Chaos near critical point

M.Asadi

School of Particles and accelerators, IPM

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Outline

Brief review

Consider a classical thermal system with phase space denoted as $\mathbf{X} = (\mathbf{q}, \mathbf{p})$.

$\mathbf{q}$: Coordinates of the phase space. $\mathbf{p}$: Momenta of the phase space.

⋆ **Is the system chaotic or not?**

√ **To Ans:** Measure the stability of a trajectory in phase space under small changes of the initial condition.

- Consider a reference trajectory in phase space $\mathbf{X}(t)$ with initial condition $\mathbf{X}(0) = \mathbf{X}_0$.
- A small change in the initial condition $\mathbf{X}_0 \to \mathbf{X}_0 + \delta\mathbf{X}_0$.
- New trajectory $\mathbf{X}(t) \to \mathbf{X}(t) + \delta\mathbf{X}(t)$.
- The system is chaotic if the distance between the new trajectory and the reference one increases exponentially with time.
- $|\delta\mathbf{X}(t)| \sim |\delta\mathbf{X}_0| e^{\lambda t}$, or $\frac{\partial \mathbf{X}(t)}{\partial \mathbf{X}_0} \sim e^{\lambda t}$.

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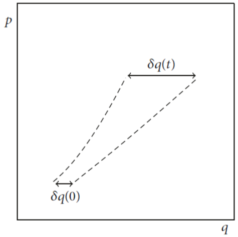

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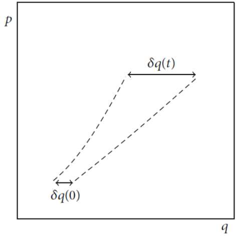

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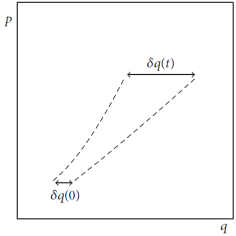

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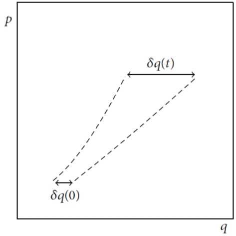

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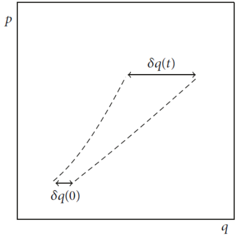

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1. The exponential increase depends on the orientation of $\delta \mathbf{X}_0$ and this leads to a spectrum of Lyapunov exponents, $\{\lambda_1, \lambda_2, ..., \lambda_K\}$, where K is the dimension of the phase space.
2. A useful parameter characterizing the trajectory instability is $\lambda_{\max} = \lim_{t\to\infty} \lim_{\delta \mathbf{X}_0 \to 0} \frac{1}{t} \log\left(\frac{\delta \mathbf{X}(t)}{\delta \mathbf{X}_0}\right)$, which is called the maximum Lyapunov exponent.
3. When the above limits exist and $\lambda_{\max} > 0$, the trajectory shows sensitivity to initial conditions and the system is said to be chaotic.

- There are two exponential behaviors in classical thermal systems that have analogues in terms of black holes physics:
  - ✓ The Lyapunov behavior, characterizing the sensitive dependence on initial conditions (correspond to blue-shift suffered by in-falling quanta).
  - ✓ The Ruelle behavior, characterizing the approach to thermal equilibrium (correspond to black hole's quasinormal modes).

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- To quantify the sensitivity to initial conditions one need to consider thermal expectation values. The classical expectation value of any function $F(\mathbf{X})$ is $\langle F \rangle_\beta = \frac{\int d\mathbf{X} e^{-\beta H(\mathbf{X})} F(\mathbf{X})}{\int d\mathbf{X} e^{-\beta H(\mathbf{X})}}$.
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- The expected behavior of this quantity is $F(t) \sim \sum_k c_k e^{2\lambda_k t}$, where c_k are constants and λ_k are the Lyapunov exponents.
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- The approach to thermal equilibrium i.e. how fast the system forgets its initial condition can be quantified by two-point functions of the form $G(t) = \langle X(t)X(0)\rangle_\beta - \langle X \rangle_\beta^2$
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- The approach to thermal equilibrium i.e. how fast the system forgets its initial condition can be quantified by two-point functions of the form $G(t) = \langle X(t) X(0) \rangle_\beta - \langle X \rangle_\beta^2$.
- The expected behavior of this quantity is $G(t) \sim \sum_j b_j e^{-\mu_j t}$, where b_j are constants and μ_j are complex parameters called Ruelle resonances.
- The late time behavior: $G \sim e^{-\mu_{\min} t}$.

