Wednesday Weekly Seminar
27 Aug 2025

Running coupling for holographic QCD with heavy and light quarks: Isotropic case

Ali Hajilou

School of Particles and Accelerators (IPM)

hajilou@mi-ras.ru

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Running coupling for holographic QCD with heavy and light quarks: Isotropic case

Irina Ya. Aref'eva,[*] Ali Hajilou,[†] Pavel Slepov,[‡] and Marina Usova[§]
Steklov Mathematical Institute, Russian Academy of Sciences, Gubkina street 8, 119991, Moscow, Russia

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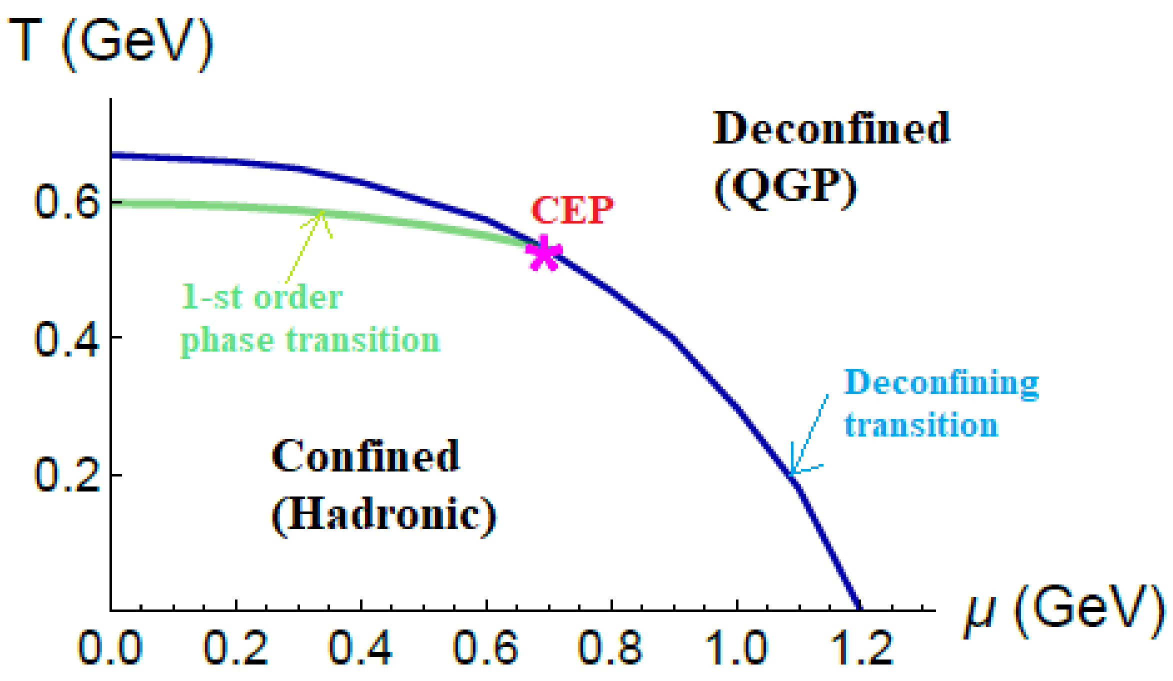
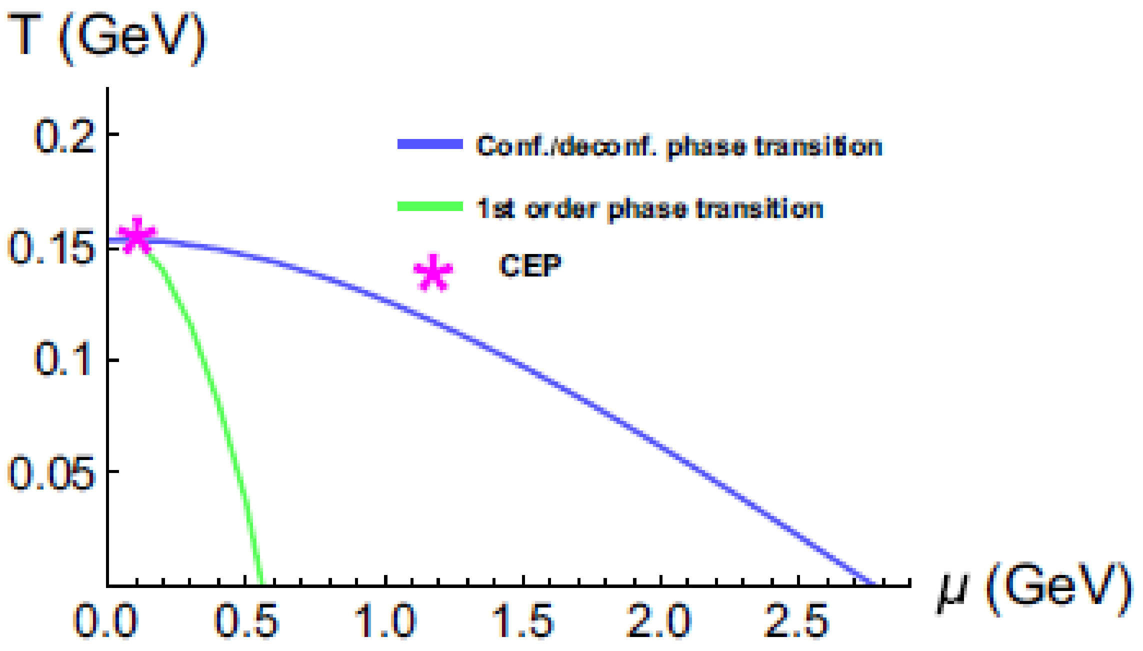
arXiv:2402.14512 **[hep-th]**

Outline:

- Introduction (Motivation!!)
- Set up a Question
- Approach: AdS/CFT or Gauge/Gravity Duality
- Results
- Summary

Introduction: *phase diagram (Isotropic case)*



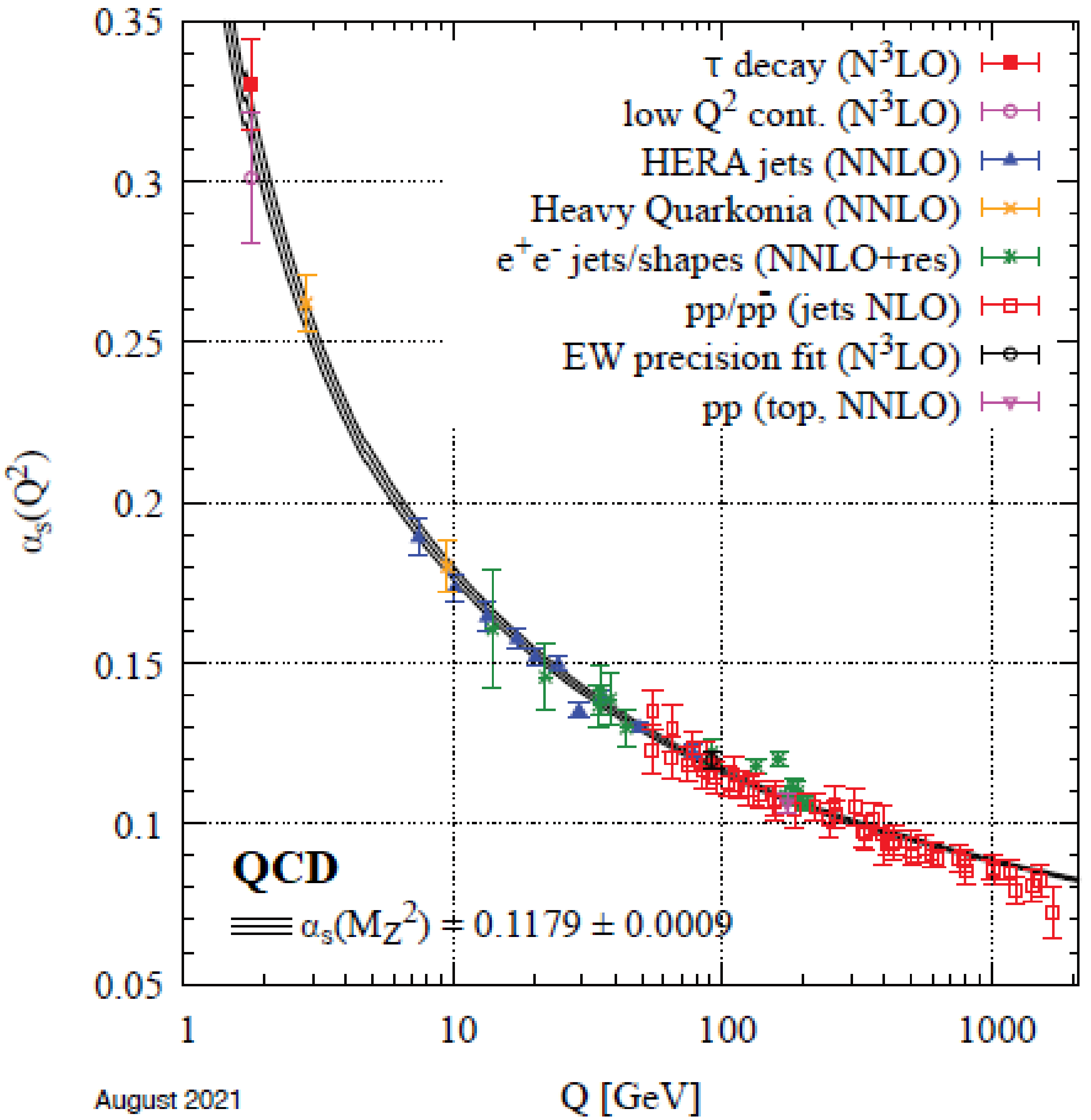


(Heavy Quarks Model)

(Light Quarks Model)

Running Coupling:

Running coupling as a function of the energy scale Q:



The respective degree of *QCD perturbation theory* used in the extraction of coupling is indicated in brackets (NLO: next-to-leading order, …)

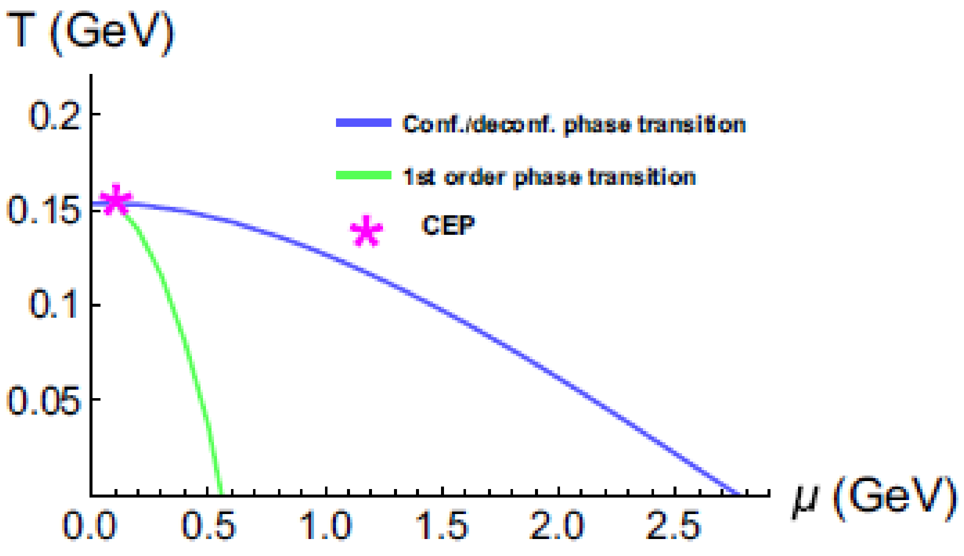
R. L. Workman et al. (Particle Data Group) *PTEP* 2022 (2022) 083C01

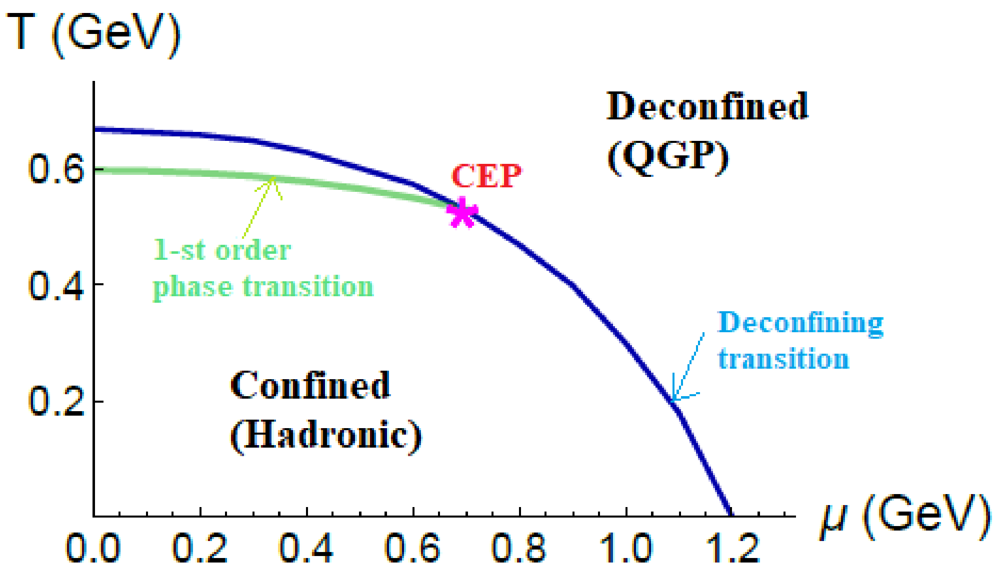
Question

Question:

What is the dependence of running coupling on temperature and chemical potential at different phases?

