Quantum Features of Gravcats

Gravitational cats

Saeed Haddadi

School of Particles and Accelerators,

Institute for Research in Fundamental Sciences (IPM)

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Headlines

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02. Gravitational cats

03. Results

05. Summary

Introduction

- Quantum features of gravcats,

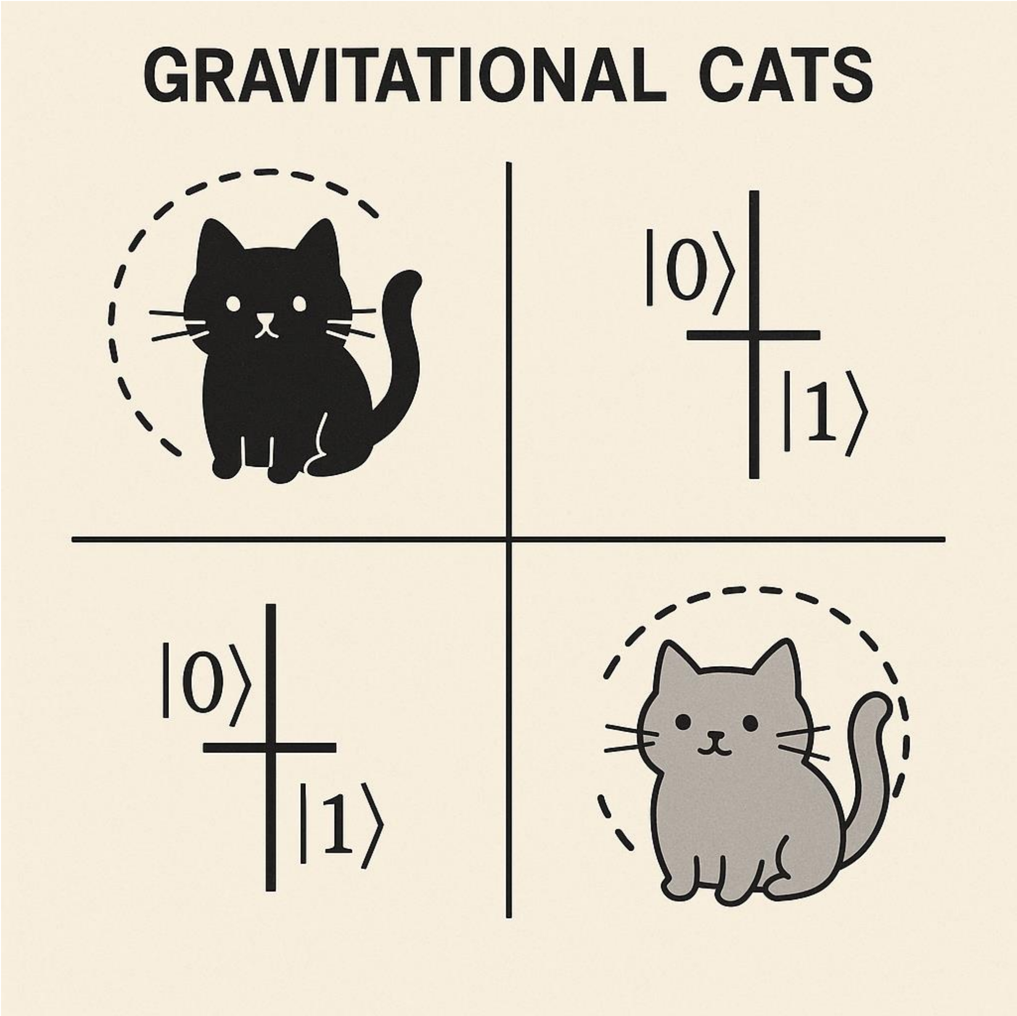
Scientific Reports **15**, 18594 (2025).

- Gravitational cat states as a resource for quantum information processing,

Phys. Rev. D **111**, 064077 (2025).

- Quantumness of gravitational cat states in correlated dephasing channels,

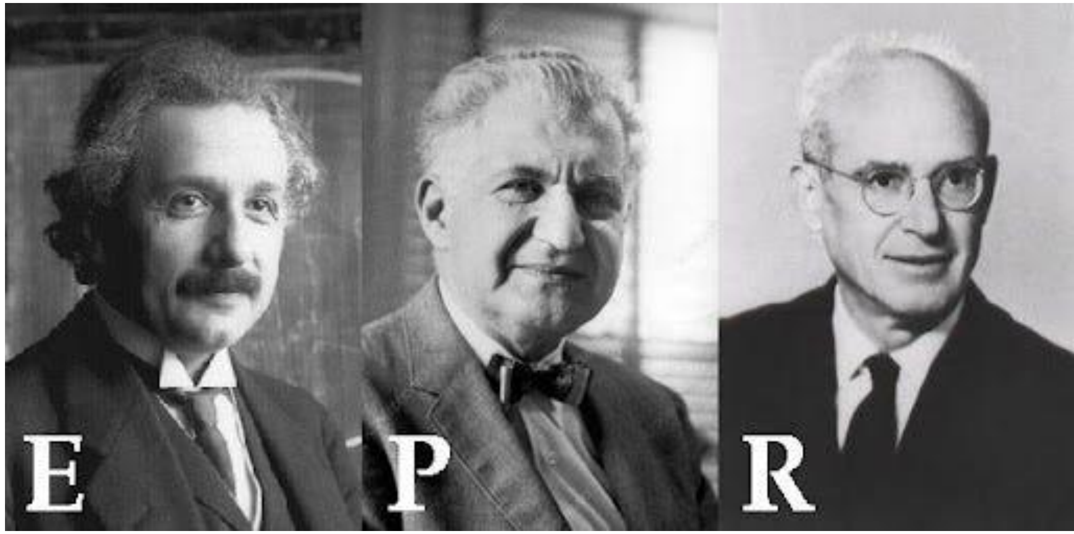
Eur. Phys. J. C **84**, 670 (2024).



Introduction

EPR - 1935

Can quantum-mechanical description of physical reality be considered complete?



