Decay of oscillating scalar field minimally coupled to gravity

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BASU, McGill & IPM

IPM, School of Particle physics, Virtual Weekly Seminar, Jun 21, 2023

Outline

Outline

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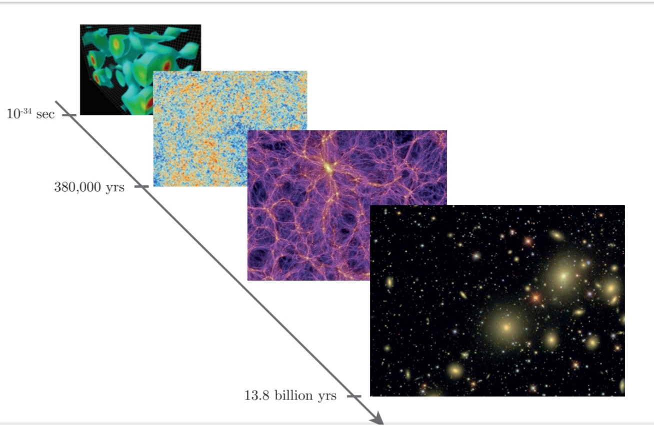
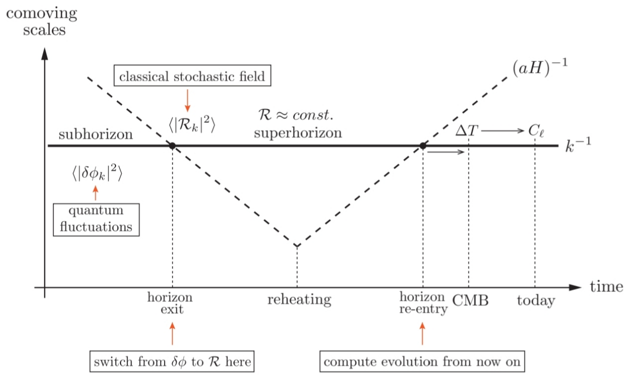
Observation

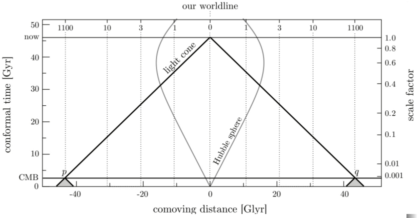


Horizon Problem



Horizon Problem

CMB Map



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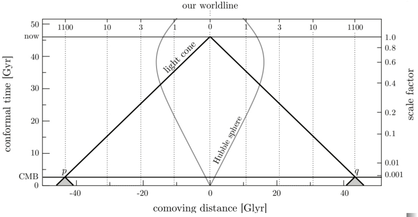
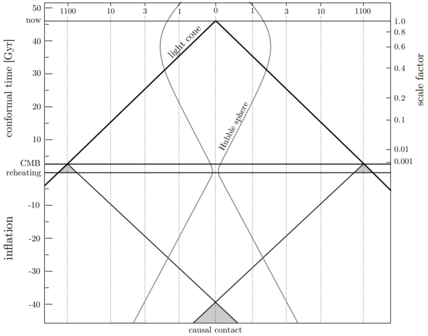




Cold inflation

Two separated epochs

Cold inflation

Two separated epochs

- Slow-roll

Cold inflation

Two separated epochs

- Slow-roll
- Reheating

Standard cosmology

Expanding universe

- Expanding homogeneous and isotropic universe

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) \quad (1)$$

- General Relativity

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (2)$$

- Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad (3)$$

- Early and late time accelerated expansions
- Inflation, Dark energy

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad P = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (4)$$

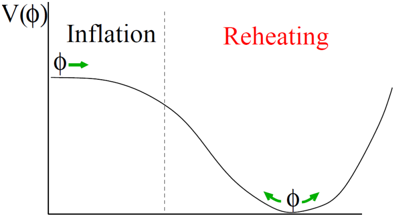
Standard Inflation

Condition

We know from G.R in order to realize inflation, it requires an equation of state $P < -\rho/3$, thus a substance with negative pressure that scalar fields can provide such an equation of state.