(For LQ and HQ)





Our Approach (bottom-up):

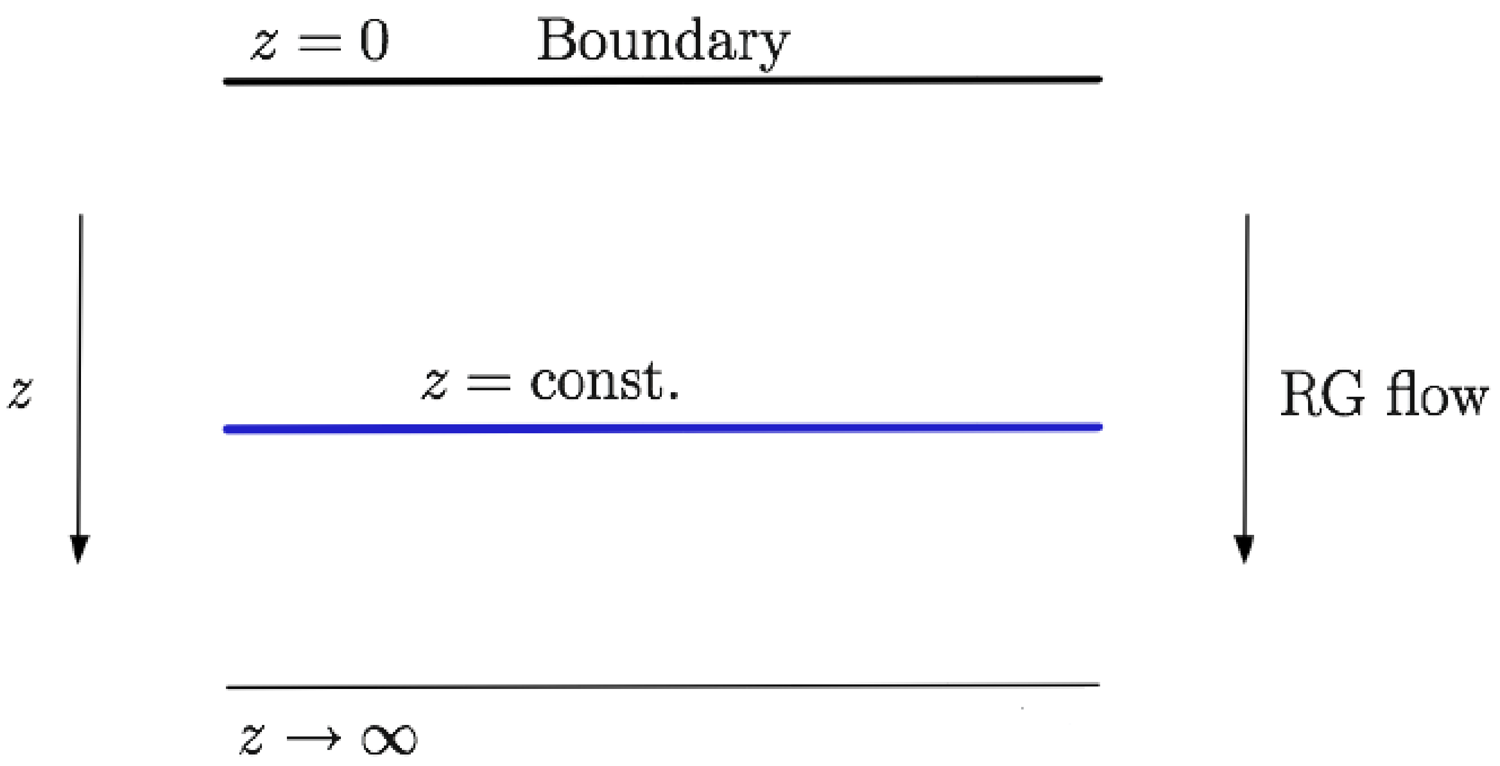
Classical gravity ⟺ Strongly coupled QFT

Anti-de Sitter Space (AdS) ⟷ Vacuum state

Black hole temperature ⟷ Temperature in QCD

Maldacena, Adv.Theor. Math.Phys. 1998; Witten, Adv.Theor.Math. Phys. 1998

Geometric picture:



J.~Casalderrey-Solana, at al, Gauge/String Duality, Hot QCD and Heavy Ion Collisions; [arXiv:1101.0618 [hep-th]]

Our Model :

$$S=\frac{1}{16\pi G_5}\int d^5x\sqrt{-\mathfrak{g}}\left[R-\frac{\mathfrak{f}_0(\varphi)}{4}F^2-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-\mathcal{V}(\varphi)\right]$$

Fields EOMs:

$$\nabla^2\varphi=\frac{\partial\mathcal{V}}{\partial\varphi}+\frac{F^2}{4}\frac{\partial\mathfrak{f}_0}{\partial\varphi}\quad,\quad\partial_\mu[\sqrt{-\mathfrak{g}}\mathfrak{f}_0(\varphi)F^{\mu\nu}]=0$$

Einstein EOMs:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{\mathfrak{f}_0(\varphi)}{2}\left(F_{\mu\rho}F_\nu^\rho-\frac{1}{4}g_{\mu\nu}F^2\right)+\frac{1}{2}\left[\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}g_{\mu\nu}(\partial\varphi)^2-g_{\mu\nu}\mathcal{V}(\varphi)\right]$$

Ansatzes for the fields:

Metric:

$$ds^2 = B^2(z)\left[-g(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{g(z)}\right]$$

$$B(z) = \frac{Le^{A(z)}}{z}$$ (Warp factor)

Gauge field: $A_\mu = (A_t(z), \vec{0}, 0)$

Dilaton field: $\varphi = \varphi(z)$

Solving EOMs: (Potential reconstruction method)

Gauge field:

$$A_t'' + \left(\frac{f_0'}{f_0} + A' - \frac{1}{z}\right) A_t' = 0$$

Dilaton field:

$$A'' - A'^2 + \frac{2}{z} A' + \frac{\varphi'^2}{6} = 0$$

Blackening function:

$$g'' + \left(3A' - \frac{3}{z}\right) g' - e^{-2A} z^2 f_0 A_t'^2 = 0$$

Dilaton potential:

$$A'' + 3A'^2 + \left(\frac{3g'}{2g} - \frac{6}{z}\right) A' - \frac{1}{z}\left(\frac{3g'}{2g} - \frac{4}{z}\right) + \frac{g''}{6g} + \frac{e^{2A} V}{3z^2 g} = 0$$

Warp factor:

$$ds^2 = B^2(z)\left[-g(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{g(z)}\right]$$

$$B(z) = \frac{e^{A(z)}}{z}$$

Has very crucial effect on the physics in the QFT side.

Light quark:

$$\mathcal{A}(z) = -a\ \ln(bz^2 + 1)$$

Li, Yang, Yuan 2015

Heavy quark:

$$\mathcal{A}(z) = -cz^2/4$$

Zakharov, Andreev, 2008

Warp factor and gauge kinetic function:

Heavy quark:

$$f_0(z) = e^{-cz^2 - A(z)}$$

$$A(z) = -\frac{c}{3}z^2 - pz^4$$

$c = 1.16\ \mathrm{GeV}^2$

$p = 0.273\ \mathrm{GeV}^4$

1- Linear Regge trajectory of J/ψ meson

2- Transition temperature at $\mu = 0$ with: lattice QCD simulation of $T_{HP} \simeq 0.6 GeV$

Warp factor and gauge kinetic function:

Light quark:

$$f_0(z) = e^{-cz^2 - A(z)}$$

$$A(z) = -a \log(bz^2 + 1)$$

$a = 4.046$

$b = 0.01613\ \mathrm{GeV}^2$

$c = 0.227\ \mathrm{GeV}^2$

1- Linear Regge trajectory of ρ meson

2- fitting the confinement-deconfinement phase transition temperature at $\mu = 0$

Back to EOMs:

Gauge field:

$$A_t'' + \left(\frac{f_0'}{f_0} + A' - \frac{1}{z}\right)A_t' = 0$$

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$$A'' + 3A'^2 + \left(\frac{3g'}{2g} - \frac{6}{z}\right)A' - \frac{1}{z}\left(\frac{3g'}{2g} - \frac{4}{z}\right) + \frac{g''}{6g} + \frac{e^{2A}V}{3z^2 g} = 0$$

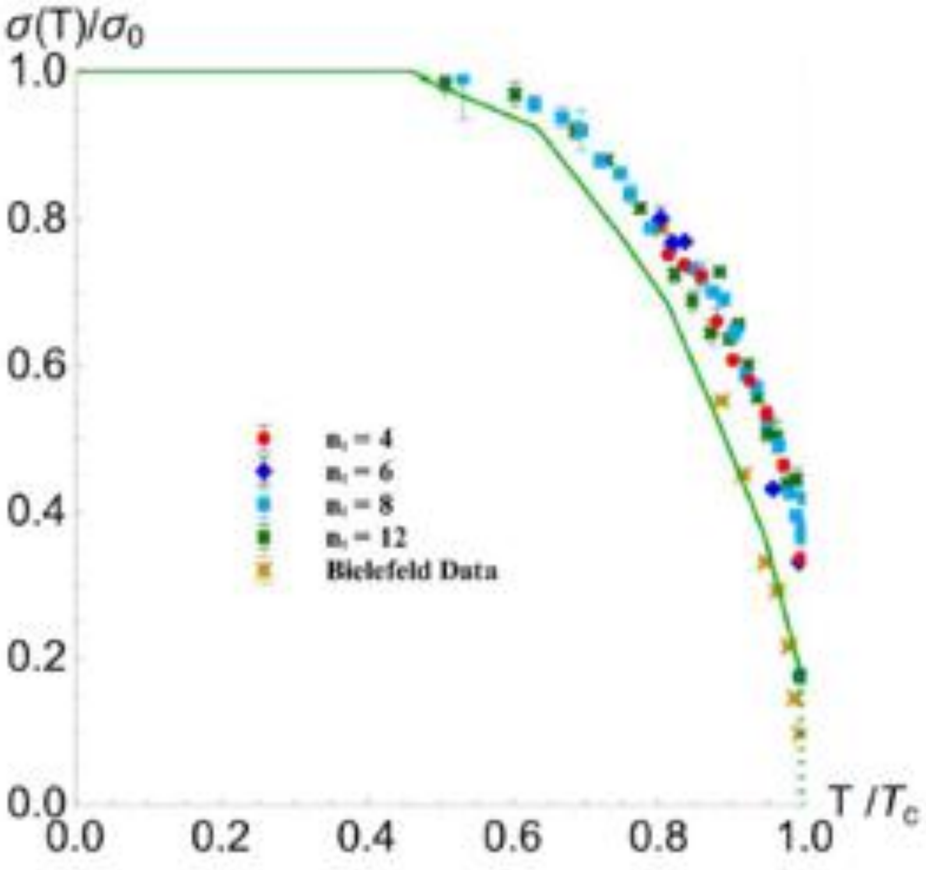
Boundary conditions:

Gauge field: $$A_t(0) = \mu, \quad A_t(z_h) = 0$$

Blackening function: $$g(0) = 1, \quad g(z_h) = 0$$

Dilaton field: $$\varphi(z, z_0)\Big|_{z=z_0} = 0 \quad \Longrightarrow \quad \begin{aligned} & z_0 = 0 \\ & z_0 = z_h \\ & z_0 = \mathfrak{z}(z_h) \end{aligned}$$

String tension: LQ Model

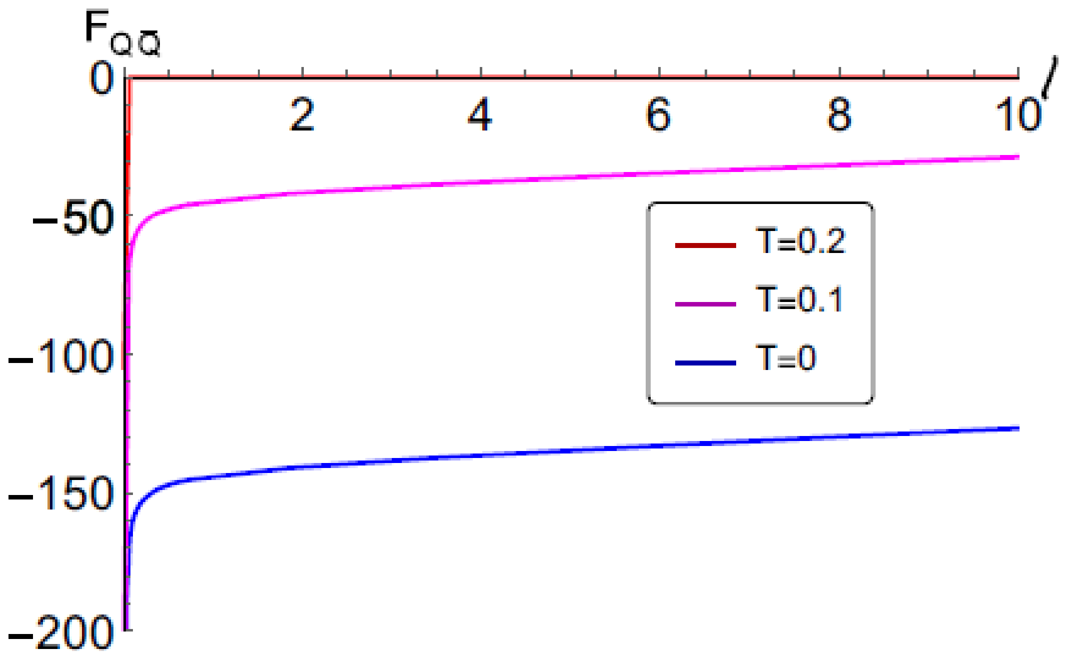


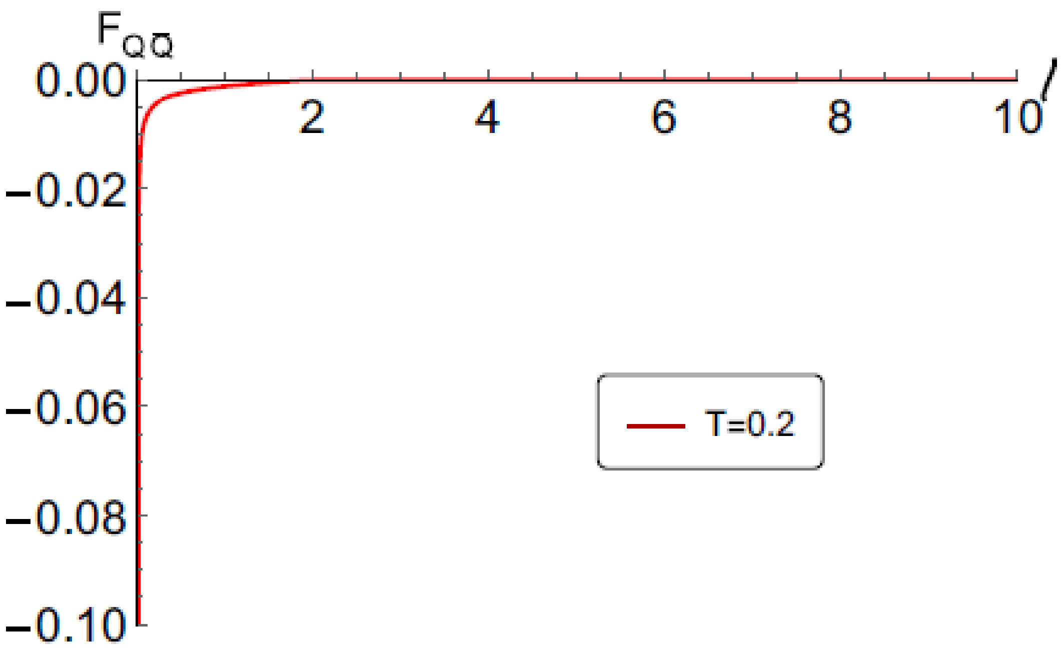
Lattice QCD computation

Cardoso and Bisudo; Phys.Rev.D 85 (2012) 077501

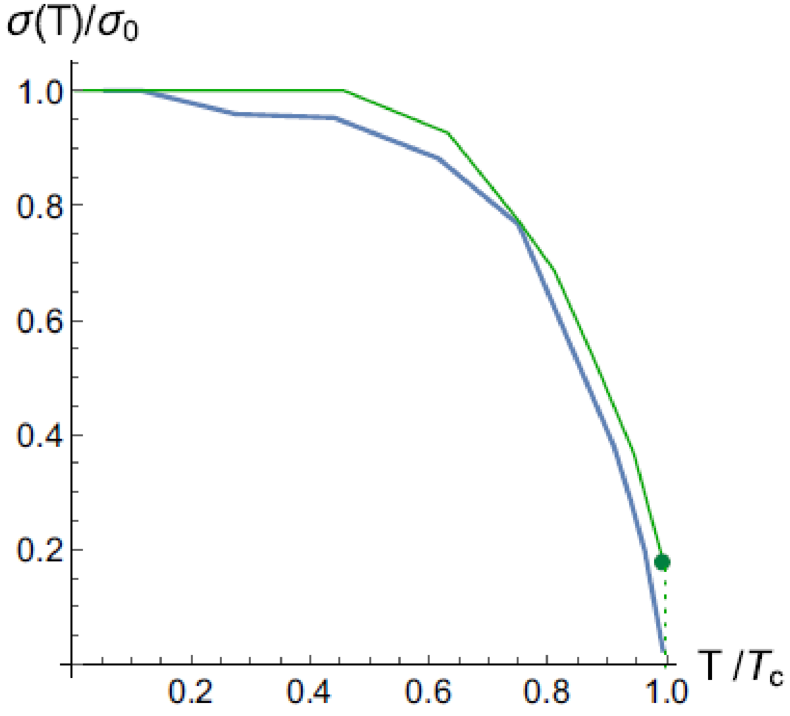
Physical boundary condition for dilaton: LQ Model

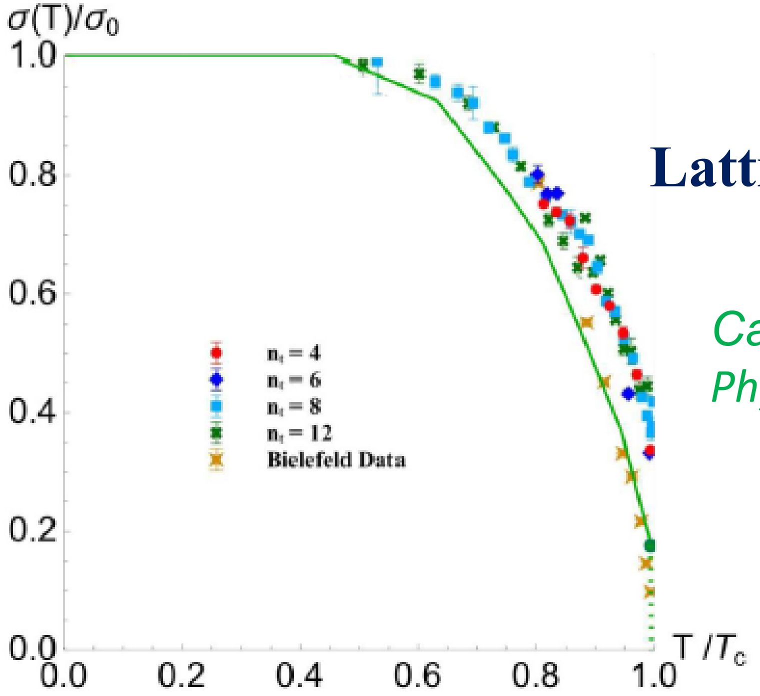
$$F_{Q\bar{Q}}(\ell) \sim C - \frac{4}{3}\frac{\alpha_{Q\bar{Q}}}{\ell} + \sigma_{Q\bar{Q}}\ell$$





Physical boundary condition for dilaton: LQ Model





Lattice QCD computation

Cardoso and Bisudo;
Phys.Rev.D 85 (2012) 077501

Physical boundary condition:

$$\varphi(z, z_0)\Big|_{z=z_0} = 0 \Longrightarrow z_0 = \mathfrak{z}_{\mathrm{LQ}}(z_h) = 10e^{(-\frac{z_h}{4})} + 0.1$$

Physical boundary condition for dilaton: HQ Model

Physical boundary condition:

$$\varphi(z, z_0)\Big|_{z=z_0} = 0 \quad \Rightarrow \quad z_0 = \mathfrak{z}_{HQ}(z_h) = e^{(-\frac{z_h}{4})} + 0.1$$

Holographic running coupling for LQ , HQ:

$$\alpha(z) = e^{\varphi(z)}$$

A.W. Peet and J. Polchinski; Phys. Rev. D 59, 065011 (1999).