A. Einstein, B. Podolsky, and N. Rosen

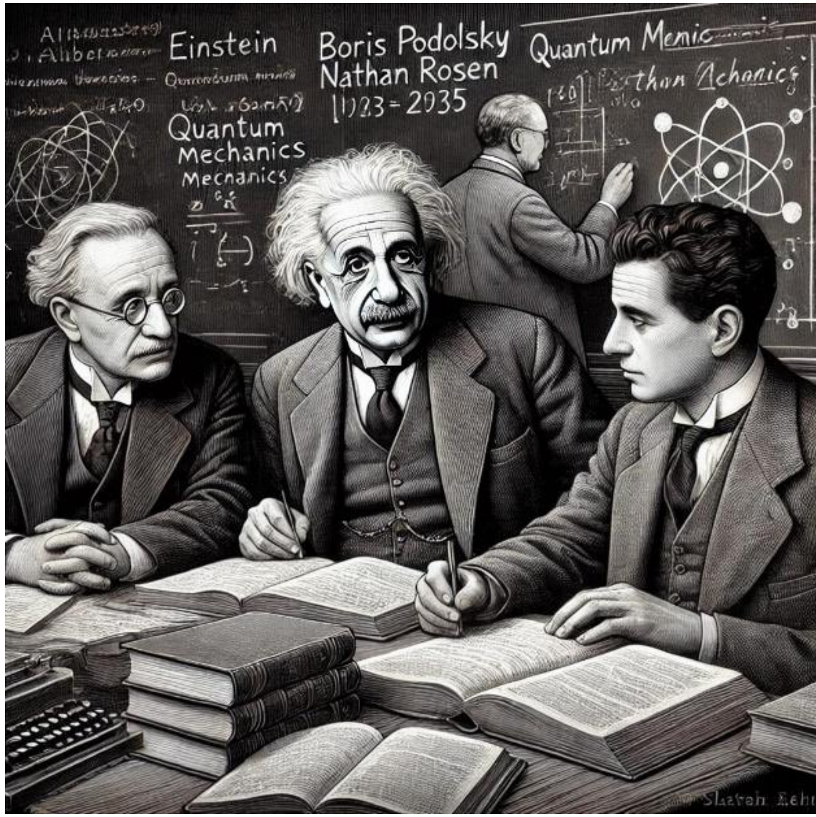
A. Einstein, B. Podolsky, and N. Rosen, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **47**, 777 (1935).

Introduction

Main ideas of EPR

- **Physical realism:** EPR assumed that if we could determine a property of a system (such as position or momentum) without directly affecting it, that property must be "real".

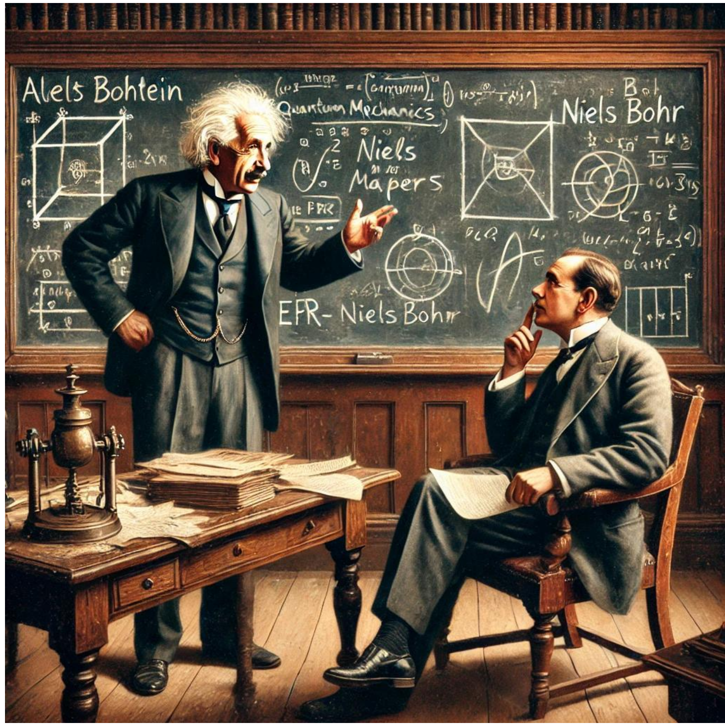
- **Locality:** They also believed that "information" could not be transmitted faster than the speed of light.



A. Einstein, B. Podolsky, and N. Rosen, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **47**, 777 (1935).

Introduction

- **Bohr's reply**



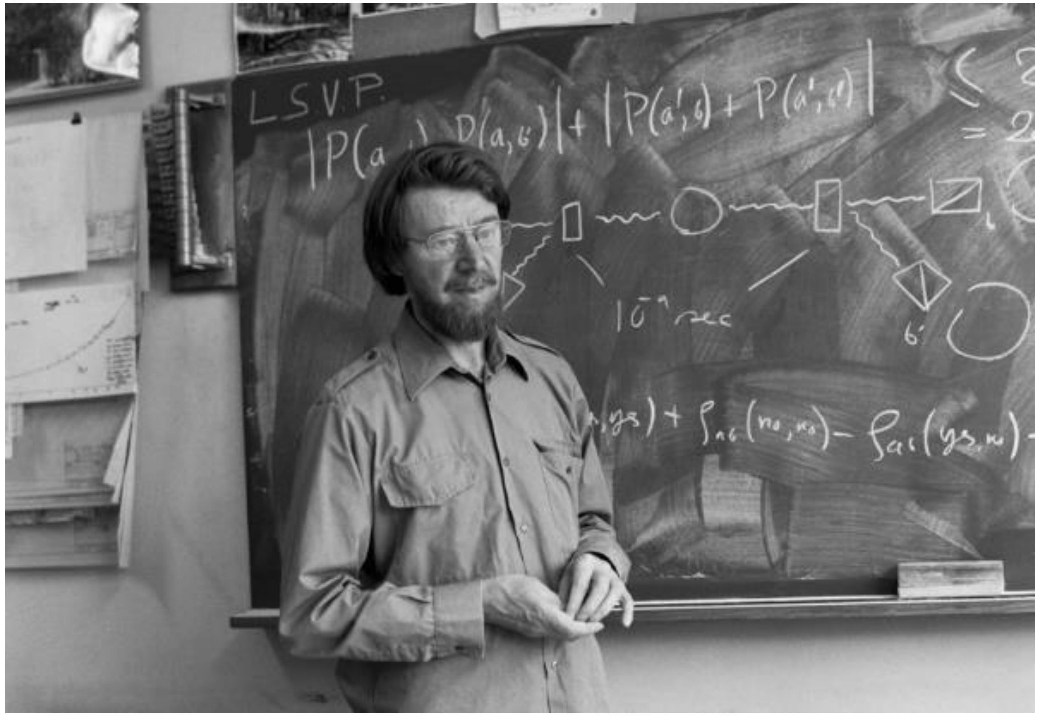
- Bohr argued that EPR had reasoned fallaciously.

- Bohr concluded that EPR's arguments do not justify their conclusion that the quantum description turns out to be essentially incomplete.

N. Bohr, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **48**, 696 (1935).

Introduction

Erwin Schrödinger
1935



John Stewart Bell
1964

Introduction

Quantum Entanglement

Quantum entanglement is the phenomenon where the quantum state of each particle in a group cannot be described independently of the state of the others, even when the particles are separated by a large distance.

Separable states of the composite system can be represented in this form: $|\psi\rangle_A \otimes |\phi\rangle_B$.

