Carroll Fermions and Supersymmetry

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Wednesday Weekly Seminars

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Based on:

K. Koutrolikos & M. Najafizadeh,

"Super-Carrollian and super-Galilean field theories",

Phys. Rev. D **108**, 125014 (2023), arXiv:2309.16786 [hep-th]

Terminology

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Ann. Inst. Henri Poincaré, Section A :
Vol. III, n° 1, 1965, p. 1-12. *Physique théorique.*

Une nouvelle limite non-relativiste du groupe de Poincaré

par

Jean-Marc LÉVY-LEBLOND
(Laboratoire de Physique Théorique, Orsay).

ABSTRACT. — It is shown that, for the Galilean approximation to the Poincaré group to be valid, not only do we have to consider pure Lorentz transformation with low velocities, but also great time-like intervals. Under the same conditions, the transformation properties of great space-like inter-

Terminology:

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P_0 le générateur des translations de temps. Il est instructif de comparer les algèbres de Lie du groupe de Carroll et des groupes de Poincaré et Galilée. On a respectivement :

$[J_i, J_j] = i\varepsilon_{ijk}J_k$	$[J_i, J_j] = i\varepsilon_{ijk}J_k$	$[J_i, J_j] = i\varepsilon_{ijk}J_k$
$[J_i, K_j] = i\varepsilon_{ijk}K_k$	$[J_i, K_j] = i\varepsilon_{ijk}K_k$	$[J_i, K_j] = i\varepsilon_{ijk}K_k$
$[K_i, K_j] = 0$	$[K_i, K_j] = -i\varepsilon_{ijk}J_k$	$[K_i, K_j] = 0$
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$[K_i, P_j] = 0$	$[K_i, P_j] = i\delta_{ij}P_0$	$[K_i, P_j] = i\delta_{ij}P_0$
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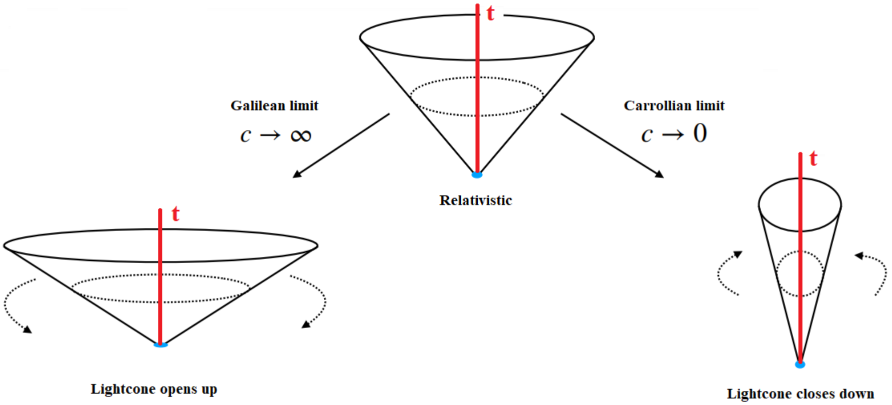
Galilée	Poincaré	Carroll

On voit clairement sur ces expressions que substituant :

Terminology:



Galilei limit: $\dfrac{v}{c} \to 0 \quad \Longleftrightarrow \quad c \to \infty$ Carroll limit: $\dfrac{v}{c} \to \infty \quad \Longleftrightarrow \quad c \to 0$

Outline:

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Flat space holography:
Conformal Carroll field theory might be dual to quantum gravity in asymptotically flat spacetime [S. Pasterski (2021), L. Donnay (2023), ...]

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Carroll symmetry in hydrodynamics:
L. Ciambelli, C. Marteau (2018), ...