U. Gursoy and E. Kiritsis; J. High Energy Phys. 02 (2008) 032.

Thermodynamics:

Temperature:

$$T = \frac{|g'|}{4\pi}\Big|_{z=z_h}$$

Entropy:

$$s = \frac{B^3(z_h)}{4}$$

Free energy:

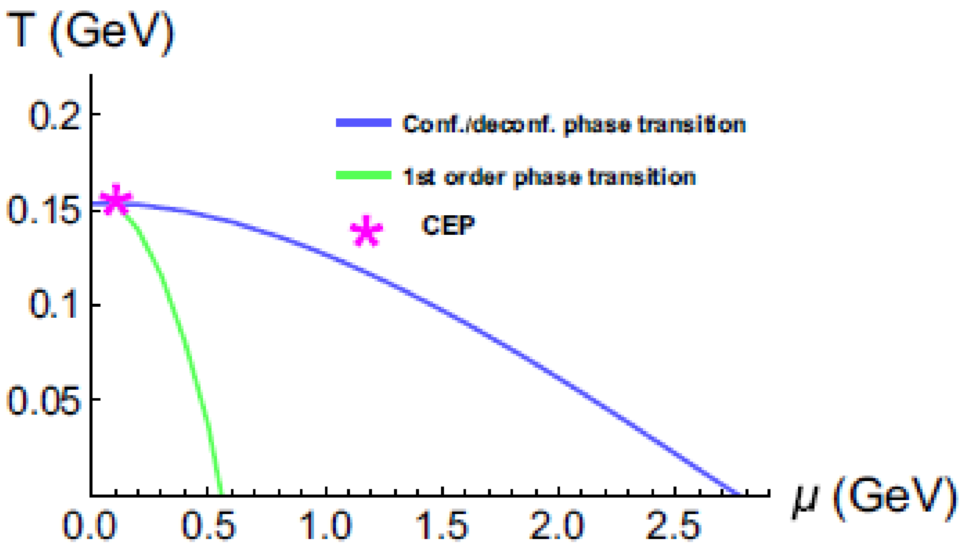
$$F = -\int s\, dT = \int_{z_h}^{\infty} s\, T' dz.$$

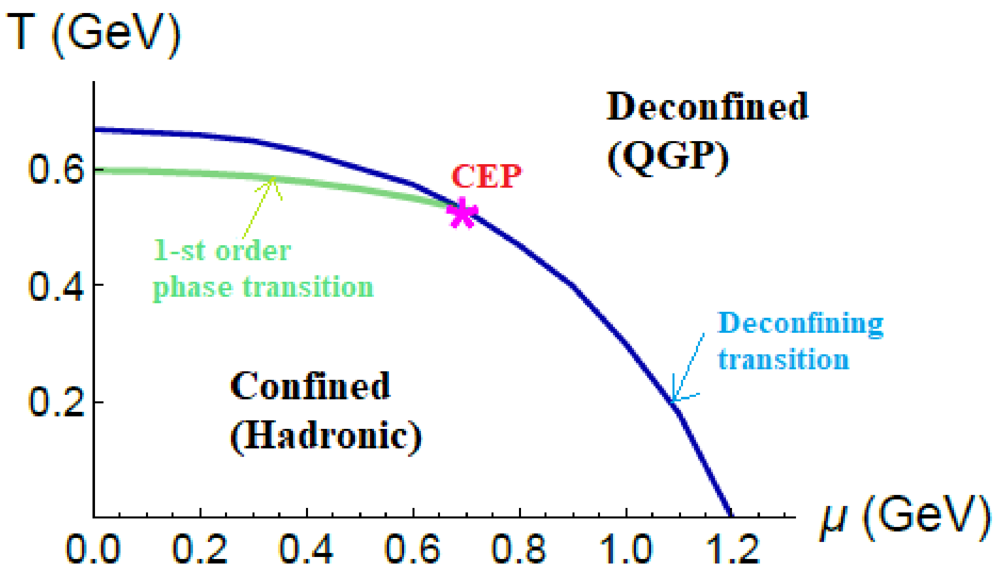
Back to Question

Question:

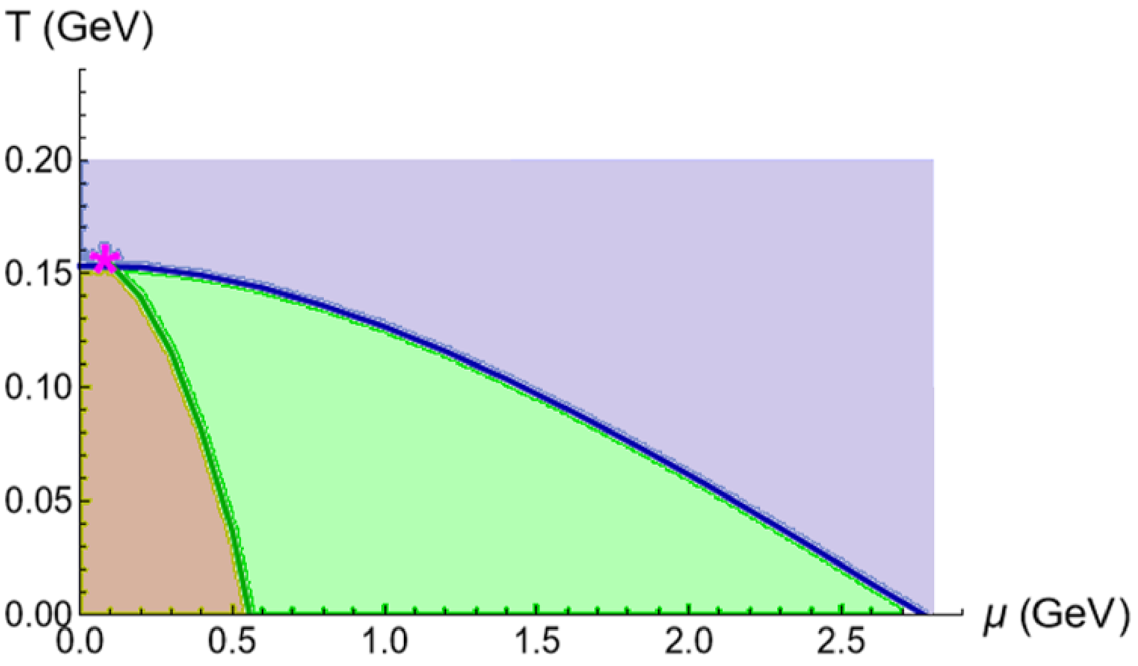
What is the dependence of running coupling on temperature and chemical potential at different phases?

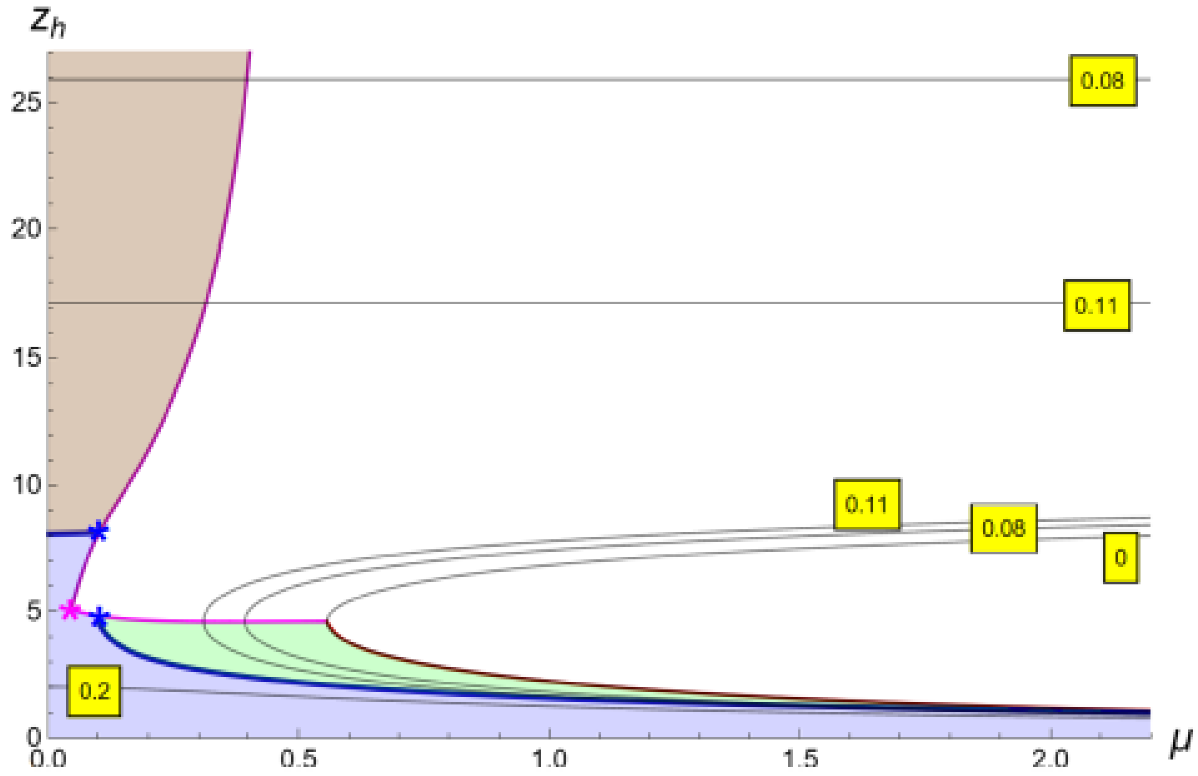
(For LQ and HQ)



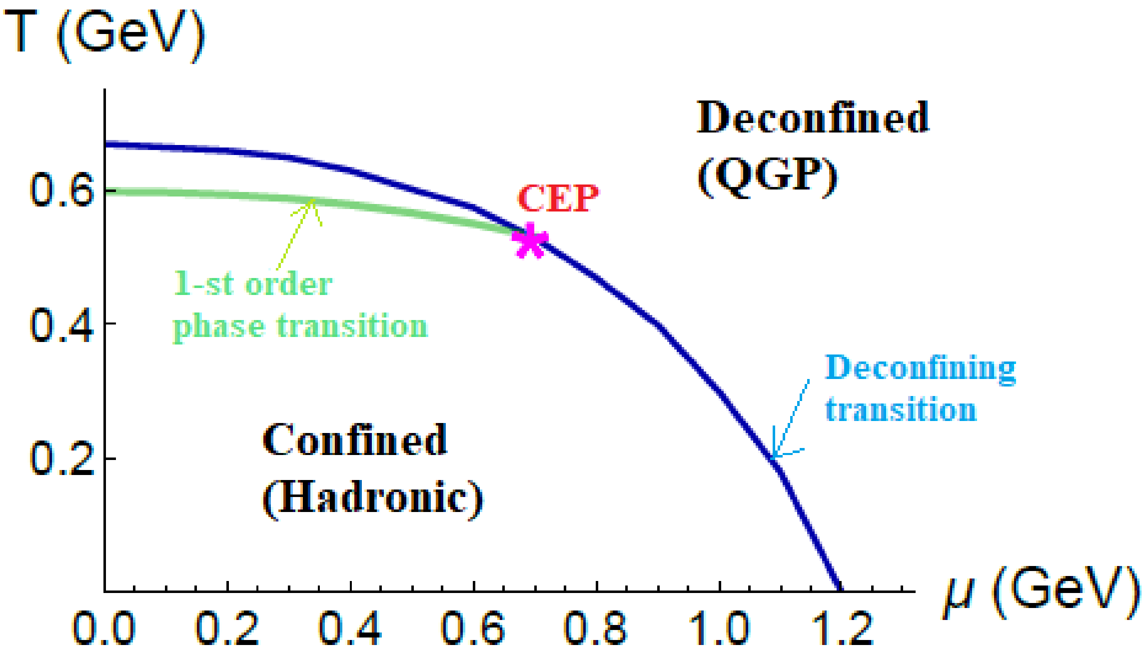


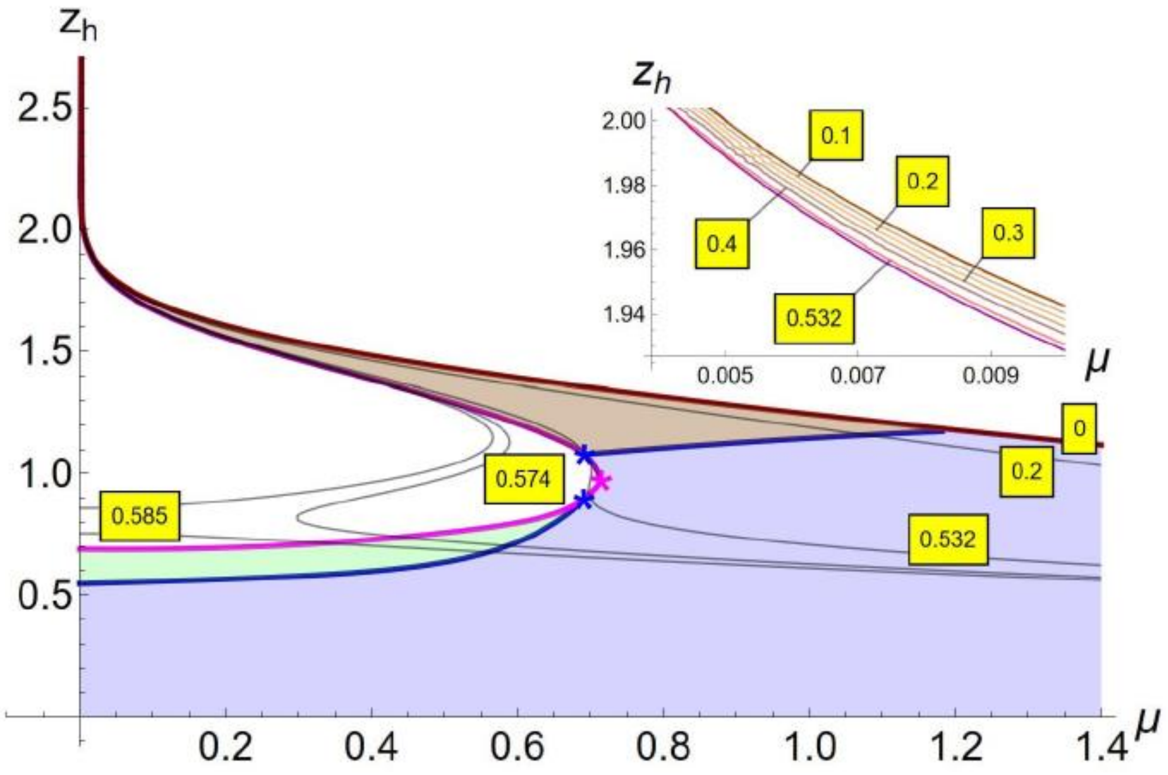
Phase diagram: (LQ Model; Isotropic case)