For example, $|\psi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$ → $|\psi\rangle = [\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)] \otimes [\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)]$

However, entagled states can be given by

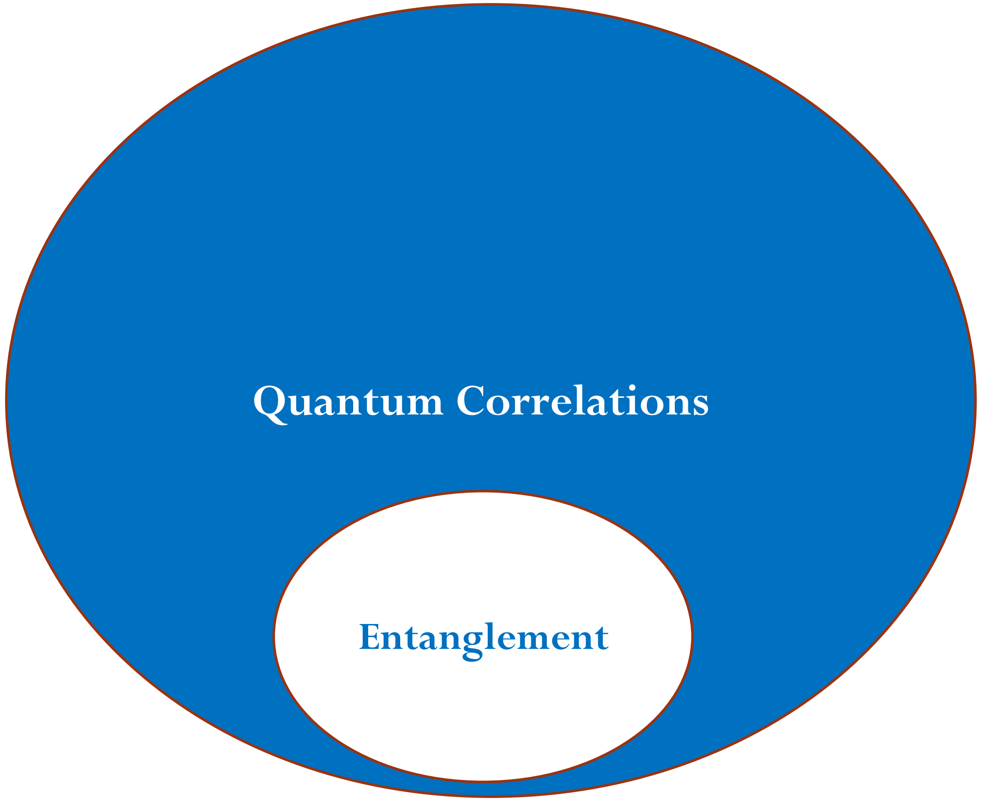
$$|\varphi\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$$

Introduction

Quantum Correlations

At the early stages of the development of quantum information and computation theory, quantum correlations were assumed to be equivalent to entanglement.

Although this assumption holds true for pure states, some mixed states possess not only entanglement but also other forms of quantum correlations.



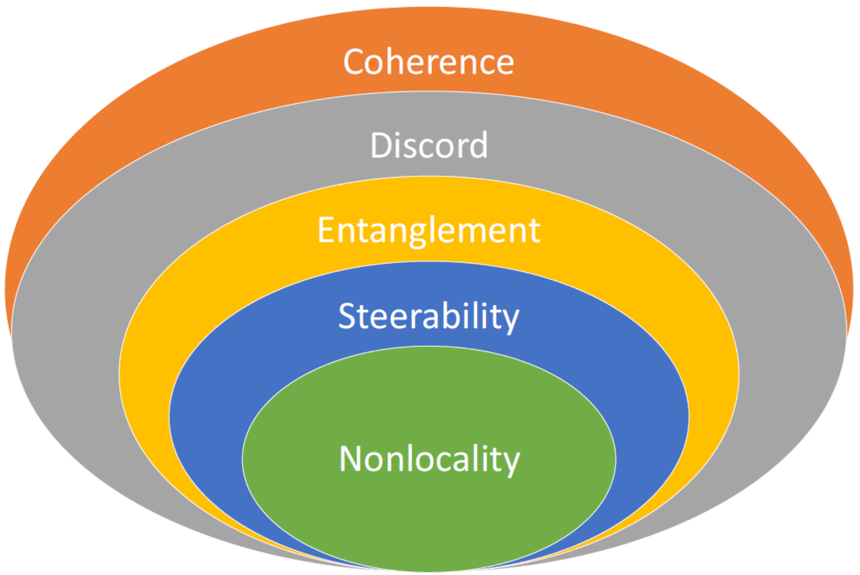
K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, The classical-quantum boundary for correlations: Discord and related measures *Rev. Mod. Phys.* **84**, 1655 (2012).

Introduction

Until now, various quantum resources have been introduced and many criteria have been proposed to measure their quantity, but

What is quantum resource?

Which measure is efficient?



App. Phys. B **130**, 177 (2024)

H. M. Wiseman, S. J. Jones, A. C. Doherty, Steering, entanglement, nonlocality, and the Einstein-Podolsky-Rosen paradox. Phys. Rev. Lett. **98**, 140402 (2007).

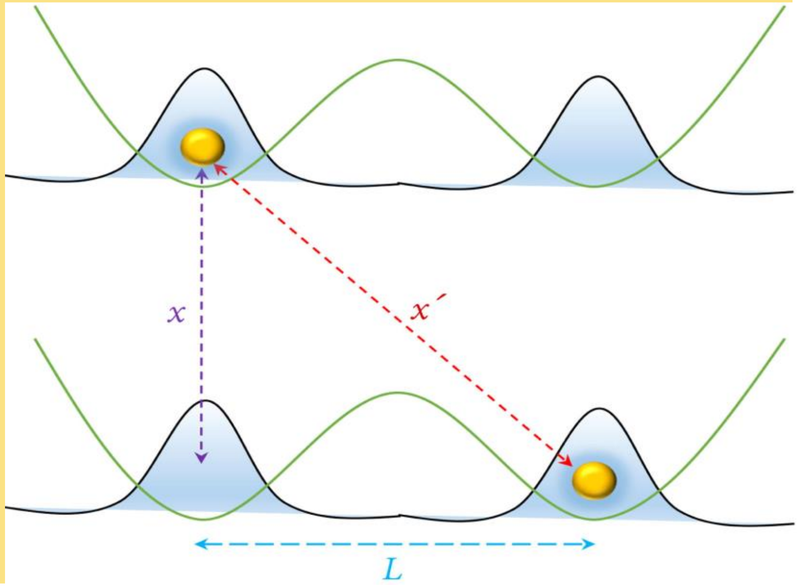
Gravitational cats

In a quantum description of matter, a single motionless massive particle can, in principle, be in a superposition state of two spatially-separated locations, i.e., a Schrodinger cat state. We use the term gravitational cat (gravcat) to refer to such states for objects that gravitate.

C. Anastopoulos and B. L. Hu, Probing a Gravitational Cat State. Class. Quant. Grav. **32**, 165022 (2015).

Gravitational cats

Gravitational cats typically refer to Schrödinger cat-like states influenced by gravitational interactions, and they are often modeled using simplified systems like two qubits to study quantum-gravitational phenomena.



In this context, each qubit might represent a localized superposition of mass at two positions (like a mass in a spatial superposition), and the interaction between them encodes the effect of gravity.