Carroll Algebra

Carroll Algebra

Recall the Poincaré algebra

$$[\,P_\mu\,,\,P_\nu\,] = 0$$

$$[\,J_{\mu\nu}\,,\,P_\rho\,] = \eta_{\mu\rho}\,P_\nu - \eta_{\nu\rho}\,P_\mu$$

$$[\,J_{\mu\nu}\,,\,J_{\rho\sigma}\,] = \eta_{\mu\rho}\,J_{\nu\sigma} - \eta_{\mu\sigma}\,J_{\nu\rho} - \eta_{\nu\rho}\,J_{\mu\sigma} + \eta_{\nu\sigma}\,J_{\mu\rho}$$

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in which

$$P_\mu \;:= \partial_\mu \qquad \text{Translation generators} \begin{cases} P_0 & \text{Hamiltonian} \\ P_i & \text{Momentum} \end{cases}$$

$$J_{\mu\nu} \;:= x_\mu\, \partial_\nu - x_\nu\, \partial_\mu \qquad \text{Lorentz generators} \begin{cases} J_{i0} & \text{Boosts} \\ J_{ij} & \text{Rotations} \end{cases}$$

are ten Poincaré generators in 4d spacetime.

Carroll Algebra

Poincare generators, $P_0,\ P_i,\ J_{i0},\ J_{ij}$, including c (i.e. $x^\mu = (ct, x^i)$) become:

$$P_0 = \frac{1}{c}\,\partial_t \qquad P_i = \partial_i \qquad J_{i0} = x_i\,\frac{1}{c}\,\partial_t + c\,t\,\partial_i \qquad J_{ij} = x_i\partial_j - x_j\partial_i$$

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Rescaling the generators, one gets

$$c\,P_0 \;\; \to \;\; H = \partial_t \qquad\qquad c\,J_{i0} \;\; \to \;\; B_i = x_i\,\partial_t + c^2\,t\,\partial_i$$

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satisfying the **Carroll algebra**:

$$\begin{aligned}
[P_i, B_j] &= \delta_{ij}\,H \\
[J_{ij}, P_k] &= \delta_{ik}P_j - \delta_{jk}P_i \\
[J_{ij}, B_k] &= \delta_{ik}B_j - \delta_{jk}B_i \\
[J_{ij}, J_{kl}] &= \delta_{ik}J_{jl} - \delta_{jk}J_{il} + \delta_{jl}J_{ik} - \delta_{il}J_{jk}
\end{aligned}$$

Carroll Particles

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Consider a free relativistic particle with energy E, mass m and momentum $\vec{p}$:

$$\vec{p} = \gamma\, m\, \vec{v} \qquad E = \gamma\, m\, c^2 \qquad E^2 = \vec{p}^{\,2} c^2 + m^2 c^4$$

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$$\begin{cases} \vec{v} = 0 \quad \to \quad \gamma = 1 \quad \to \quad \vec{p} = 0 \qquad E = mc^2 \qquad \textbf{electric}\ \text{Carroll particles} \\ \\ \end{cases}$$

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Jan de Boer et al. [arxiv:2110.02319]

Carroll Scalar Field Theories ($s = 0$)

There are 4 methods in the literature:

- Lagrange multiplier method ✓
- Hamiltonian method ✓
- Field expansion method ✓
- Seed Lagrangian method

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This Lagrangian has also an alternative by adding a Lagrange multiplier χ:

$$\mathcal{L}' = -\frac{1}{2}\,c^2\,\chi^2 + \chi\,\partial_t\phi - \frac{1}{2}\,(\partial_i\phi)^2 \qquad\qquad -\,c^2\,\chi + \partial_t\phi = 0 \tag{2}$$

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Hamiltonian method: Let us consider a massless scalar field Lagrangian in which the canonical momentum conjugate to the field reads

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$$S\,[\phi, \pi_\phi] = \int dt\, d^3x \left\{\pi_\phi\,\dot{\phi} - \mathcal{H}\right\} = \int dt\, d^3x \left\{\pi_\phi\,\dot{\phi} - \frac{1}{2}\,c^2(\pi_\phi)^2 - \frac{1}{2}\,(\partial_i\phi)^2\right\}$$

