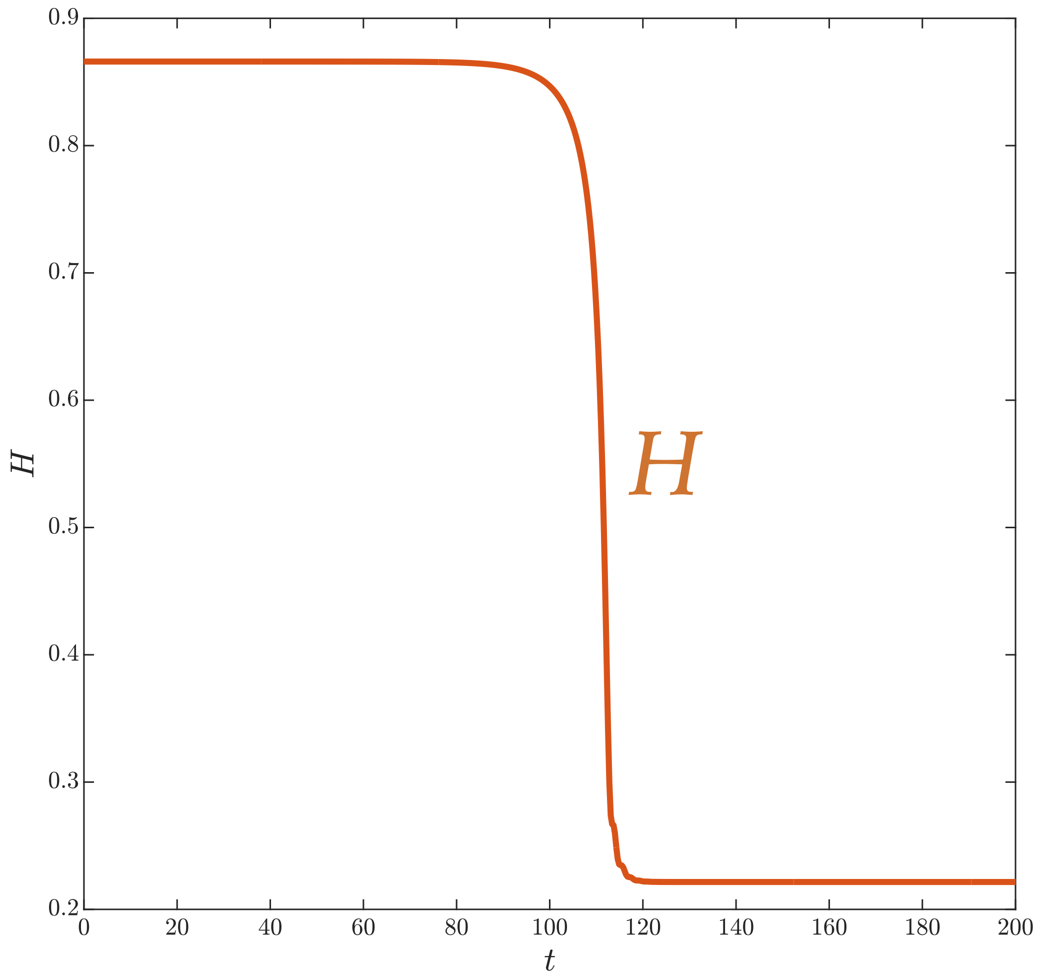
$f = -M\Omega$ **SCALARON**

$$V(f,\phi) = \frac{9M^4}{4\alpha} + f^2\phi^2\left(-\frac{3\xi}{2\alpha} + \frac{K}{M^4}f^2\phi^2\right)$$

$$K = \frac{1}{4}\left(\lambda + \frac{\xi^2}{\alpha}\right)$$







SCALE-INVARIANT MODIFIED GRAVITY MODEL

FIELDS REDEFINITION

Single-field inflation can be attained via field redefinition.

GOLDSTONE BOSON

$$\boxed{\rho = \frac{M}{2}\log\left[\frac{\phi^2}{2M^2} + 3\frac{M^2}{f^2}\right]}$$

$$\boxed{\zeta = \sqrt{6}M\text{ArcSinh}\left[\frac{f\phi}{\sqrt{6}M^2}\right]}$$

INFLATON

$$\mathscr{L}_E = \sqrt{-g}\left(\frac{M^2}{2}R - \frac{1}{2}\partial_\mu\zeta\,\partial^\mu\zeta - 3\text{Cosh}\left[\frac{\zeta}{\sqrt{6}M}\right]^2\partial_\mu\rho\partial^\mu\rho - U(\zeta)\right);$$

$$U(\zeta) = \frac{9M^4}{4\alpha}\left(1 - 4\,\xi\,\text{Sinh}\left[\frac{\zeta}{\sqrt{6}M}\right]^2 + 4\,\underset{\big\downarrow}{\Omega}\,\text{Sinh}\left[\frac{\zeta}{\sqrt{6}M}\right]^4\right);$$

$$\Omega = \alpha\lambda + \xi^2$$