√ Lyapunov behavior in a thermal system ($\frac{\partial \mathbf{X}(t)}{\partial \mathbf{X}_0} \sim e^{\lambda t}$):

- To quantify the sensitivity to initial conditions one need to consider thermal expectation values. The classical expectation value of any function $F(\mathbf{X})$ is $\langle F\rangle_\beta = \frac{\int d\mathbf{X} e^{-\beta H(\mathbf{X})} F(\mathbf{X})}{\int d\mathbf{X} e^{-\beta H(\mathbf{X})}}$.
- Note that $\partial \mathbf{X}(t)/\partial \mathbf{X}(0)$ can be both positive and negative. To avoid cancellations in a thermal expectation values, we consider $F(t) = \left\langle \left(\frac{\partial \mathbf{X}(t)}{\partial \mathbf{X}(0)}\right)^2 \right\rangle_\beta$
- The expected behavior of this quantity is $F(t) \sim \sum_k c_k e^{2\lambda_k t}$, where c_k are constants and λ_k are the Lyapunov exponents.
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Quantum Chaos

Now consider a 1D thermal system with phase space variables (q, p).

Classically, for a chaotic system, $\partial q(t)/\partial q(0)$ grows exponentially with time.

⋆ **What is the quantum version of this quantity?**

- $\frac{\partial q(t)}{\partial q(0)} = \{q(t), p(0)\}_{P.B.} \rightarrow \frac{\partial \hat{q}(t)}{\partial \hat{q}(0)} = \frac{1}{i\hbar}[\hat{q}(t), \hat{p}(0)]$
- In the thermal systems, we must calculate the expectation value of $[\hat{q}(t), \hat{p}(0)]$ in a thermal state.
- This commutator might be positive or negative in a thermal expectation value and this might lead to cancellations.
- Therefore, we consider the expectation value of the square of this commutator $C(t) = \langle -[\hat{q}(t), \hat{p}(0)]^2 \rangle_\beta$.
- More generally, one can replace $\hat{q}(t)$ and $\hat{p}(0)$ by two generic Hermitian operators V and W and quantify chaos with the double commutator $C(t) = \left\langle -[W(t), V(0)]^2 \right\rangle_\beta$.
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Chaos means sensitive dependence on initial conditions. So we expect $C(t)$ to be 'small' in nonchaotic system and 'large' if the dynamics is chaotic.

- For some class of chaotic systems, which include holographic systems, $C(t)$ is expected to behave as:
- $$C(t) \sim \begin{cases} N_{\text{dof}}^{-1} & t < t_d \\ N_{\text{dof}}^{-1} e^{\lambda_L t} & t_d \ll t \ll t_* \\ \mathcal{O}(1) & t > t_* \end{cases},$$

where N_{dof} is the number of degrees of freedom of the system and we have assumed V and W to be unitary and Hermitian operators, so that $VV = WW = 1$.

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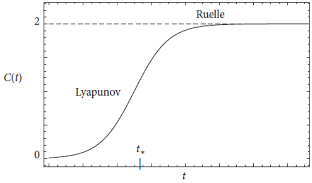

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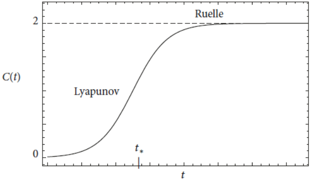

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- The exponential growth of $C(t)$ is characterized by the Lyapunov exponent λ_L and takes place at intermediate time scales bounded by the dissipation time t_d and the scrambling time t_*.