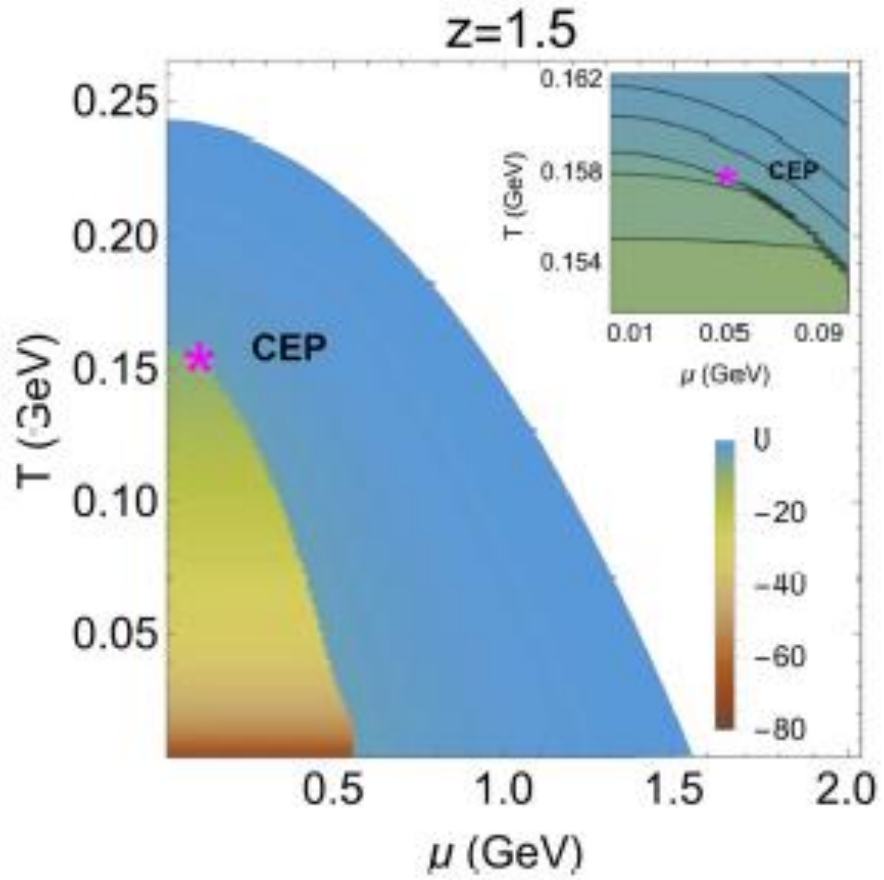


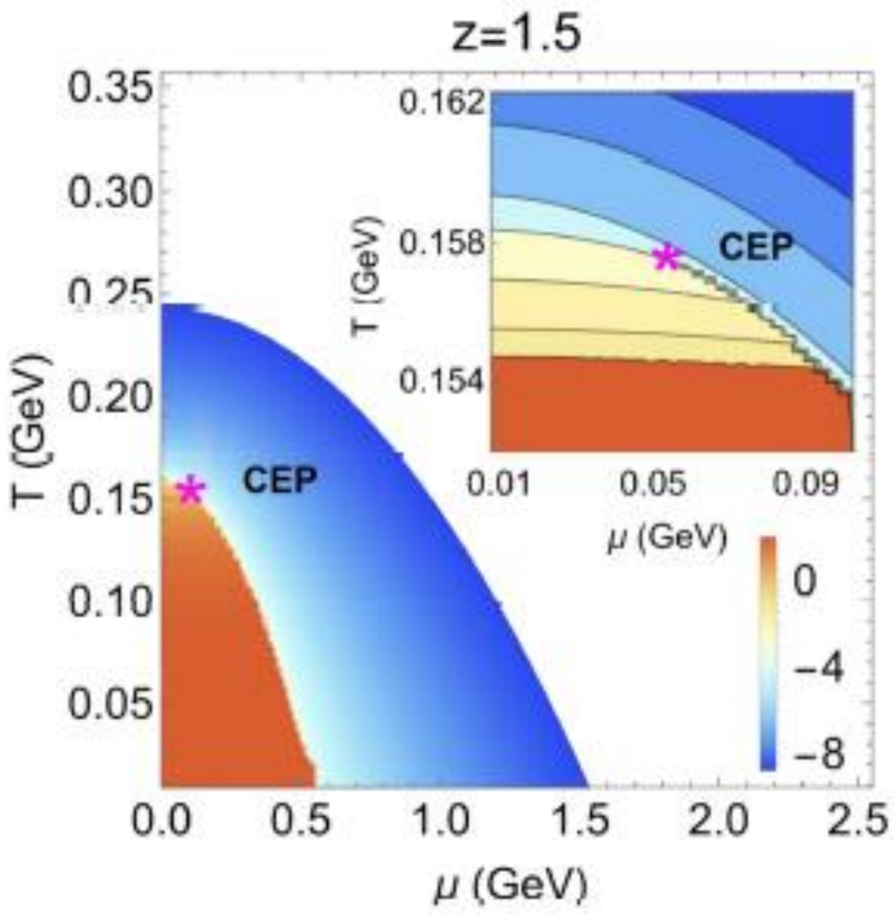
Phase diagram: (HQ Model; Isotropic case)





Logarithm of running coupling: (LQ phase diagram)

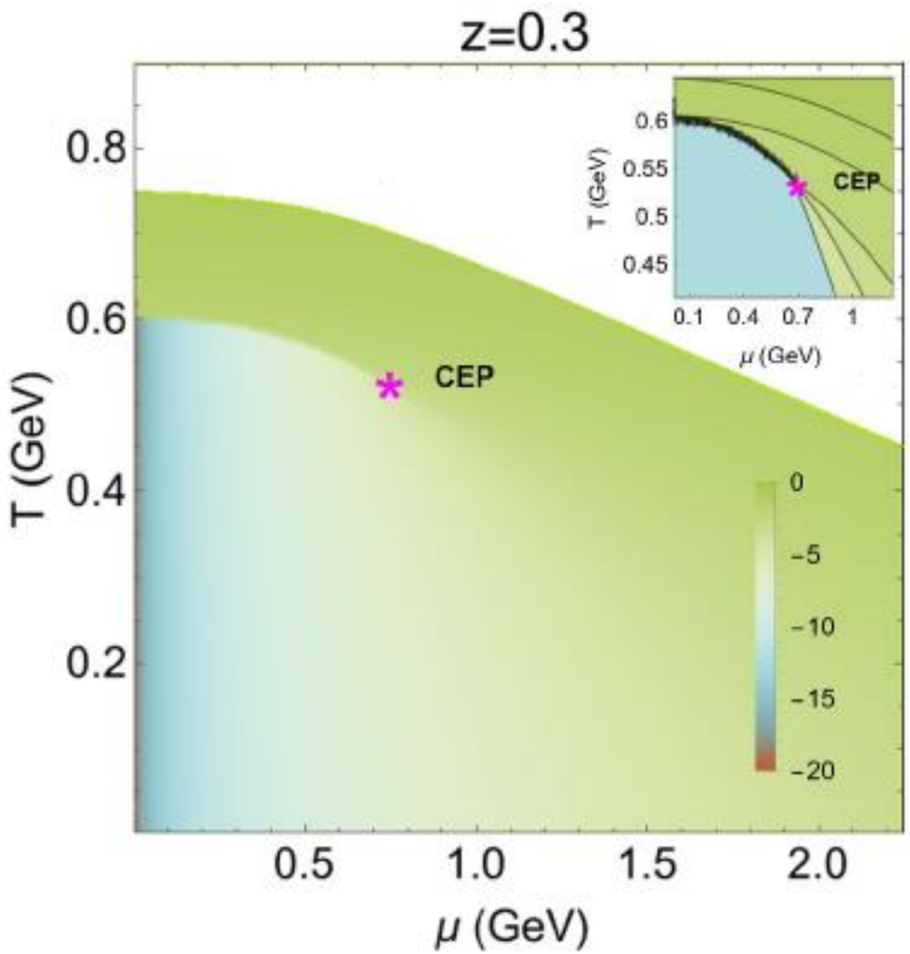


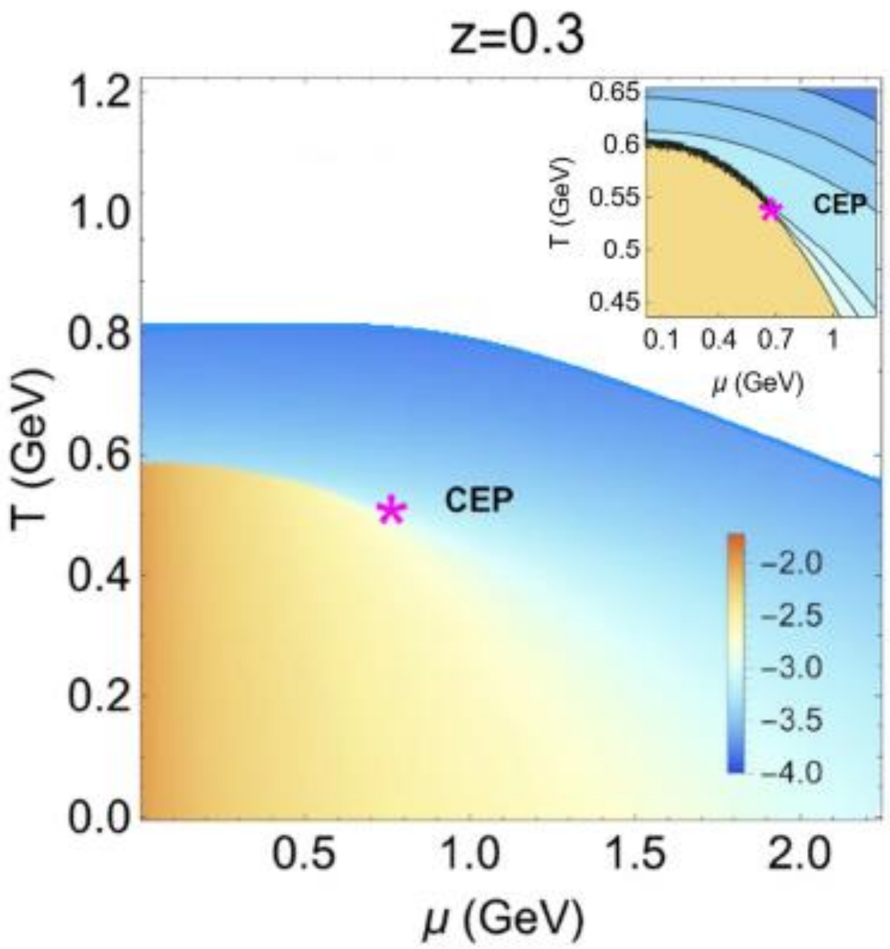


Boundary conditions: $z_0 = z_h$

$$z_0 = \mathfrak{z}_{LQ}(z_h) = 10e^{(-\frac{z_h}{4})} + 0.1$$

Logarithm of running coupling: (HQ phase diagram)