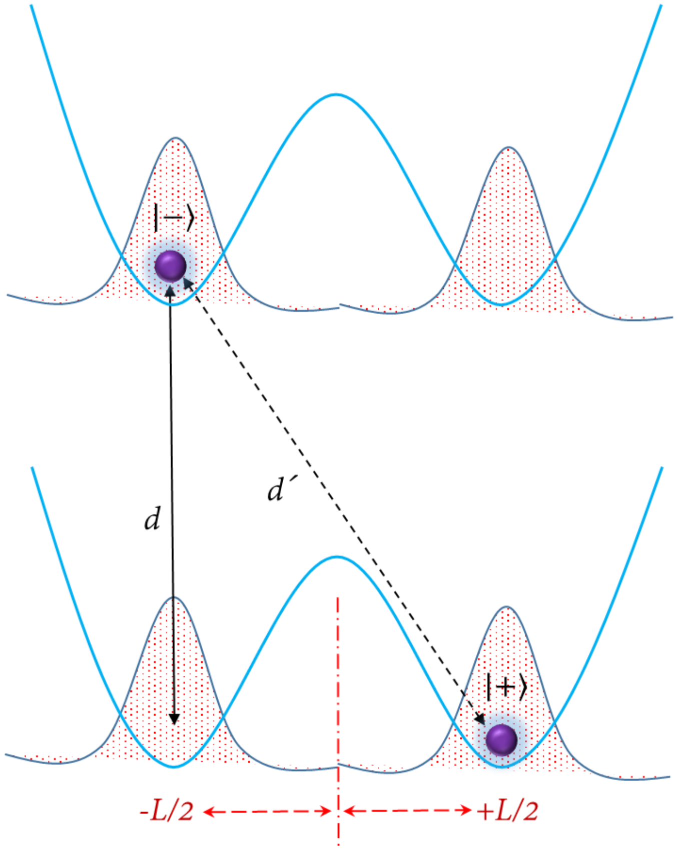
Gravitational cats

Gravitational interaction of qubits

Consider a particle of mass m in one dimension with Hamiltonian $\hat{H}_0 = \frac{1}{2m}\hat{p}^2 + U(\hat{x})$. The potential $U(x)$ corresponds to a symmetric double well, with local minima at $x = \pm\frac{1}{2}L$. We assume that $U(x)$ is even, so all eigenstates of $\hat{H}$ are parity definite. The lowest energy eigenstate $|g\rangle$ is always parity symmetric and the first excited $|e\rangle$ is parity antisymmetric. We denote the energy difference between $|e\rangle$ and $|g\rangle$ by ω.

We define the states

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle \pm |e\rangle\right)$$



C. Anastopoulos and B. L. Hu, Quantum superposition of two gravitational cat states, Class. Quant. Grav. **37**, 235012 (2020).

Gravitational cats

Physical scenario

In the non-relativistic limit, the following Hamiltonian describes the gravitational interaction between two gravcats

$$H = \frac{\omega}{2}(\sigma_z \otimes \mathbb{I} + \mathbb{I} \otimes \sigma_z) - \gamma(\sigma_x \otimes \sigma_x)$$

where $\gamma = \frac{Gm^2}{2}\left(\frac{1}{d} - \frac{1}{d'}\right)$ regulates the intensity of the gravitational interaction between the two gravcats, in which G is a universal gravitational constant and the terms d and d' represent the relative separations between the two gravcats.

Results

In matrix representation, the Hamiltonian (1) based on the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ within a two-dimensional Hilbert space takes the following form

$$H = \frac{\omega}{2}(\sigma_z \otimes \mathbb{I} + \mathbb{I} \otimes \sigma_z) - \gamma(\sigma_x \otimes \sigma_x)$$

$$H = \begin{pmatrix} \omega & 0 & 0 & -\gamma \\ 0 & 0 & -\gamma & 0 \\ 0 & -\gamma & 0 & 0 \\ -\gamma & 0 & 0 & -\omega \end{pmatrix}.$$

After diagonalizing the above Hamiltonian, we arrive at a set of specific eigenvalues

$$\begin{cases} \lambda_{1,2} = \mp\gamma \\ \lambda_{3,4} = \mp\sqrt{\omega^2 + \gamma^2} \end{cases}$$

and associated eigenvectors

$$\begin{cases} |\psi_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\ |\psi_2\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \\ |\psi_3\rangle = \cos(\kappa_+)|00\rangle + \sin(\kappa_+)|11\rangle \\ |\psi_4\rangle = \cos(\kappa_-)|00\rangle + \sin(\kappa_-)|11\rangle \end{cases}$$

where $\kappa_\pm$ are defined by

$$\kappa_\pm = \arctan\left(\frac{\gamma}{\omega \pm \sqrt{\omega^2 + \gamma^2}}\right).$$

Thermal state

$$\varrho(T) = \frac{e^{-\beta H}}{Z}$$

$$Z = tr[\exp(-\beta H)]$$

$$\beta = 1/k_B T$$

Results

Thermal state

$$\varrho_T = \begin{pmatrix} \varrho_{1,1} & 0 & 0 & \varrho_{1,4} \\ 0 & \varrho_{2,2} & \varrho_{2,3} & 0 \\ 0 & \varrho_{3,2} & \varrho_{3,3} & 0 \\ \varrho_{4,1} & 0 & 0 & \varrho_{4,4} \end{pmatrix}$$

$$\varrho_{1,1} = \frac{\cos^2(\kappa_+)\exp(-\beta\lambda_3) + \cos^2(\kappa_-)\exp(-\beta\lambda_4)}{Z},$$

$$\varrho_{2,2} = \varrho_{3,3} = \frac{\exp(-\beta\lambda_1) + \exp(-\beta\lambda_2)}{2Z},$$

$$\varrho_{2,3} = \varrho_{3,2} = \frac{\exp(-\beta\lambda_1) - \exp(-\beta\lambda_2)}{2Z},$$

$$\varrho_{1,4} = \varrho_{4,1} = \frac{\sin(2\kappa_+)\exp(-\beta\lambda_3) + \sin(2\kappa_-)\exp(-\beta\lambda_4)}{Z},$$

$$\varrho_{4,4} = \frac{\sin^2(\kappa_+)\exp(-\beta\lambda_3) + \sin^2(\kappa_-)\exp(-\beta\lambda_4)}{Z},$$

$$H = \frac{\omega}{2}(\sigma_z \otimes \mathbb{I} + \mathbb{I} \otimes \sigma_z) - \gamma(\sigma_x \otimes \sigma_x)$$

$$Z = 2\cosh[\beta\gamma] + 2\cosh\left[\beta\sqrt{\omega^2 + \gamma^2}\right]$$

Results

Quantum steering is a phenomenon in which the measurement performed on a particle at one location A seems to instantaneously influence the state of another particle at a distant location B .

$$\varrho_{A\to B} = \frac{1}{\sqrt{3}}\varrho_{AB} + \left(1 - \frac{1}{\sqrt{3}}\right)\hat{\varrho}_B$$

$$\varrho_{B\to A} = \frac{1}{\sqrt{3}}\varrho_{AB} + \left(1 - \frac{1}{\sqrt{3}}\right)\hat{\varrho}_A,$$

where $\hat{\varrho}_B = \frac{\mathbb{I}}{2} \otimes \varrho_B$ and $\hat{\varrho}_A = \varrho_A \otimes \frac{\mathbb{I}}{2}$ with $\varrho_B = tr_A(\varrho_{AB})$ and $\varrho_A = tr_B(\varrho_{AB})$ which are the reduced states at Bob's and Alice's sides, respectively.