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  - ✓ is related to the classical Ruelle resonances ($t_d \sim \mu^{-1}$).
  - ✓ characterizes the exponential decay of two-point correlators $\langle V(0)V(t)\rangle \sim e^{-t/t_d}$
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- Now we write the double commutator as (assumption: V and W are Hermitian and unitary operators)
  $C(t) = \left\langle -[W(t), V(0)]^2 \right\rangle_\beta = 2 - 2\langle W(t)V(0)W(t)V(0)\rangle_\beta$.
- All the relevant information about $C(t)$ is contained in the OTOC:
  $OTOC(t) = \langle W(t)V(0)W(t)V(0)\rangle$
- $C(t)$ approaches 2 at later times which implies that the $OTOC(t)$ should vanish in that limit. This is related to chaos.
- In quantum field theories we can upgrade $C(t)$ to
  $C(t,x) = \left\langle -[V(0,0), W(t,x)]^2 \right\rangle_\beta$
- In many examples the above commutator is roughly given by
  $C(t,x) \sim e^{\lambda_L\left(t - t_* - \frac{|x|}{v_B}\right)}$, where v_B is the butterfly velocity.
- This velocity describes the growth of the operator W in physical space and sets a bound for the rate of transfer of quantum information.
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- $C(t)$ approaches 2 at later times which implies that the $OTOC(t)$ should vanish in that limit. This is related to chaos.
- In quantum field theories we can upgrade $C(t)$ to
  $C(t,x) = \left\langle -[V(0,0), W(t,x)]^2 \right\rangle_\beta$
- In many examples the above commutator is roughly given by
  $C(t,x) \sim e^{\lambda_L\left(t - t_* - \frac{|x|}{v_B}\right)}$, where v_B is the butterfly velocity.
- This velocity describes the growth of the operator W in physical space and sets a bound for the rate of transfer of quantum information.
- There is an additional delay in scrambling due to the physical separation between the operators.

Perturbations of the TFD state & Shockwave geometries

- Consider a states of the form $W(t)|TFD\rangle$, where W is a thermal scale operator that acts on the right boundary theory.

- This state can be describe by a 'particle' (field excitation) in the bulk that comes out of the past horizon, reaches the boundary at time t producing the perturbation $W(t)$, and then falls into the future horizon. We will refer to this bulk excitation as the W-particle.

- If $|t|$ is not too large, the state $W(t)|TFD\rangle$ will represent just a small perturbation of the TFD state and the corresponding description in the bulk will be just an eternal two-sided black hole geometry slightly perturbed by the presence of a probe particle.

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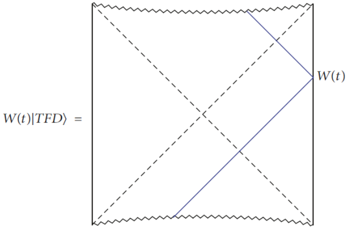

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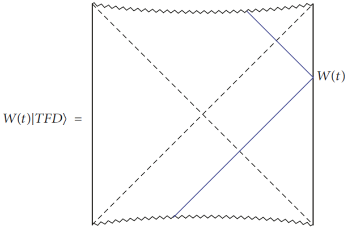

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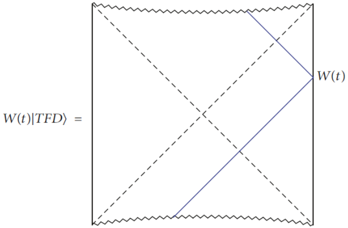

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- In the large $|t|$ case, there is a nontrivial modification of the geometry.
- A very early perturbation is described in the bulk in terms of a particle that falls towards the future horizon for a very long time and gets highly blue-shifted in the process.
- If the particle's energy is E_0 in the asymptotic past, this energy will be exponentially larger from the point of view of the $t = 0$ slice of the geometry, $E = E_0 e^{(2\pi/\beta)t}$.
- Therefore, for large enough $|t|$, the particle's energy will be very large and one needs to include the corresponding backreaction.
- The back-reaction of a very early (or very late) perturbation corresponds to a shock wave geometry.
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- For large enough $|t|$ we can approximate the stress tensor of the W-particle as $T_{VV} \sim P^U \delta(V) a(\vec{x})$.
- $P^u \sim e^{(2\pi/\beta)t}/\beta$ is the momentum of the W-particle in the $U-$direction.
- $a(\vec{x})$ is some generic function that specifies the location of the perturbation in the spatial directions of the right boundary.
- T_{VV} is completely localized at $V = 0$ and homogeneous along the $U-$direction.
- The shock wave geometry produced by the W-particle is described by the metric $ds^2 = 2A(UV)dUdV + G_{ij}(UV)dx^i dx^j - 2A(UV)h(t, \vec{x})\delta(v)dV^2$.
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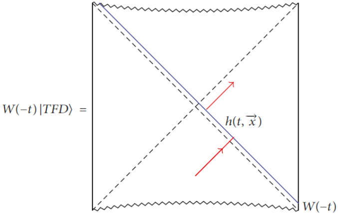

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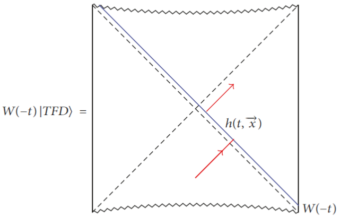