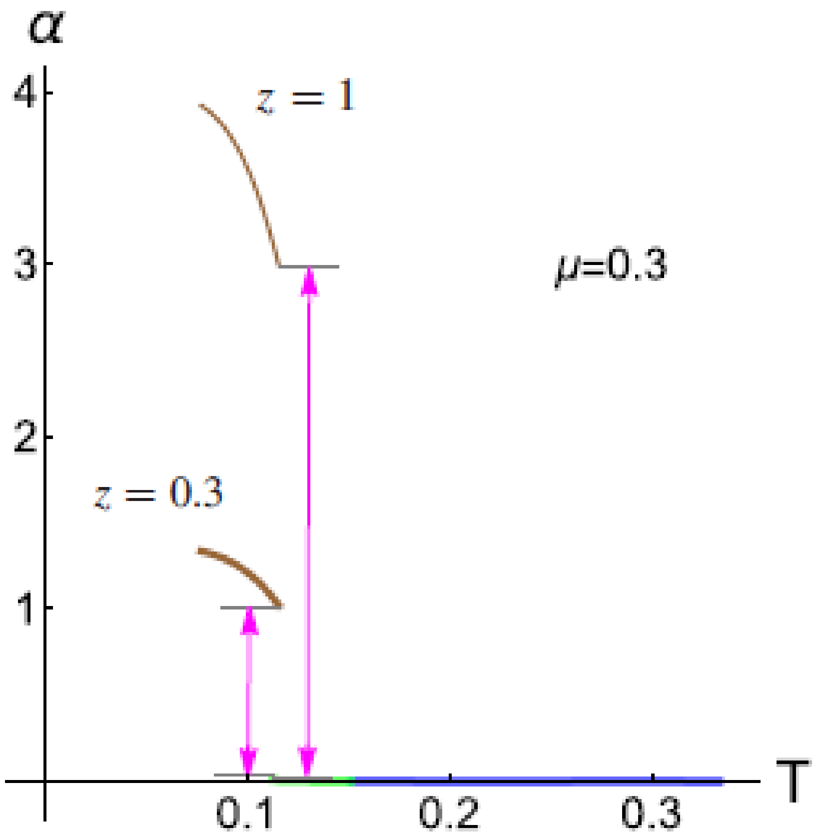


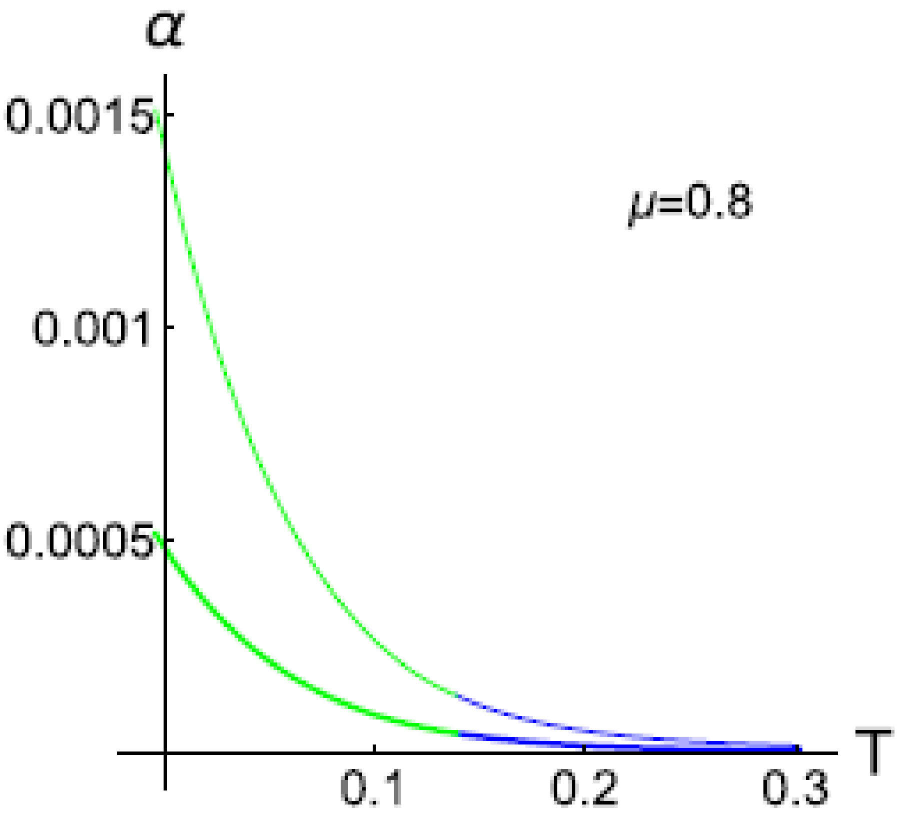


Boundary conditions: $z_0 = z_h$ $\qquad$ $z_0 = \mathfrak{z}_{HQ}(z_h) = e^{(-\frac{z_h}{4})} + 0.1$

Running coupling vs T: (LQ Model)

Running coupling senses the jump at 1-st order phase transition line.





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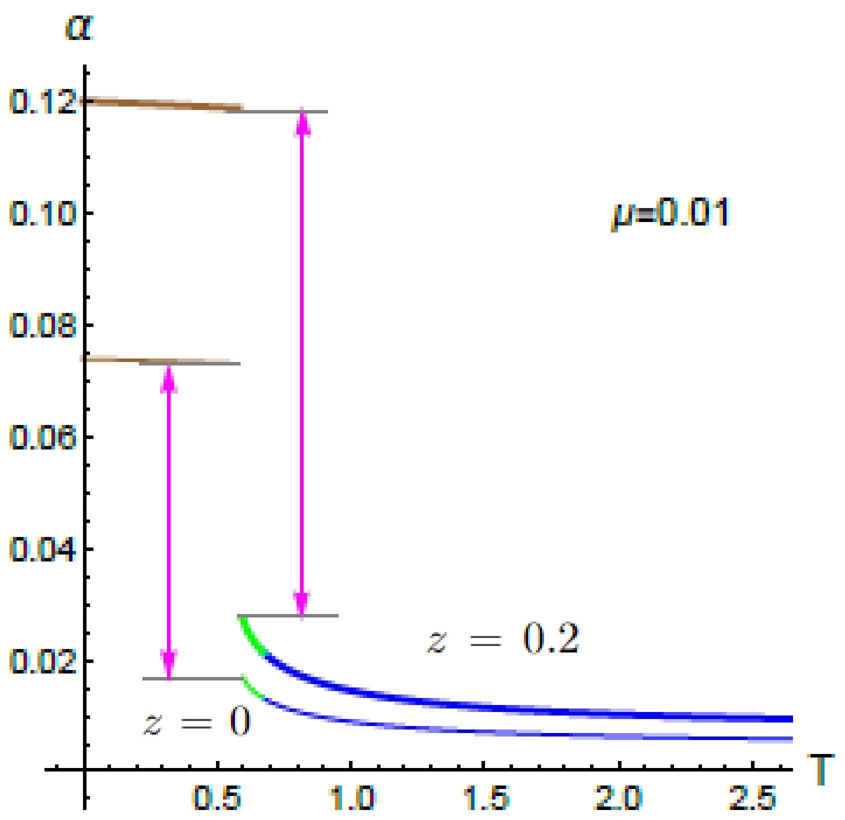
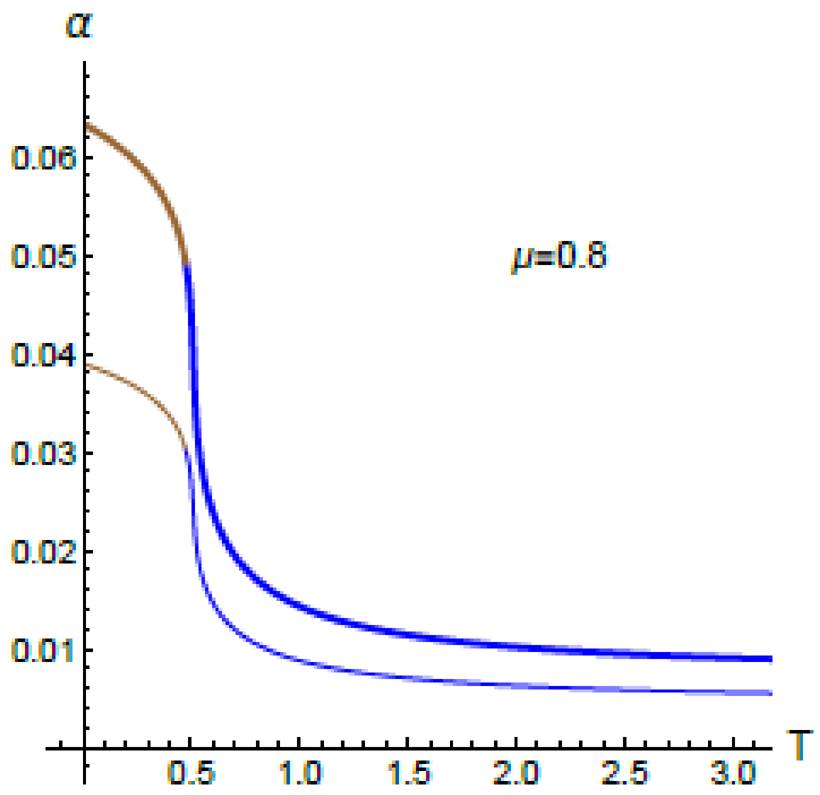
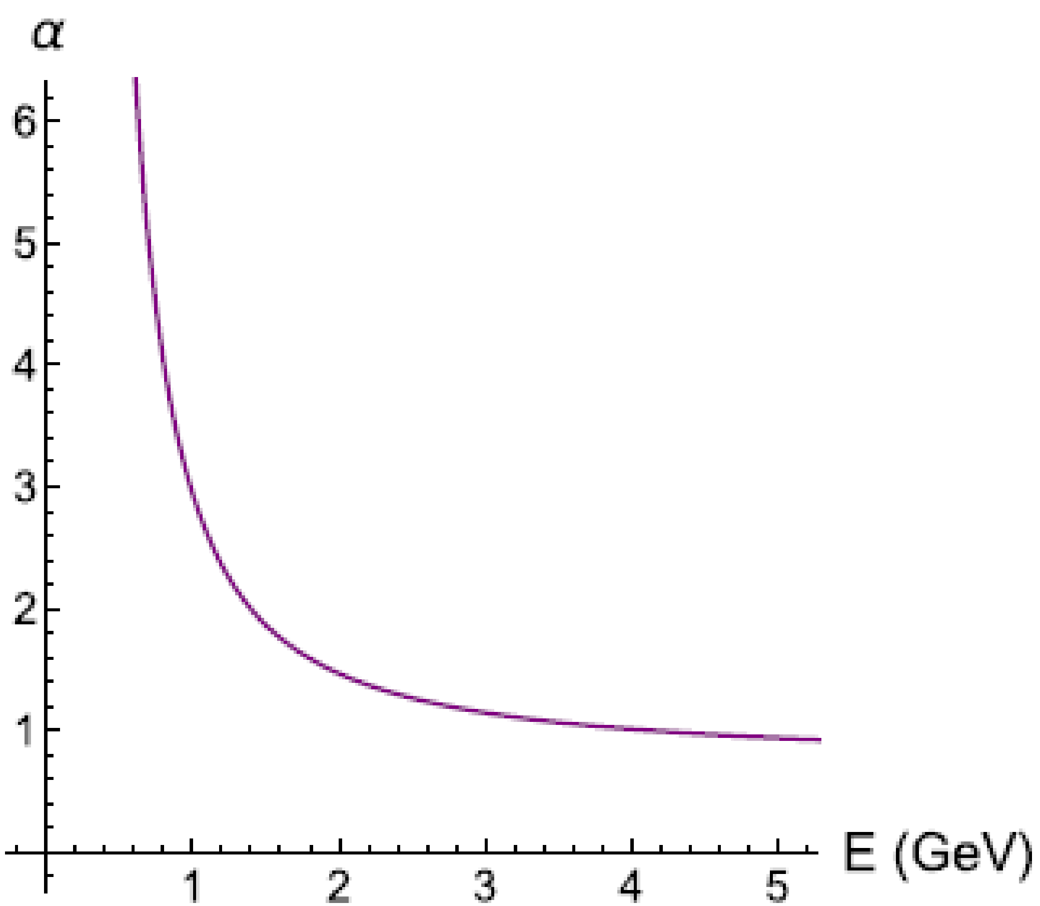
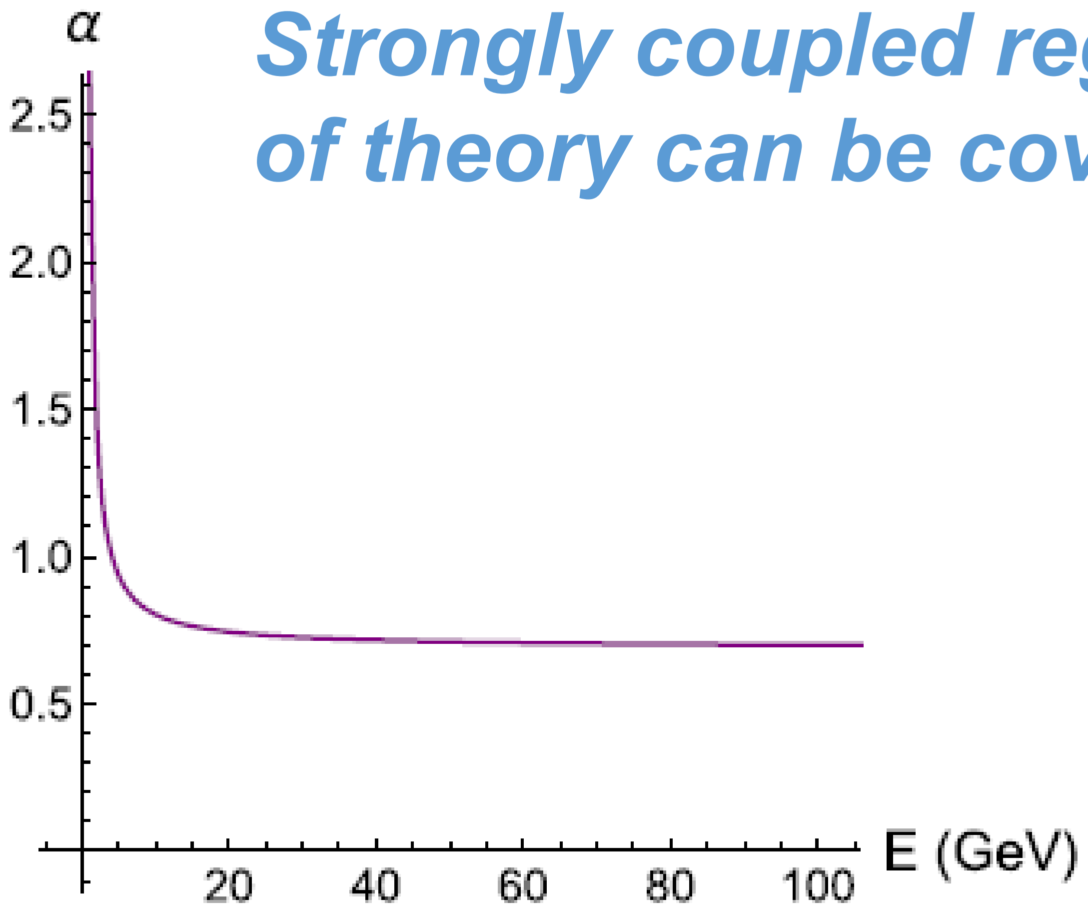