Results

Through simple calculations, the matrix $\varrho_{B\to A}$ of an X-state can be expressed as

$$\varrho^X_{B\to A} = \begin{pmatrix} \frac{\sqrt{3}}{3}\varrho_{1,1} + a & 0 & 0 & \frac{\sqrt{3}}{3}\varrho_{1,4} \\ 0 & \frac{\sqrt{3}}{3}\varrho_{2,2} + a & \frac{\sqrt{3}}{3}\varrho_{2,3} & 0 \\ 0 & \frac{\sqrt{3}}{3}\varrho_{3,2} & \frac{\sqrt{3}}{3}\varrho_{3,3} + b & 0 \\ \frac{\sqrt{3}}{3}\varrho_{4,1} & 0 & 0 & \frac{\sqrt{3}}{3}\varrho_{4,4} + b \end{pmatrix}$$

where $a = \frac{3-\sqrt{3}}{6}(\varrho_{1,1} + \varrho_{2,2})$ and $b = \frac{3-\sqrt{3}}{6}(\varrho_{3,3} + \varrho_{4,4})$.

The above state is entangled if it satisfies one of the following inequalities: $|\varrho_{1,4}|^2 > f_a - f_b, \ |\varrho_{2,3}|^2 > f_c - f_b$

Using a similar method, we see that the steering from Alice to Bob would be verified by one of the following inequalities:

$$|\varrho_{1,4}|^2 > f_a + f_b, \qquad |\varrho_{2,3}|^2 > f_c + f_b.$$

Results

Based on the above inequalities, the steerability of Bob to Alice $S_{B\to A}$ is equal to

$$S_{B\to A} = \max\left\{0,\ \frac{8}{\sqrt{3}}\left[|\varrho_{1,4}|^2 - f_a + f_b,\ |\varrho_{2,3}|^2 - f_c + f_b\right]\right\}$$

By exchanging *A* and *B*, one can obtain the steerability of Alice to Bob $S_{A\to B}$ as follows

$$S_{A\to B} = \max\left\{0,\ \frac{8}{\sqrt{3}}\left[|\varrho_{1,4}|^2 - f_a - f_b,\ |\varrho_{2,3}|^2 - f_c - f_b\right]\right\}$$

The steering asymmetry is defined as the absolute difference between the steering measures from Alice to Bob and from Bob to Alice, namely

$$\Delta_{12} = |S_{A\to B} - S_{B\to A}|$$

Results

$$\varrho_T = \begin{pmatrix} \varrho_{1,1} & 0 & 0 & \varrho_{1,4} \\ 0 & \varrho_{2,2} & \varrho_{2,3} & 0 \\ 0 & \varrho_{3,2} & \varrho_{3,3} & 0 \\ \varrho_{4,1} & 0 & 0 & \varrho_{4,4} \end{pmatrix}$$

$$S_{A\to B} = S_{B\to A} = \max\left\{0,\ \frac{8}{\sqrt{3}}[|\varrho_{1,4}|^2 - f_a,\ |\varrho_{2,3}|^2 - f_c]\right\}$$

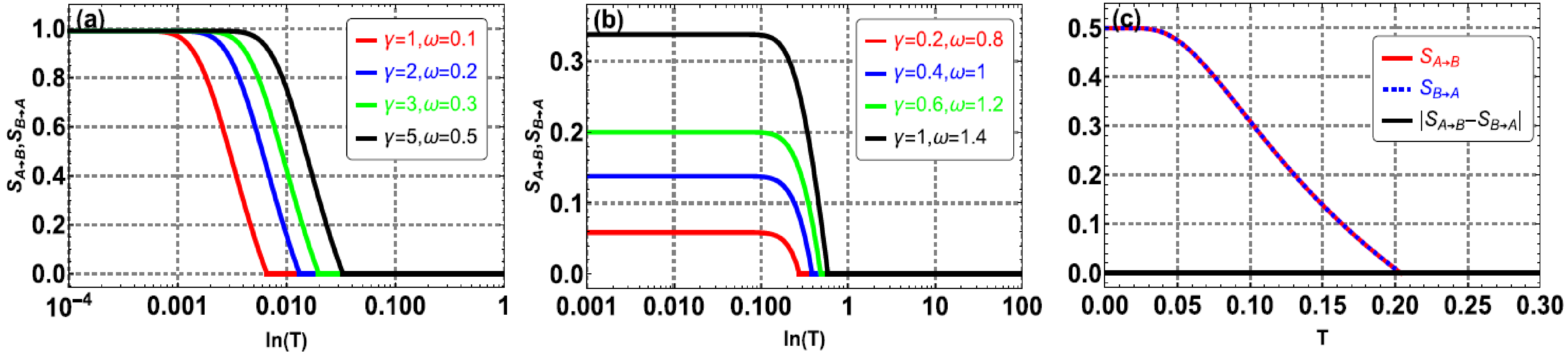


Fig. 2. Quantum steering (30) as a function of temperature in the logarithmic scale for different values of γ and ω such that (**a**) $\gamma > \omega$ and (**b**) $\gamma < \omega$. (**c**) Comparison of the quantum steearbilities $S_{A\to B}$ and $S_{B\to A}$ for $\gamma = \omega = 0.5$.

Results

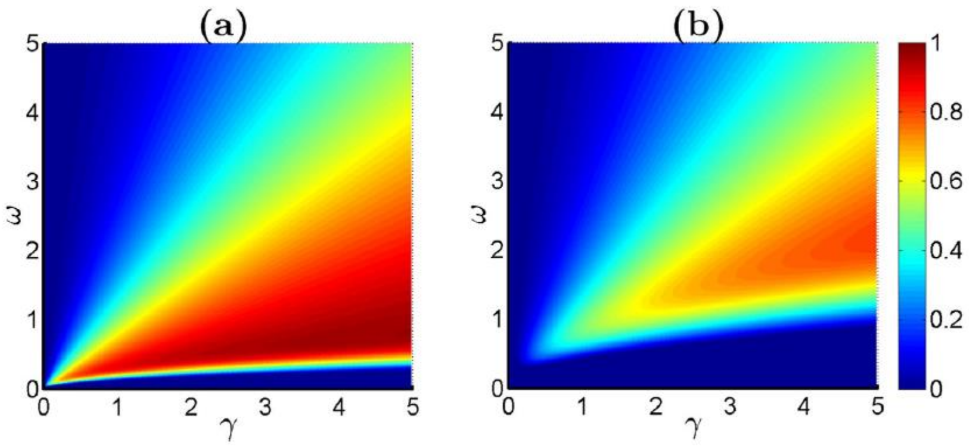


Fig. 3. Quantum steering in two gravcats versus ω and γ with (a) $T = 0.01$ and (b) $T = 0.1$.