$$\rho(\phi) = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad P(\phi) = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{5}$$

Evolution of the Inflaton



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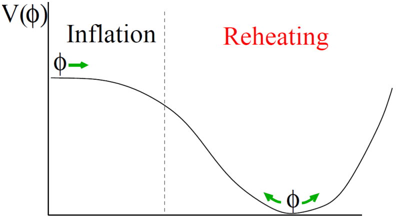
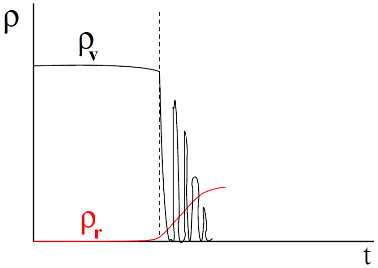


Evolution of Energy Density



Evolution of the Scalar Field

Related Equations

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- Energy Density Conservation

$$\dot{\rho} + 3H(\rho + P) = 0 \ , \tag{6}$$

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- Equations Describing the Evolution of Scalar Field

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \; , \; \epsilon = \frac{1}{2} M_p^2 (\frac{V'}{V})^2,$$

where H is Hubble parameter

Reheating

Interacting Potential

Reheating

Interacting Potential

- Non-thermal phase

$$V(\phi,\chi) = \frac{\lambda}{4}(\phi^2 - \phi_0^2)^2 + \frac{1}{2}g^2\phi^2\chi^2 \tag{7}$$

$$V(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{\alpha}{4}(\chi^2 - \frac{M^2}{\alpha})^2 + \frac{1}{2}g^2\phi^2\chi^2$$

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where σ, h, k are coupling constants

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- Thermalization

Warm inflation

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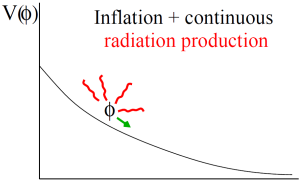

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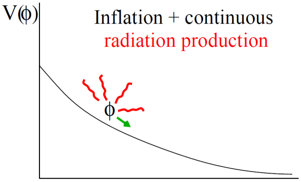

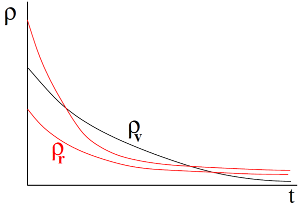

Condition of Warm inflation

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- Conservation equation

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = -\Upsilon\dot{\phi}^2 \\ \dot{\rho}_\gamma + 4H\rho_\gamma = \Upsilon\dot{\phi}^2; \tag{9}$$

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- Equation of motion of warm inflation

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{10}$$

$$\epsilon = \frac{M_p^2}{2(1+Q)}\left(\frac{V'}{V}\right)^2 \qquad Q = \frac{\Upsilon}{3H} \qquad \Upsilon(T,\phi) = C\frac{T^3}{\phi^2}.$$

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- Perturbation of the scalar field

$$\delta\phi^2_{warm} \sim \sqrt{HT} \qquad \delta\phi_{cold} \sim H \quad T > H \qquad (11)$$

Perturbation

Scalar perturbation of warm inflation, growing modes

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- Power-spectrum

$$\Delta_{\mathcal{R}}(k/k_*) = \left(\frac{H_*^2}{2\pi\dot{\phi}_*}\right)^2 \mathcal{F}(k/k_*), \tag{12}$$

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- Modification coefficient

$$\mathcal{F}(k/k_*) \equiv \left(1 + 2n_* + \frac{2\sqrt{3}\pi Q_*}{\sqrt{3+4\pi Q_*}} \frac{T_*}{H_*}\right) G(Q_*), \quad (13)$$

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- where n_* denotes the inflaton statistical distribution due to the presence of the radiation bath and $G(Q_*)$ accounts for the effect of the coupling of the inflaton fluctuations to radiation

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- Tensor perturbation of warm inflation model is the same as cold (standard) model of inflation.