Running coupling vs E in QFT: (LQ Model)

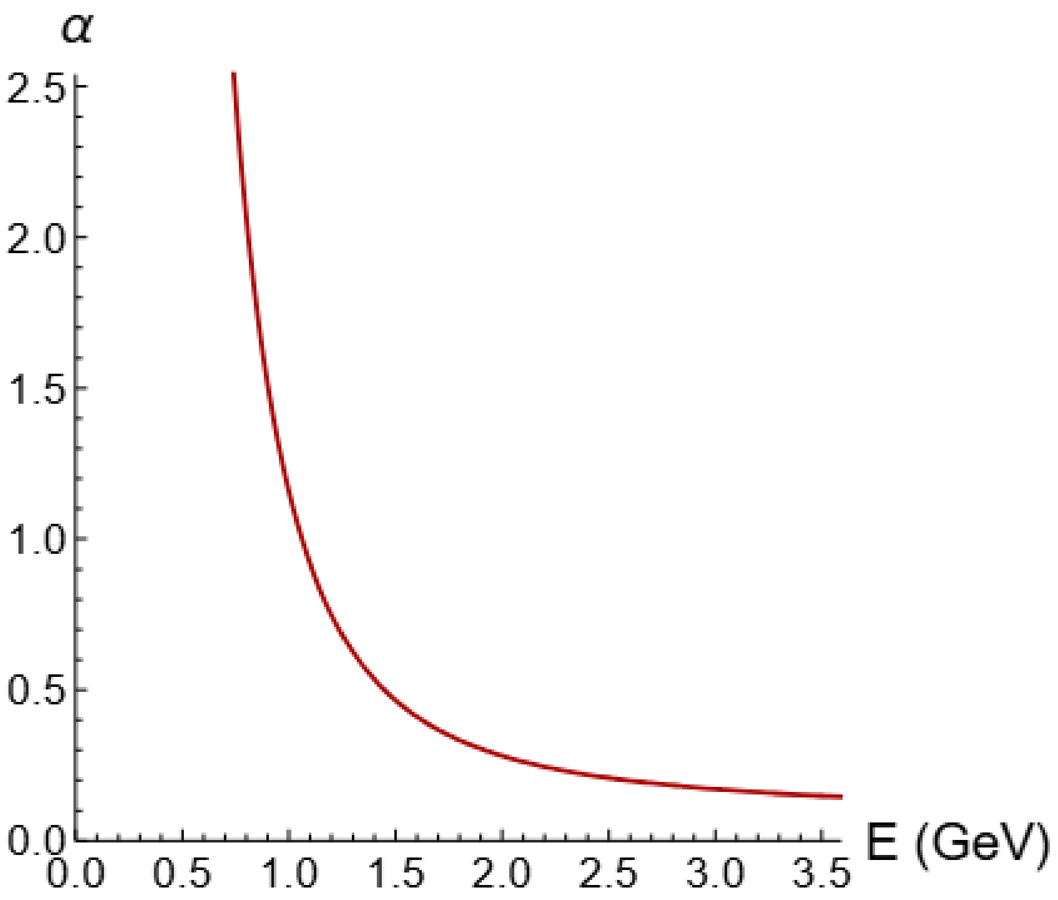


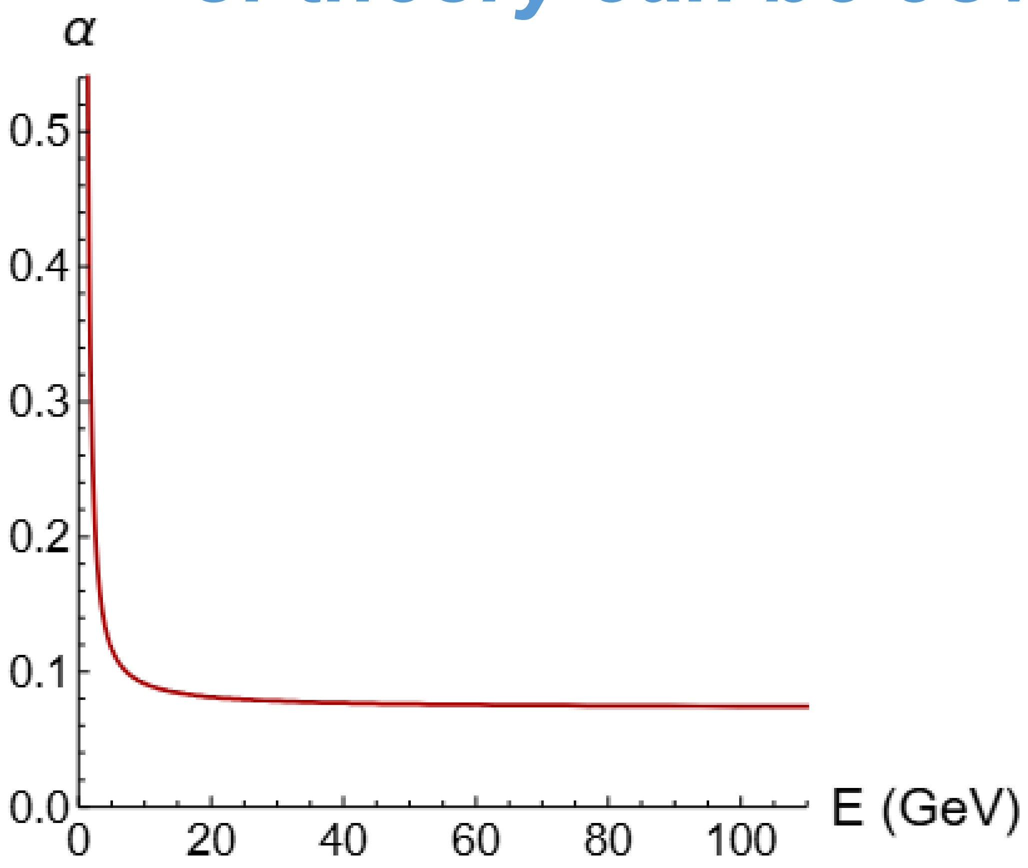


Strongly coupled regime of theory can be covered!

Running coupling vs E in QFT: (HQ Model)

Strongly coupled regime of theory can be covered!





Summary:

- Obtaining the physical-boundary condition for dilaton field
- Describing the running coupling behavior in phase diagram of LQ, HQ models.
- Phase structure of QCD is independent of the boundary conditions.
- Coupling senses the 1^{st} order phase transition with a jump.

Future plan:

1- Extending to anisotropic models

2- Considering the hybrid model

Thank you for your attention!