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$$\boxed{S_m = \int dt\, d^3x \left\{\pi_\varphi\,\dot{\phi} - \frac{1}{2}\,(\partial_i \phi)^2\right\}}$$

electric: $\phi \to c\phi, \pi \to \frac{1}{c}\pi, c \to 0$

$$\boxed{S_e = \int dt\, d^3x \left\{\pi_\phi\,\dot{\phi} - \frac{1}{2}\,(\pi_\phi)^2\right\}}$$

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$$S\left[\phi, \pi_\phi\right] = \int dt\, d^3x \left\{\pi_\phi\,\dot{\phi} - \mathcal{H}\right\} = \int dt\, d^3x \left\{\pi_\phi\,\dot{\phi} - \frac{1}{2}\,c^2(\pi_\phi)^2 - \frac{1}{2}\,(\partial_i\phi)^2\right\}$$

- **magnetic:** $c \to 0$ $\qquad$ $\boxed{S_m = \int dt\, d^3x \left\{\pi_\varphi\,\dot{\phi} - \frac{1}{2}\,(\partial_i\phi)^2\right\}}$
- **electric:** $\phi \to c\phi, \pi \to \frac{1}{c}\pi, c \to 0$ $\qquad$ $\boxed{S_e = \int dt\, d^3x \left\{\pi_\phi\,\dot{\phi} - \frac{1}{2}\,(\pi_\phi)^2\right\}}$

M. Henneaux (2021)

Carroll Scalar Field Theories

- Under a Lorentz boost transformation, the scalar field Φ transforms as

$$\delta\,\Phi = i\,\beta_i\,J^{0i}\,\Phi = \beta_i\left(x^0\partial^i - x^i\partial^0\right)\Phi = \left(c\,t\,\beta^i\,\partial_i + \frac{1}{c}\,\beta^i\,x_i\,\partial_t\right)\Phi$$

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$$\textbf{Carroll boost trans.:}\begin{cases}\delta\phi &= b^i\,x_i\,\partial_t\,\phi\\ \delta\chi &= b^i\,x_i\,\partial_t\,\chi + b^i\,\partial_i\,\phi\end{cases}$$

Carroll Fermion Field Theories ($s = \frac{1}{2}$)

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Let us consider the Lagrangian density of a massless Dirac field including c

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[K. Koutrolikos, M.N., PRD (2023)]

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These Lagrangians are invariant under the following Carroll boost transformations:

$$\textbf{Carroll boost trans.}: \begin{cases} \delta\psi \;=\; b^i x_i\,\partial_t\,\psi \\ \delta\eta \;=\; b^i x_i\,\partial_t\,\eta + \frac{1}{2}\,\gamma^0\,\gamma^i\,b_i\,\psi \end{cases}$$

- Carroll fermions can be of **Dirac**, **Majorana**, or **Weyl** spinors

On Supersymmetry

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- In the Carroll supersymmetry, the SUSY algebra reads

Carroll SUSY algebra: $\boxed{[\,\delta_1\,,\,\delta_2\,] = 2\,(\bar{\epsilon}_2\,\gamma^0\,\epsilon_1)\,\partial_t}$

Carrollian Supersymmetry: $(\,0\,,\ \frac{1}{2}\,)$

Super-electric Carroll theory

We introduce the **super-electric Carroll** action by [K.K, M.N., PRD (2023)]:

$$\boxed{\mathrm{S}^{\mathrm{SUSY}}_{\mathrm{eC}} = \frac{1}{2}\int dt\, d^3x \left\{ (\partial_t \phi_R)^2 + (\partial_t \phi_I)^2 + {F_R}^2 + {F_I}^2 - \bar{\psi}\,\gamma^0 \partial_t\, \psi \right\}}$$

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and find that the action is invariant under the Super-electric Carroll transformations

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\delta\phi_R &= \bar{\epsilon}\,\psi \\
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We show that these transformations close off-shell for every field

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Note

We also derived

Galilei fermions

and generalized them to

Supersymmetry

[K. Koutrolikos, M.Najafizadeh, PRD (2023)]

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Thank you for your attention!