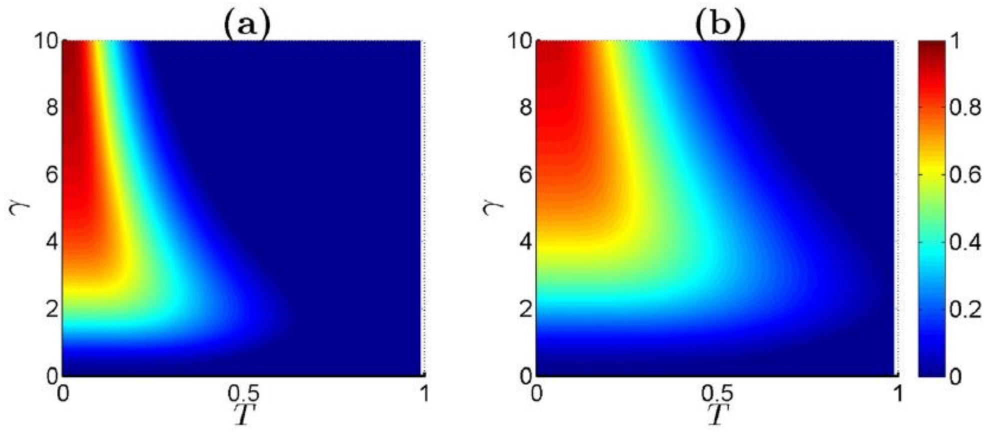


Fig. 4. Quantum steering in two gravcats versus γ and T with (a) $\omega = 2$ and (b) $\omega = 3$.

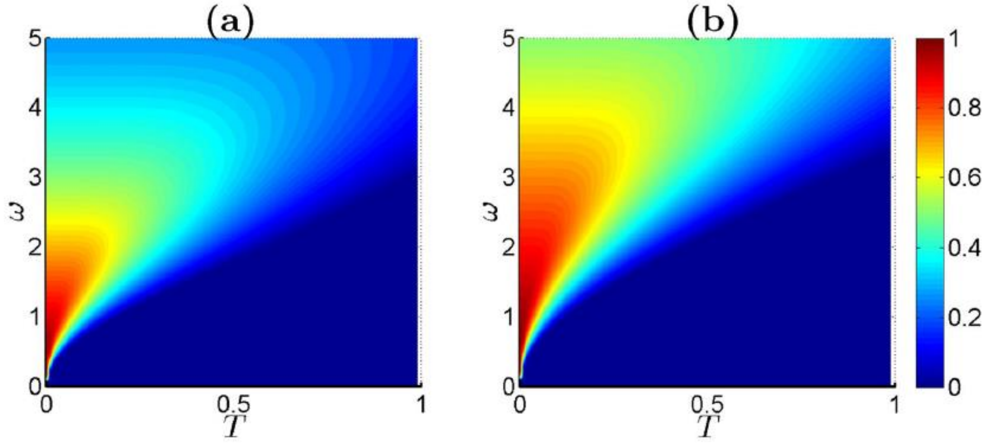


Fig. 5. Quantum steering in two gravcats versus ω and T with (a) $\gamma = 3$ and (b) $\gamma = 5$.

Results

Quantum entanglement W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).

As we all know, the quantum entanglement of the bipartite states can be effectively identified by some entanglement measures such as concurrence, negativity, etc. Here, let us use concurrence to determine the entanglement of the arbitrary state ϱ_{AB}

$$C(\varrho_{AB}) = \max\left\{\sqrt{\xi_1} - \sqrt{\xi_2} - \sqrt{\xi_3} - \sqrt{\xi_4}, 0\right\},$$

in which ξ_i(a non-negative real number) represents the eigenvalues, in decreasing order, of the matrix ϱ, given by

$$\varrho = \varrho_{AB}(\sigma_y \otimes \sigma_y)\varrho^*_{AB}(\sigma_y \otimes \sigma_y),$$

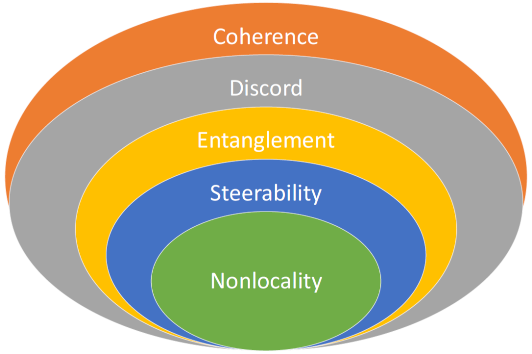
where ϱ^*_{AB} denotes the complex conjugation of ϱ_{AB} and σ_y is the y-component of Pauli matrices. Nevertheless, for our purposes in this work, the concurrence of an X-state can be formulated by

$$C(\varrho_X) = 2\max\left\{|\varrho_{2,3}| - \sqrt{\varrho_{1,1}\varrho_{4,4}}, |\varrho_{1,4}| - \sqrt{\varrho_{2,2}\varrho_{3,3}}, 0\right\}.$$

Geometric quantum discord →

$$Q_G(\varrho_{AB}) = \min_{\varrho_c \in \Omega} ||\varrho_{AB} - \varrho_c||_1$$

Results



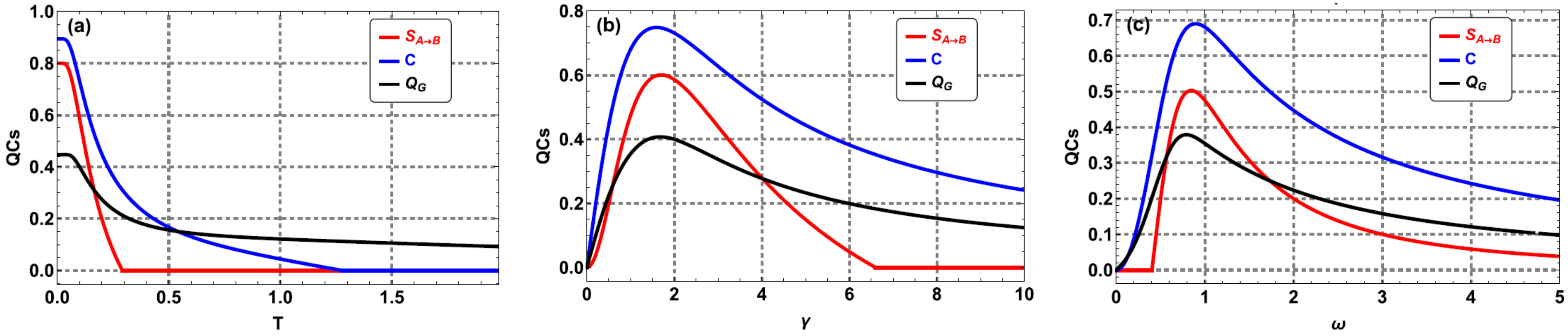


Fig. 8. Quantum steering $S_{A\to B}$, concurrence C and GQD Q_G (**a**) as a function of temperature with $\gamma = 2$ and $\omega = 1$, (**b**) as a function of γ with $T = 0.1$ and $\omega = 1$, (**c**) as a function of ω with $T = 0.1$ and $\gamma = 1$.