Complementary slides

String tension of Light quarks:

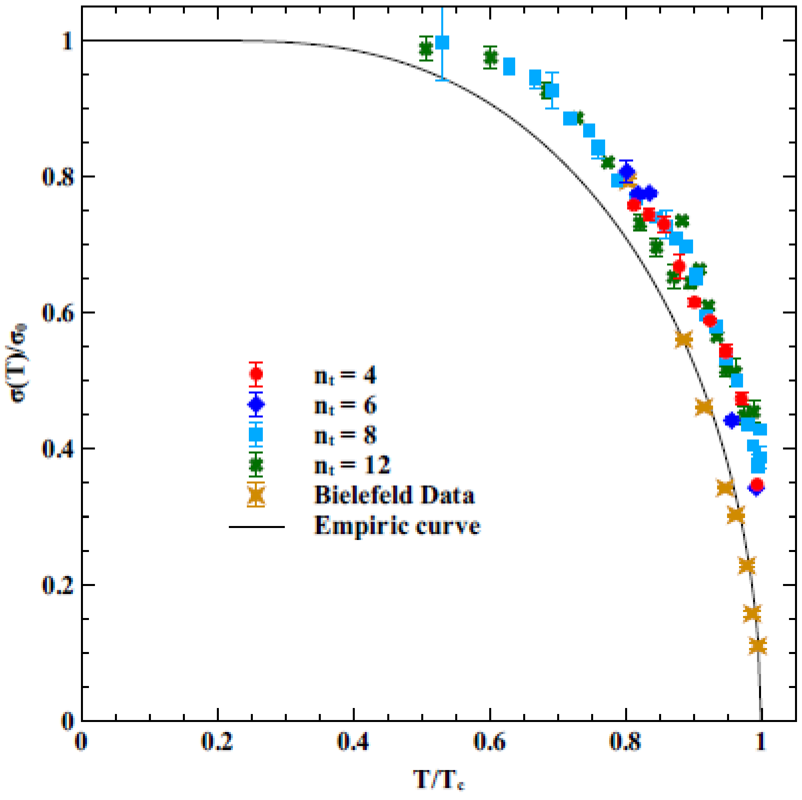


Figure 4: String tension as function of the temperature for $N_t = 4, 6, 8, 12$ combined with the results from the Bielefeld group. The black line corresponds to the ferromagnet magnetization $M/Msat$ critical curve, [1].

arXiv: 1111.1317

Light quarks:

4. Dependence of energy scale E on the holographic z coordinate

To obtain the dependence of the running coupling on the energy scale E or the square of the transferred momenta Q^2, it is necessary to relate the holographic coordinate z to the E or Q^2 [45]. The energy scale E in the boundary field theory can be identified with the warp factor in the metric (2.4) [45]. For the light quark model we have

$$E = B(z) = \frac{1}{z(1 + bz^2)^a}, \qquad (2.30)$$

Our Model: ***Einstein-Maxwell-dilaton action***

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{f_0(\phi)}{4}F_0^2 - \frac{f_1(\phi)}{4}F_1^2 - \frac{f_3(\phi)}{4}F_3^2 - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right]$$

$$\phi = \phi(z),$$

$$\text{Electric anzats } F_0: \quad A_0 = A_t(z), \quad A_{i=1,2,3,4} = 0,$$

$$\text{Magnetic anzats } F_k: \quad F_1 = q_1\, dx^2 \wedge dx^3, \quad F_3 = q_3\, dx^1 \wedge dx^2.$$

F_0 ⟷ Chemical potential

F_1 ⟷ Spatial anisotropy

F_3 ⟷ Magnetic field

Aref'eva, Rannu, Slepov, JHEP, 2021

Our ansatz for the metric:

$$ds^2 = \frac{L^2}{z^2}\mathfrak{b}(z)\left[-g(z)\,dt^2 + dx_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_2^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_3^2 + \frac{dz^2}{g(z)}\right]$$

$$\mathfrak{b}(z) = e^{2A(z)}$$

Warp factor

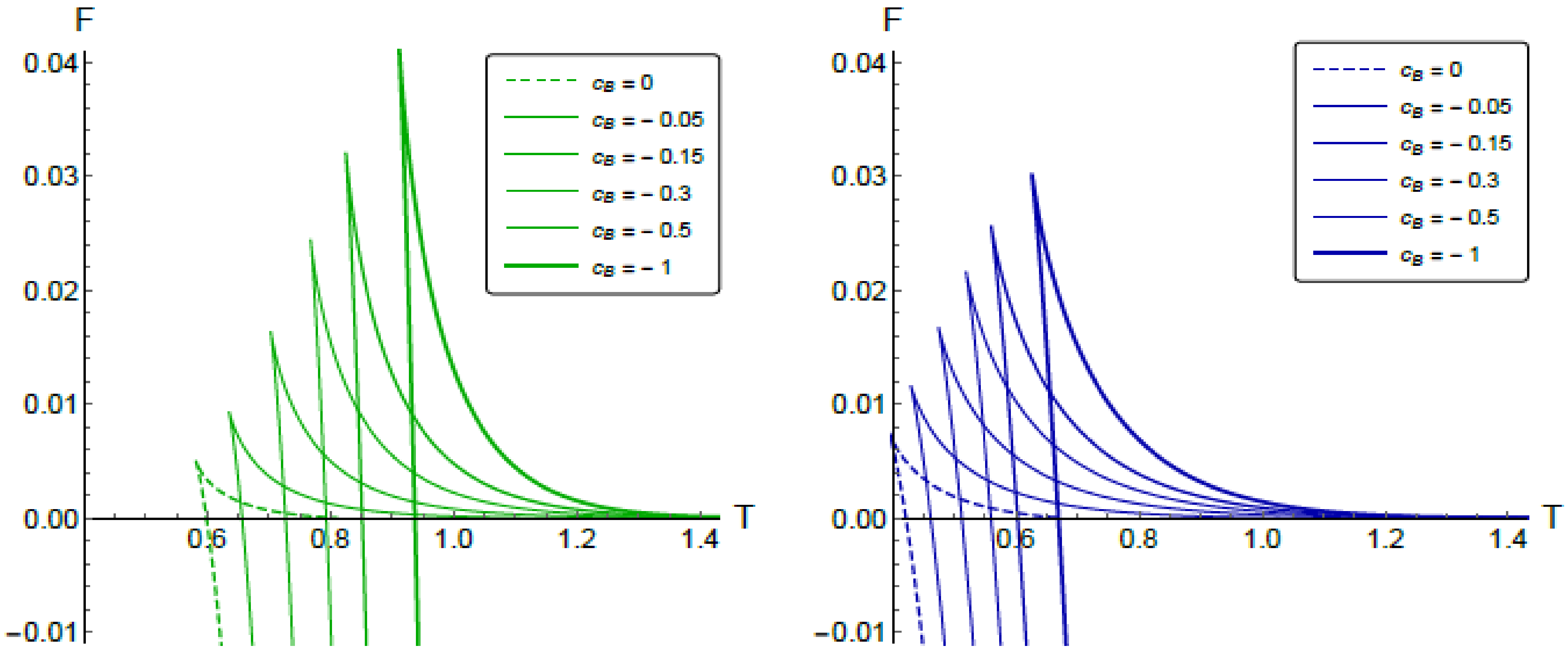
Isotropic $\nu = 1$

Anisotropic $\nu = 4.5$

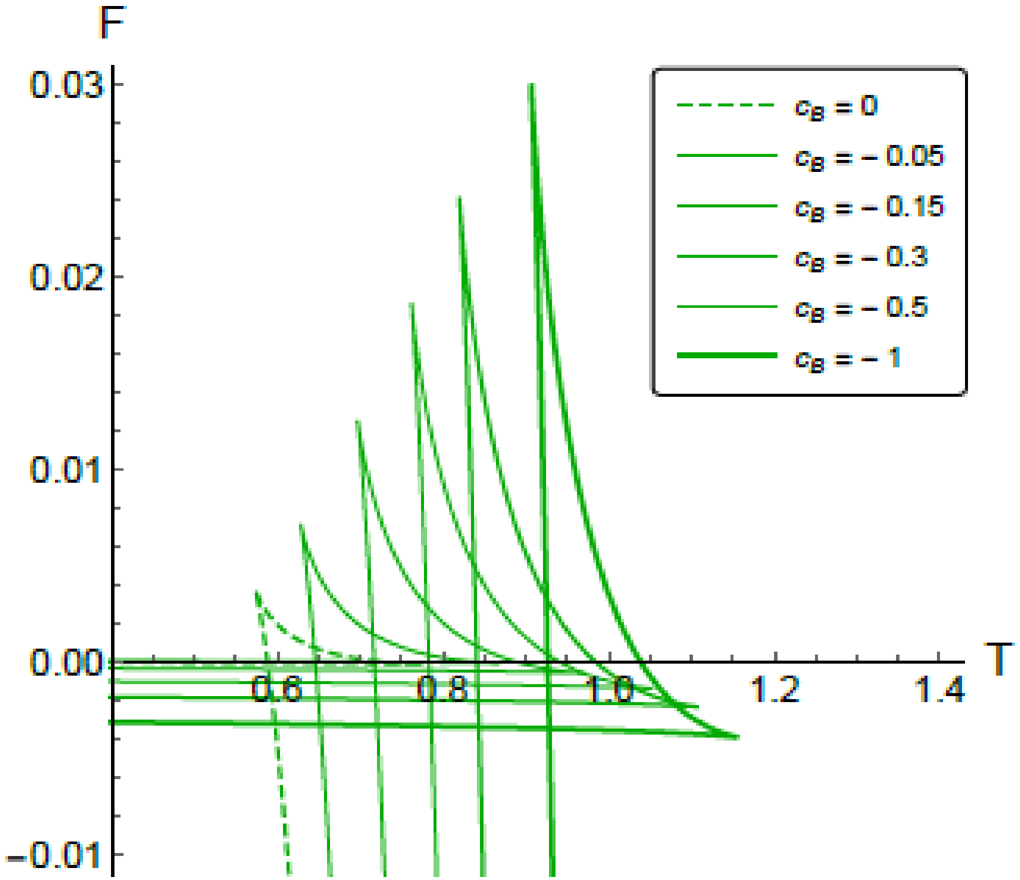
$$\mathcal{A}(z) = -cz^2/4 - (p - c_B q_3)z^4$$

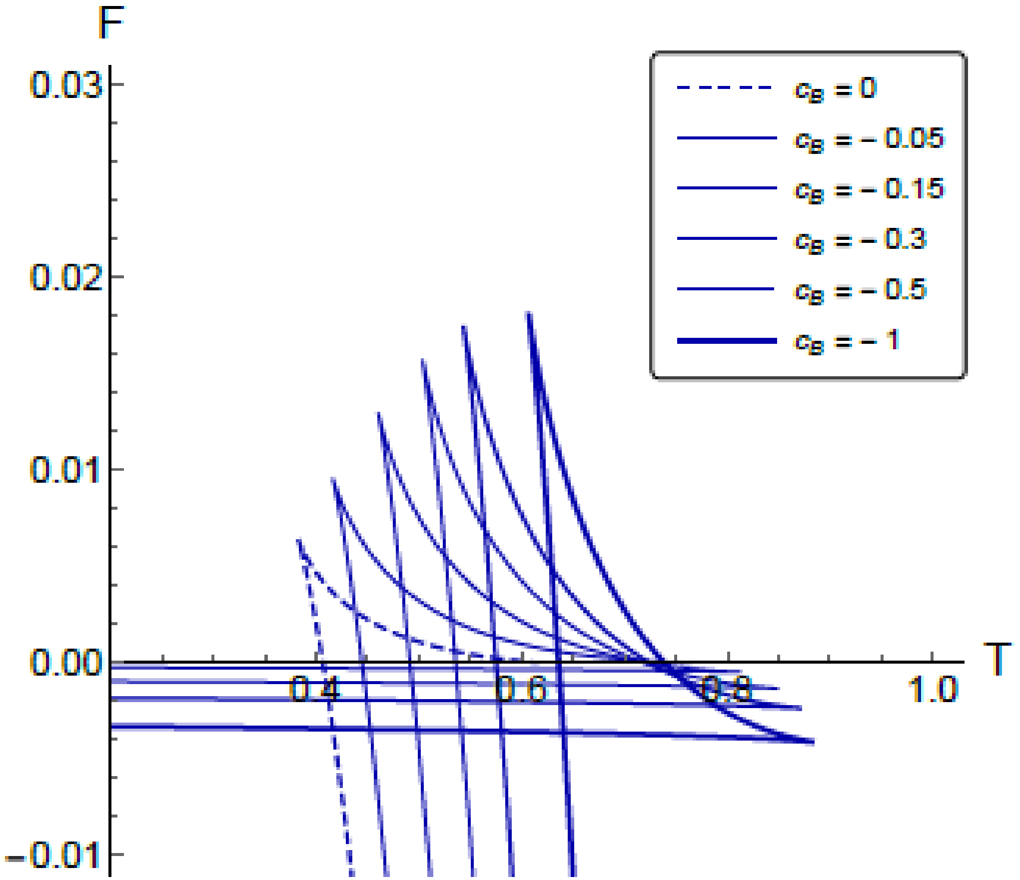
(Heavy Quarks)

Free energy: $\mu = 0$



Free energy: $\mu = 0.3$





Phase diagram of heavy quarks:

(considering spatial anisotropy)