Model building of warm inflation

Problems

- Inflation field with direct coupling to light fields has problems. For example Yukawa interaction $\lambda\phi\bar{\psi}\psi$ leads to Fermion's mass $m_\psi = \lambda\phi$. Typically large inflaton values are needed in the slow-roll limit. But light fermions are the case in radiation sector. On the other hands when small coupling λ is considered the thermal bath condition $T > H$ may not be sustained.

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- Direct coupling to light fields may add large thermal correction to inflaton mass λT, that can stop slow-roll condition $\dot{\phi}^2 < V(\phi)$ in warm inflation regime $T > H$.

Axion and Warm inflation

- Warm (Pseudo)Scalar Inflation

$$\mathcal{L}_{int} = \frac{\phi}{8M} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \tag{16}$$

$$F^a_{\mu\nu} = \frac{1}{ig}[D^a_\mu, D^a_\nu] \qquad \tilde{F}^{\mu\nu}_a = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{a\mu\nu}$$

$$D^a_\mu = \partial_\mu - igA_\mu J^a \qquad Tr[J_a, J_b] = \frac{1}{2}\delta_{ab}$$

  which leads to constant Q
- Minimal warm inflation which leads to T-dependent dissipation parameter

Reheating

Interacting Potential

- Non-thermal phase

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where σ, h, k are coupling constants

- Thermalization

Minimal reheating

The model

$$\mathcal{L}_m = \int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\phi,\chi)\right], \quad (19)$$

$$V(\phi,\chi) = \frac{m_\phi^2}{2}\phi^2 + \frac{m_\chi^2}{2}\chi^2 \quad .$$

Note that inflaton self couplings or couplings between the inflaton and matter would open the usual preheating channels and make the overall preheating process more efficient

Evolution of momentum mode

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2\right)\chi_k = 0\,. \tag{20}$$

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Rescale mode

$$\psi_k \equiv a^{3/2}\chi_k\,, \quad (21)$$

$$\ddot{\psi}_k + \omega_k^2(t)\psi_k = 0, \quad (22)$$

$$\omega_k^2(t) = \frac{k^2}{a^2} + m_\chi^2 - \frac{3}{4}\left(\frac{\dot{a}}{a}\right)^2 - \frac{3}{2}\frac{\ddot{a}}{a}\,. \quad (23)$$

The contribution to the mass of χ due to the expansion in Eq. (23) can be expressed as

$$-\frac{3}{4}\left(\frac{\dot{a}}{a}\right)^2 - \frac{3}{2}\frac{\ddot{a}}{a} = -\frac{9}{4}H^2 + \frac{3}{2}\dot{H}^2 = 6\pi G p_\phi, \quad (24)$$

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Oscillating scalar field

The oscillating homogeneous inflaton field can be parameterized as

$$\phi(t) = A(t)\cos(m_\phi t)\,, \tag{26}$$

with a decreasing time dependent amplitude $A(t)$.

$$A(t) = \mathcal{A}\,\frac{t_R}{t}\,, \tag{27}$$

Pressure

This leads to the following expression for the pressure

$$p_\phi = -\frac{1}{2}m_\phi^2 A^2 \cos(2m_\phi t) - A^2\frac{m_\phi}{t}\sin(2m_\phi t) + \frac{A^2}{t^2}\cos^2(m_\phi t)\,. \tag{28}$$

In the above, the first term dominates for $m_\phi t > 1$, i.e.

time dependence frequency for the field modes becomes

$$\omega_k^2(t) = k^2\left(\frac{t_R}{t}\right)^{4/3} + m_\chi^2 - 3\pi G \mathcal{A}^2 \cos(2m_\phi t) m_\phi^2 \left(\frac{t_R}{t}\right)^2 . \tag{29}$$