Results

Gravcats in correlated dephasing channel

we focus on a two-qubit system initially prepared in state ρ(0). Let's imagine these two qubits sequentially pass through a channel. If the channel operates independently and uniformly on each qubit, the resulting state of the system can be represented by the map

$$\varrho(t) = \mathcal{E}[\varrho(0)] = \sum_{i,j=0}^{3} E_{i,j}\,\varrho(0)E_{i,j}^{\dagger}$$

The Kraus operators are defined by $E_{i,j} = \sqrt{p_i p_j}\sigma_i \otimes \sigma_j$, where $\sigma_0 = \mathbb{I}_2$ and $\sigma_{1,2,3}$ represent respectively the identity and the three Pauli operators. The probability distribution p_i satisfies $p_i \geq 0$ for all i and $\sum_i p_i = 1$. It is clear that $E_{i,j}$ satisfy the completely positive and trace-preserving condition, i.e. $\sum_{i,j} E_{i,j}^{\dagger} E_{i,j} = \mathbb{I}_4$.

M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (publisher Cambridge University Press, 2000).

Results

$$\varrho(t) = \mathcal{E}[\varrho(0)] = \sum_{i,j=0}^{3} E_{i,j}\varrho(0)E_{i,j}^{\dagger}$$

The described map assumes that the quantum channel is memoryless. In other words, when qubit A passes through this channel, the effect of the channel on A does not influence the way the channel acts on the subsequent qubit, B. However, in practice, this independence is not always true.

A quantum channel can have partial memory, meaning it can partially retain information about how it acted on previous qubits.

Consequently, there are classical correlations between the channel's actions on successive qubits.

In this model, the Kraus operators, which describe the channel, are modified to account for these correlations. The new Kraus operators are: $E_{i,j} = \sqrt{p_{i,j}}\sigma_i \otimes \sigma_j$ with a joint probability as

$$p_{i,j} = (1-\mu)p_i p_j + \mu p_i \delta_{i,j},$$

where $\delta_{i,j}$ denotes the Kronecker delta, and $\mu \in [0,1]$ represents the degree of classical correlations between the successive actions of quantum channel on the two qubits.

C. Macchiavello and G. M.Palma, Entanglement-enhanced information transmission over a quantum channel with correlated noise. Phys. Rev. A 65, 050301 (2002).

Results

$$p_{i,j} = (1-\mu)p_i p_j + \mu p_i \delta_{i,j},$$

To be explicit, we focus on the dephasing channel with the probability distribution: $p_0=1-p$, $p_1=p_2=0$, and $p_3=p$.

$$p = \frac{1}{2}\left[1 - \Phi(t)\right].$$

In the non-Markovian regime (i.e. $\tau > 1/4$), we have

$$\Phi(t) = e^{-t/2\tau}\left[\cos\frac{ut}{2\tau} + \frac{1}{u}\sin\frac{ut}{2\tau}\right],$$

with $u = \sqrt{|1-16\tau^2|}$. For the Markovian regime ($\tau < 1/4$), we follow

$$\Phi(t) = e^{-t/2\tau}\left[\cosh\frac{ut}{2\tau} + \frac{1}{u}\sinh\frac{ut}{2\tau}\right].$$

Results

Output state

One can now derive the thermal time-dependent state for two qubits (gravcats) passing through a correlated dephasing channel. By considering the input gravcat state $\varrho_T = \varrho(0)$ expressed in (6), and using the above equations, the following output state can be obtained as

$$\varrho_T(t) = \begin{pmatrix} \varrho_{1,1} & 0 & 0 & \eta\varrho_{1,4} \\ 0 & \varrho_{2,2} & \eta\varrho_{2,3} & 0 \\ 0 & \eta\varrho_{2,3} & \varrho_{2,2} & 0 \\ \eta\varrho_{1,4} & 0 & 0 & \varrho_{4,4} \end{pmatrix},$$

where $\eta = \Phi^2(t) + \left[1 - \Phi^2(t)\right]\mu$.

Results

$$\varrho_T(t) = \begin{pmatrix} \varrho_{1,1} & 0 & 0 & \eta\varrho_{1,4} \\ 0 & \varrho_{2,2} & \eta\varrho_{2,3} & 0 \\ 0 & \eta\varrho_{2,3} & \varrho_{2,2} & 0 \\ \eta\varrho_{1,4} & 0 & 0 & \varrho_{4,4} \end{pmatrix}$$

$$\eta = \Phi^2(t) + \left[1 - \Phi^2(t)\right]\mu$$

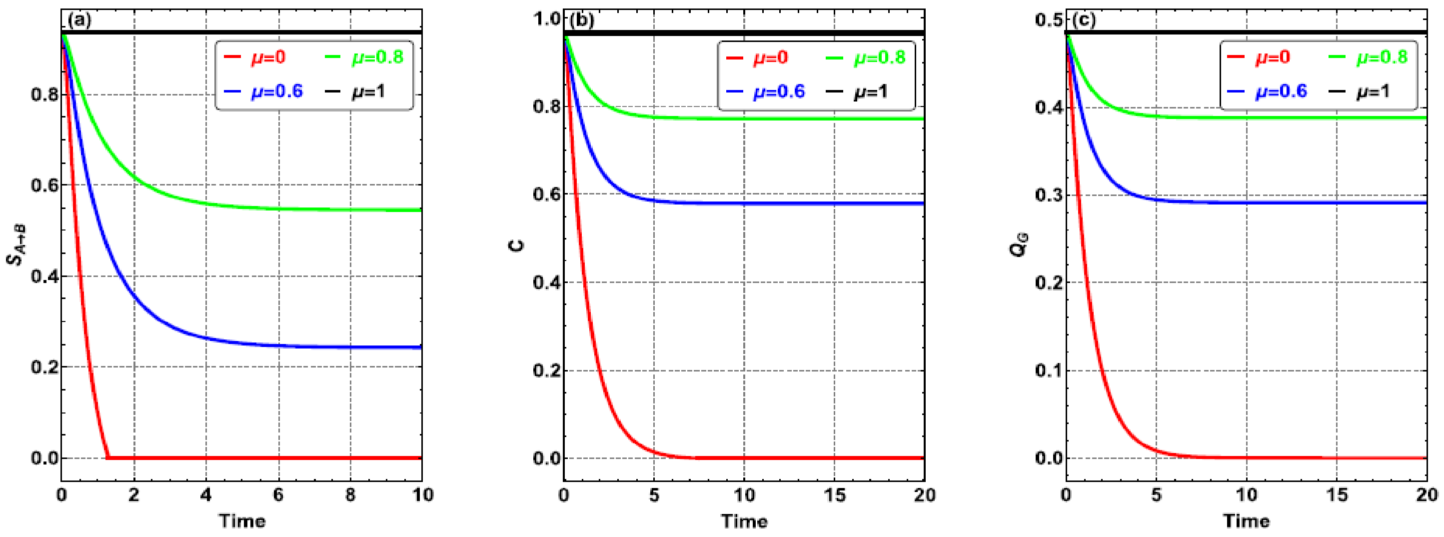


Fig. 9. Time (t) dependence of quantum steering (a), concurrence (b), and GQD (c) in the Markovian regime for different values of μ with $4\omega = \gamma = 2, T = 0.01$ and $\tau = 0.1$.