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- The precise form of $h(t,\vec{x})$ can be determined by solving the $VV-$component of Einsteins equation.
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  (for simplicity, G_{ij} has been assumed tobe diagonal and isotropic.)
- The shock wave profile contains information about the parameters characterizing the chaotic behavior of the boundary theory. So, we have $C(t,\vec{x}) \sim h(t,\vec{x})$.
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- The quantum Lyapunov exponent λ_L is bounded from above, while the scrambling time t_* is bounded from below.
  - √ For a generic many-body quantum system, the scrambling time is bounded from below as $t_* \geqslant C(\beta) \log N_{\mathrm{dof}}$,
    where $C(\beta)$ is some function of the β. In the case of black holes this function is simply given by $C(\beta) = \frac{\beta}{2\pi}$.
- For black holes, one expects the dissipation time to be given by the black hole quasinormal modes $t_d \sim \beta$, while the scrambling time is parametrically larger $t_* \sim \beta \log N_{\mathrm{dof}}$.
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Bounds on chaos

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1RCBH Background

- Metric of this background:
  $ds^2 = \exp^{2A(R)}(-h(r)dt^2 + d\overrightarrow{x}^2) + \frac{\exp^{2B(r)}}{h(r)}dr^2,$
  where
  $A(r) = \ln\left(r(1+\frac{Q^2}{r^2})^{\frac{1}{6}}\right), \quad B(r) = -\ln\left(r(1+\frac{Q^2}{r^2})^{\frac{1}{3}}\right), \quad h(r) = 1 - \frac{M^2}{r^2(r^2+Q^2)}.$
- Temperature & chemical potential:
  $T = \frac{2r_h^2+Q^2}{2\pi\sqrt{Q^2+r_h^2}}, \quad \mu = \frac{Qr_h}{\sqrt{Q^2+r_h^2}}.$
- It was shown that there is a critical point at $\frac{\mu}{T} = (\frac{\mu}{T})_* = \frac{\pi}{\sqrt{2}}$ $(\frac{Q}{r_h} = \sqrt{2})$ and the solutions are thermodynamically stable for $\frac{Q}{r_h} < \sqrt{2}$.

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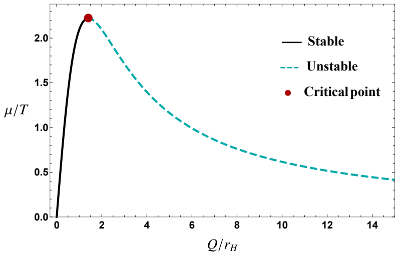

Butterfly velocity and critical exponent

- v_B in 1RCBH model:
  $v_B^2 = \frac{4}{7 \mp \sqrt{1 - \left(\frac{\mu / T}{\pi / \sqrt{2}} \right)^2}}$,
  $-(+)$ indicates the stable(unstable) black hole solutions.

- By expanding v_B and $\frac{dv_B}{d\left(\frac{\mu}{T}\right)}$ in power of $\left(\left(\frac{\mu}{T}\right)^* - \frac{\mu}{T}\right)$ we obtain:
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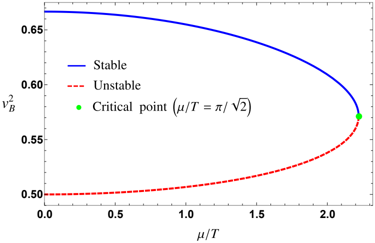

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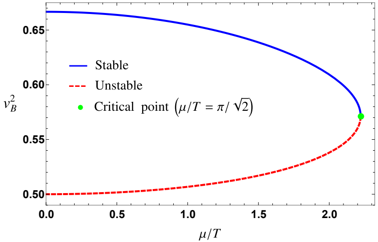

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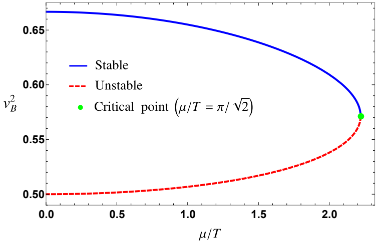

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A bound on butterfly velocity

- In addition to the 1RCBH model, we consider AdS-Reisner-Nordstrom black hole which is holographically dual to a strongly coupled field theory with a U(1) charge.
- In the both this models, we obtain the inequality between the butterfly velocity and speed of sound for every black hole solution:

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Thanks for your attention!