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$$k < \sqrt{3\pi G\mathcal{A}^2}\left(\frac{t_R}{t}\right)^{1/3} m_\phi \,, \quad (30)$$

and provided that the mass of the χ particles is small

$$m_\chi < \sqrt{3\pi G\mathcal{A}^2}\,\frac{t_R}{t} m_\phi \,, \quad (31)$$

Parametric resonance

In this case the equation of motion (22) becomes (introducing the rescales time variable $z = 2m_\phi t$)

$$\psi_k'' - \frac{3\pi}{4} G\mathcal{A}^2 \frac{z_R^2}{z^2} \cos(z)\psi_k = 0, \quad (32)$$

where a prime denotes the derivative with respect to z,

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If it were not for the z^{-2} factor, Equation (32) would be of the form of a Mathieu equation and lead to broad band parametric resonance (i.e. all infrared modes obeying (30) would be exponentially excited.

Linear solution for ψ

The growth of ψ can hence be approximated as a linear growth in time

$$\psi_k(t) \simeq (t - t_R)\tilde{\mu}_{k,eff}\psi_k(t_R)\,. \quad (33)$$

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Energy density

A lower bound on the energy density of χ particles at time t can then be obtained (30)

$$\rho_\chi(t) \sim 4\pi \frac{1}{(2\pi)^3}\int_{k=0}^{k_{max}(t_R)} dk k^4 \frac{a^2(t_R)}{a^2(t)}\chi_k(t)^2\,, \tag{34}$$

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$$T_{RH} = \left(\frac{1}{\sqrt{2}} m_\phi \mathcal{A}\right)^{1/2} \left(\frac{3}{266}\right)^{3/4} \left(G\mathcal{A}^2\right)^{3/4} \left(G m_\phi^2\right)^{3/4}\,. \quad (35)$$

The first term on the right hand side of (35) is the maximal temperature T_{max} after inflation

$$T_{RH} \sim 10^{-6} T_{max}\,. \quad (36)$$

Numerical Analysis

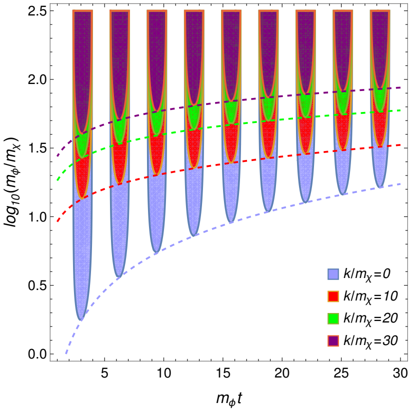


Figure: Where $\omega_k^2 < 0$, Roughly speaking, only for χ masses smaller than the ϕ mass can there be an instability. The different colors show the results for different values of k. For values of k larger than m_χ, it is the value of k which determines if an instability is possible. It is infrared modes which can undergo the resonant excitation. The dashed lines indicate the lowest value of m_ϕ/m_χ for which there can be a resonance.

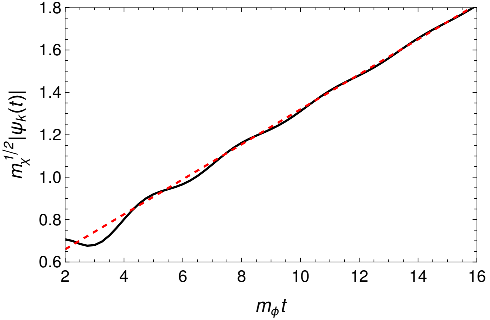
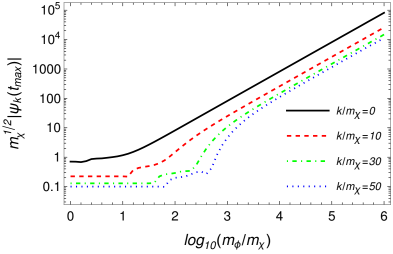
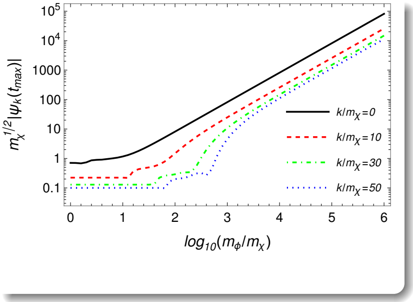


Figure: Time evolution of ψ_k for $k = 0$ and $m_\chi = 10^{-6} m_\phi$ (solid black curve). The fit to a linear growth (red dashed curve) as a function of time is a good one, and it is found to be given by $|\psi_k(t)| \sim 0.0825(m_\phi t + 6)/m_\chi^{1/2}$.