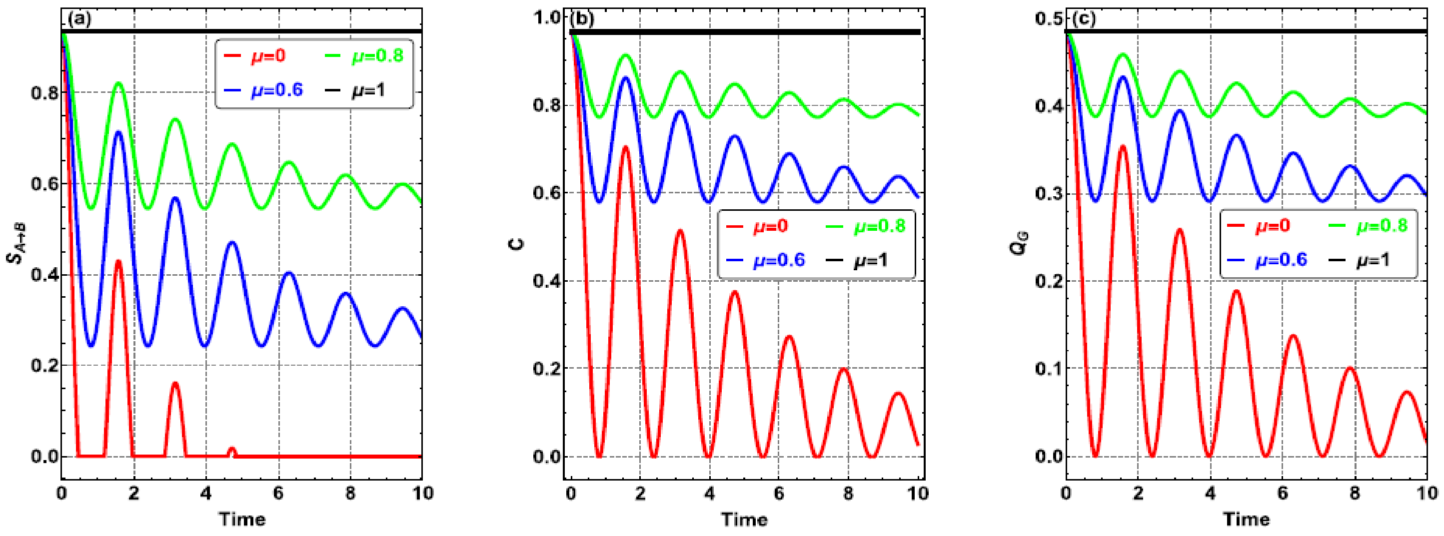
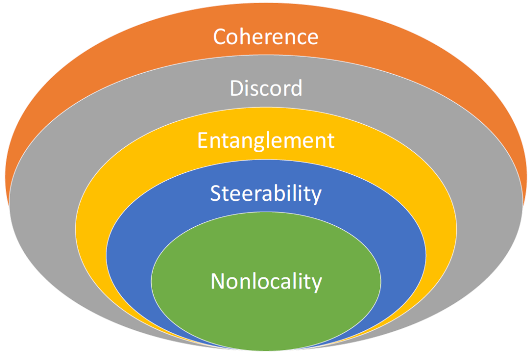


Fig. 10. Time (t) dependence of quantum steering (a), concurrence (b), and GQD (c) in the non-Markovian regime for different values of μ with $4\omega = \gamma = 2, T = 0.01$, and $\tau = 5$.

Results



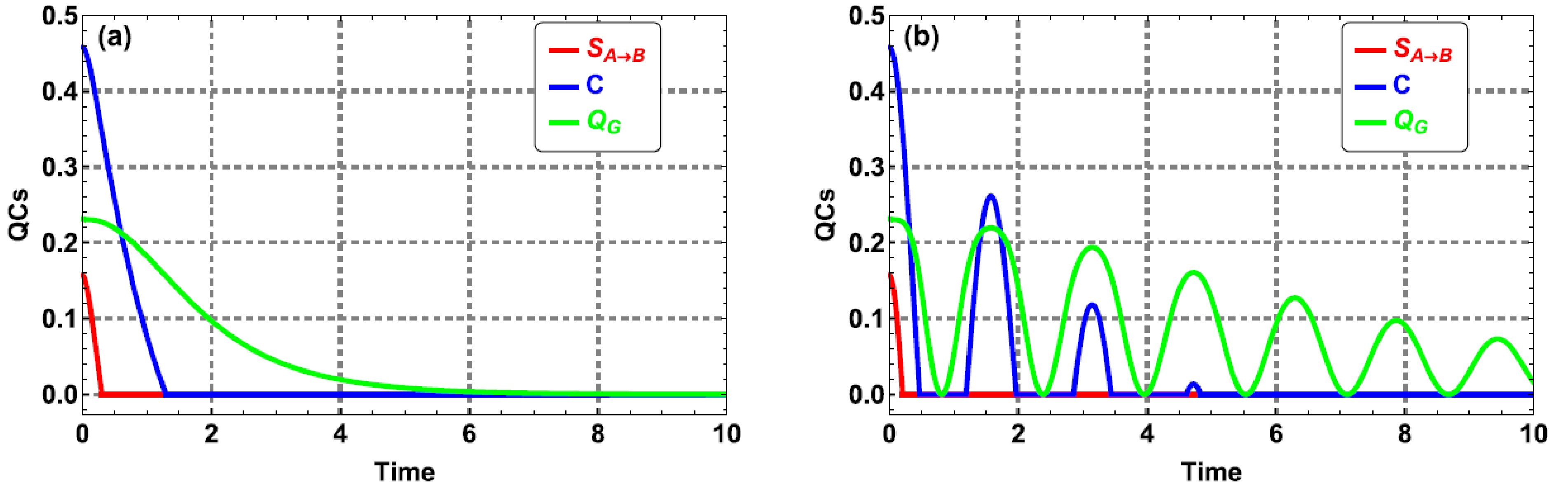


Fig. 11. Comparison of the time (t) dependence of quantum steering, concurrence, and GQD in (**a**) Markovian regime with $\tau = 0.1$ and in (**b**) non-Markovian regime with $\tau = 5$. Fixed values are: $\gamma = 2$, $\omega = 0.2$, $\mu = 0$, and $T = 0.01$.

Summary

- ✓ Reviewing some basic concepts.
- ✓ The results of this study demonstrated the complex relationships between quantum correlations and various parameters such as temperature, gravitational interaction strength, and excitation energy within the gravcat model.
- ✓ We observed that quantum steerability decreases with temperature due to thermal noise but can be enhanced through stronger gravitational interactions and correlated dephasing channels.
- ✓ Classical correlations play a crucial role in maintaining quantum resources, with higher values of classical correlations leading to increased stability in both Markovian and non-Markovian regimes.

Titles of some results

- **Quantum coherence of gravcats**
- **Bell non-locality in gravcats**
- **Weak measurement protocol**
- **Extension of the model to electrostatic notion**
- **Impact of independent channels and nonuniform fields**
- **The implication of non-Markovian master equation**

- Scientific Reports **15**, 18594 (2025).; Phys. Rev. D **111**, 064077 (2025).; Eur. Phys. J. C **84**, 670 (2024).

Thank you

Thanks for your attention.

Saeed Haddadi

haddadi@ipm.ir