$m_\chi^{1/2}|\psi_k(t_{max})|$
10^5
10^4
1000
100
10
1
0.1
0 1 2 3 4 5 6
$log_{10}(m_\phi/m_\chi)$
k/m_χ=0
k/m_χ=10
k/m_χ=30
k/m_χ=50



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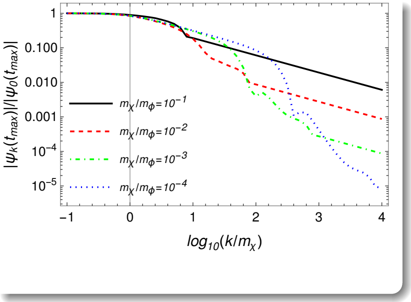
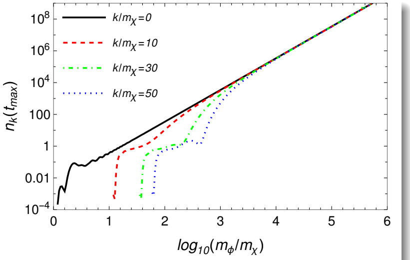
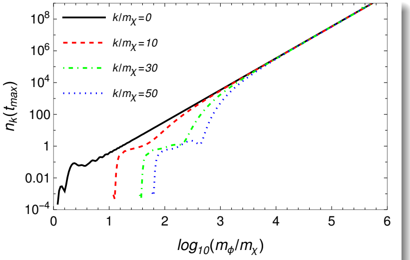
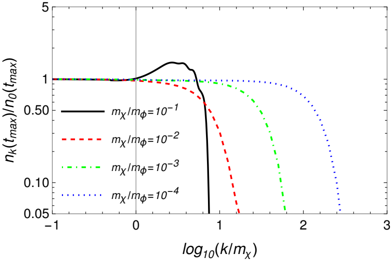


$|\psi_k(t_{max})|/|\psi_0(t_{max})|$
1
0.100
0.010
0.001
10^{-4}
10^{-5}
−1 0 1 2 3 4
$log_{10}(k/m_\chi)$
$m_\chi/m_\phi=10^{-1}$
$m_\chi/m_\phi=10^{-2}$
$m_\chi/m_\phi=10^{-3}$
$m_\chi/m_\phi=10^{-4}$



$k/m_\chi=0$
$k/m_\chi=10$
$k/m_\chi=30$
$k/m_\chi=50$
$n_k(t_{max})$
10^8
10^6
10^4
100
1
0.01
10^{-4}
0 1 2 3 4 5 6
$log_{10}(m_\phi/m_\chi)$


10^8
10^6
10^4
100
1
0.01
10^{-4}
0
1
2
3
4
5
6
$n_k(t_{max})$
$log_{10}(m_\phi/m_\chi)$
k/m_χ=0
k/m_χ=10
k/m_χ=30
k/m_χ=50


5
1
0.50
0.10
0.05
−1
0
1
2
3
$n_k(t_{max})/n_0(t_{max})$
$log_{10}(k/m_\chi)$
$m_\chi/m_\phi=10^{-1}$
$m_\chi/m_\phi=10^{-2}$
$m_\chi/m_\phi=10^{-3}$
$m_\chi/m_\phi=10^{-4}$

Our mechanism may have implications for the Dark Matter coincidence problem, namely the question of why the current energy densities of dark and visible matters are comparable. If Dark Matter is a very weakly inter- acting low mass (compared to the inflaton mass) scalar field, then during the phase of oscillations of the infla- ton condensate the same energy density of dark matter and Higgs particles will be generated.

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Thanks for your attention